Symmetric Tensor Networks and Topological Phases

Congratulations to KITS!

KITS, Mar. 2017

Two types of symmetric topological phases

SPT --- symmetry protected topological phases: (Pollmann, Berg, Turner, Oshikawa, Chen, Liu, Gu, Wen…)

Example: Haldane spin chain, integer quantum Hall states, topological insulators…

Features:

- no topological order
- anomalous edge states protected by symmetry

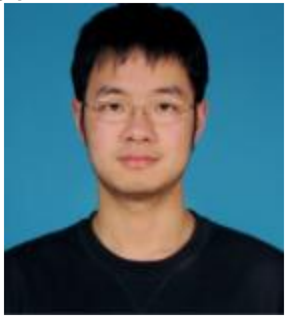
SET --- symmetry enriched topological phases: (Wen, Essin, Hermele, Mesaros,YR, Barkeshli….)

Example: toric code, gapped quantum spin liquids, fractional quantum Hall states…

Features:

- topological order (anyon excitations in 2d)
- symmetry can be fractionalized (e.g. e/3 quasiparticle in Laughlin's state).

Acknowledgement:



- **Collaborators:**
  - (Boston College) Shenghan Jiang Xu Yang
  - Panjin Kim, Hyungyong Lee, Jung Hoon Han (Sungkyunkwan University)
  - Brayden Ware, Chao-Ming Jian, Michael Zaletel (StationQ)
- **References:**
  - arXiv: 1505.03171, S. Jiang, Y. Ran
  - arXiv: 1509.04358, P. Kim, H. Lee, S. Jiang, B. Ware, C. Jian, M. Zaletel, J. Han, Y. Ran
  - arXiv: 1610.02024, S. Jiang, P. Kim, J. Han, Y. Ran
  - arXiv: 1611.07652, S. Jiang, Y. Ran

Motivations

- We focus on bosonic topological (SET or SPT) phases, which require strong interactions to realize.

(1) Conceptual issues:

-- Classification problems

(SPT phases with spatial symmetries.)

(2) “Practical” issues: How to realize them?

-- Physical intuitions/guiding principles?

(Are there criteria like the band-inversion picture in topological insulators?)

-- Numerical methods suitable for searching for these topological phases in models?

(How to write down generic variational wavefunctions?)

Main result

Based on tensor-network formulation, we develop a machinery to:

(1) systematically (but partially) classify topological phases

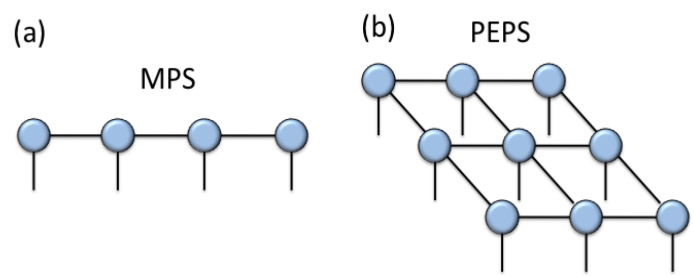
(2) construct generic variational wavefunctions for these phases

(onsite and spatial) symmetries of the system → (Partially) classification of topological phases and construction of generic wavefunctions for each class

- This machinery answers:

How many classes of symmetric tensor-network wavefunctions that cannot be smoothly deformed into each other under certain assumptions?

1D-MPS, 2D-PEPS, and 3D generalizations



figures from R. Orus, Annals Phys. (2014)

Main result

Based on tensor-network formulation, we develop a machinery to:

(1) systematically (but partially) classify topological phases

(2) construct generic variational wavefunctions for these phases

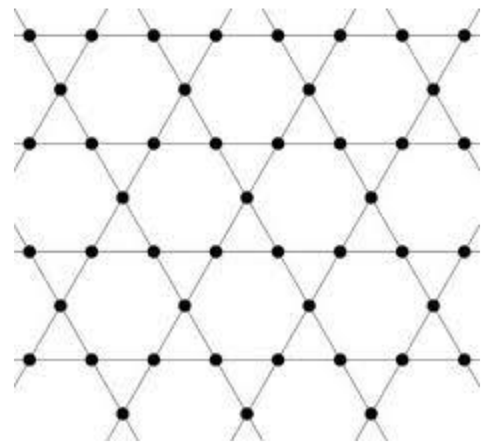
(onsite and spatial) symmetries of the system → (Partially) classification of topological phases and construction of generic wavefunctions for each class

Combining with tensor-based variational numerical algorithms → Numerically simulating Topological phases in practical models

Some applications of this machinery

(1) Classification and simulation of competing spin liquids in the spin-1/2 Heisenberg model on the kagome lattice.

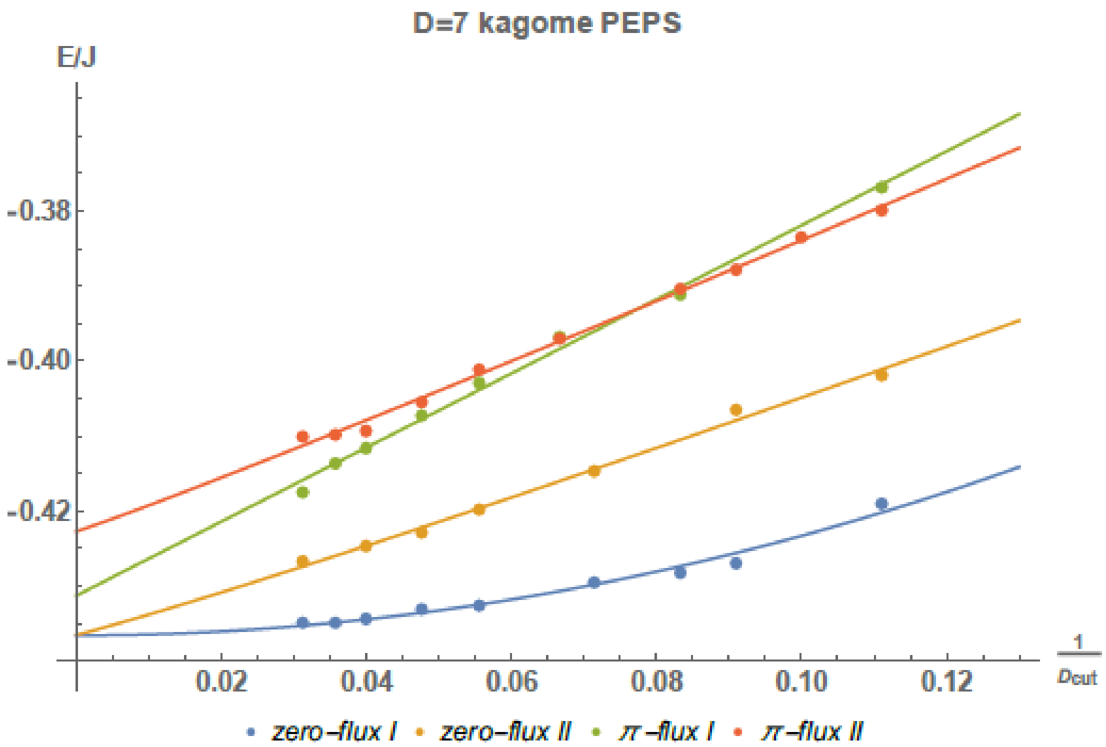
$$H = J \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j$$



Sachdev, Marston, Senthil, Singh, Evenbly, Vidal, Ran, Hermele, Wen, Lee, Wang, Vishwanath, Weng, Xiang, Sheng, Iqbal, Becca, Sorella, Poilblanc, White, Huse, Depenbrock, McCulloch, Schollwock, Jiang, Balents, Mei, He, Zaletel, Oshikawa, Pollmann… and many more

Some applications of this machinery

(1) Classification and simulation of competing spin liquids in the spin-1/2 Heisenberg model on the kagome lattice.



D_{cut} ~ virtual states kept when performing tensor contraction

Some applications of this machinery

(1) Classification and simulation of competing spin liquids in the spin-1/2 Heisenberg model on the kagome lattice.

(2) Classification of bosonic cohomological SPT: $H^{d+1}(SG, U(1))$

- SG: on-site and lattice symmetries (onsite (Chen, Liu, Gu, Wen…), lattice (Chen, Hermele, Fu, Qi, Furusaki, Cheng…))
- T and P(mirror) should be treated as “anti-unitary”
- Generic tensor wavefunctions for every class (if SG is discrete)

Some applications of this machinery

(1) Classification and simulation of competing spin liquids in the spin-1/2 Heisenberg model on the kagome lattice

(2) Classification of bosonic cohomological SPT: $H^{d+1}(SG, U(1))$

- SG: on-site and lattice symmetries (onsite (Chen, Liu, Gu, Wen…), lattice (Chen, Hermele, Fu, Qi, Furusaki, Cheng…))
- T and P(mirror) should be treated as "anti-unitary"
- Generic tensor wavefunctions for every class (if SG is discrete)

(3) A by-product: a general connection between "conventional" fractionalized phases and SPT phases in 2D via anyon condensation.

Plan

- Anyon condensation mechanism:

  “conventional” fractionalized phases → SPT phases

A by-product: anyon condensation

An example:

Z_2 gauge theory $\xrightarrow{\text{Condense gauge fluxes}}$ Trivial confined phase

This is the well-studied deconfinement-confinement phase transition.

I will show:
implementing symmetry into such a transition can lead to SPT phases

A by-product: anyon condensation

An example:

Z_2 gauge theory $\xrightarrow{\text{Condense gauge fluxes}}$ Trivial confined phase

Z_2 gauge theory with Ising symmetry *g* *(an SET phase)* $\xrightarrow{\text{Condense gauge fluxes}}$???

SET: symmetry-enriched topological phases

A by-product: anyon condensation

An example:

Z_2 gauge theory $\xrightarrow{\text{Condense gauge fluxes}}$ Trivial confined phase

Z_2 gauge theory with Ising symmetry *g* *(an SET phase)*

$\xrightarrow{\text{Condense gauge fluxes}}$???

- Consider a particular SET phase:

  (e: gauge charge, m: gauge flux)

$$[g(e)]^2 = -1, [g(m)]^2 = 1$$

Namely: Ising symmetry is fractionalized on the e-particle

This is a rather conventional fractionalized phase without gapless edge states

A by-product: anyon condensation

An example:

$\mathbb{Z}_2$ gauge theory $\xrightarrow{\text{Condense gauge fluxes}}$ Trivial confined phase

$\boxed{\text{Z}_2 \text{ gauge theory with Ising symmetry } g \text{ (an SET phase)}}$ $\xrightarrow{\text{Condense gauge fluxes}}$???

- Consider a particular SET phase:

  (e: gauge charge, m: gauge flux)

$$[g(e)]^2 = -1, [g(m)]^2 = 1$$

  - Condense m with $g(m) = 1$ → trivial Ising paramagnet
  - Condense m with $g(m) = -1$ → nontrivial Ising SPT

Schematic phase diagram

General criteria for anyon condensation

$$\text{an SET phase} \xrightarrow{\text{Condense gauge fluxes}} \text{an SPT phase}$$

- Gauge group: $Z_{N_1} \times Z_{N_2} \times \cdots$ & symmetry group: SG
- e-particles feature nontrivial symmetry fractionalization
- m-particles have trivial fractionalization, but can carry usual quantum numbers

$\Omega_{g_1} \cdot \Omega_{g_2} = \boldsymbol{\lambda}(\boldsymbol{g_1}, \boldsymbol{g_2}) \cdot \Omega_{g_1 g_2}$ Ω_g ~ symmetry defect, λ ~ certain m particle

- Condensing m-particles without breaking symmetry, which requires:

  1. Condensed m's carry 1D symmetry irrep: $\boldsymbol{\chi_m}(\boldsymbol{g})$
  2. $\chi_m(g) \cdot \chi_{m'}(g) = \chi_{mm'}(g)$

- After condensing those m's, we get an SPT phase

$$\omega(g_1, g_2, g_3) \equiv \chi_{\lambda(g_2,g_3)}(g_1), \qquad [\omega] \in H^3(SG, U(1))$$

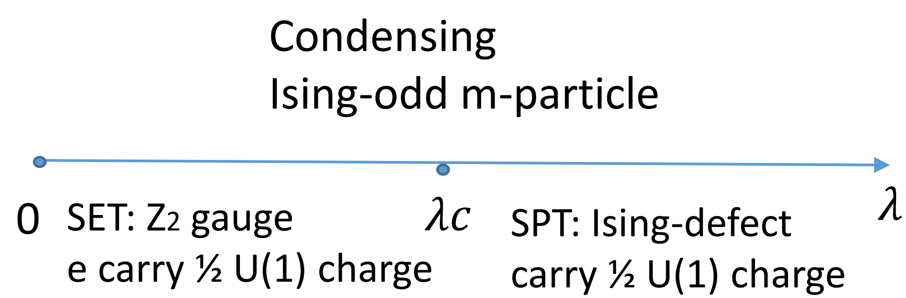
A somewhat simple model realizing SPT

- Following the anyon-condensation mechanism, we can design somewhat simple models realizing bosonic SPT phases. (need 3-spin interactions)

- The model looks like this:

Global symmetry: U(1) x Ising

$$H = H_{U(1)} + H_{Ising} + \lambda \cdot W$$



U(1)-Layer

W

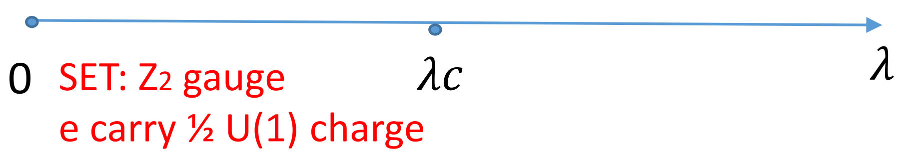
Ising-Layer

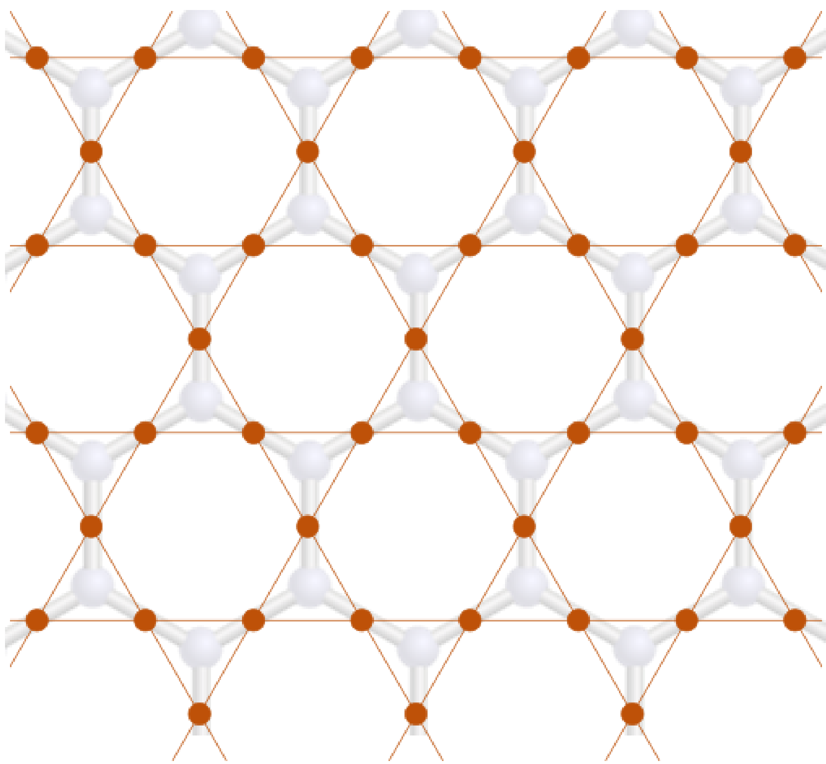
$$H_{Ising} = h \cdot \Sigma \sigma^x$$

A somewhat simple model realizing SPT

Global symmetry: U(1) x Ising

$$H = H_{U(1)} + H_{Ising} + \lambda \cdot W$$





U(1)-layer: Half-filled hard-core bosons on the kagome lattice

$$H_{U(1)} = -t\Sigma b_i^+ b_j + V_1 \Sigma n_i n_j + V_2 \Sigma n_i n_j + V_3 \Sigma n_i n_j$$

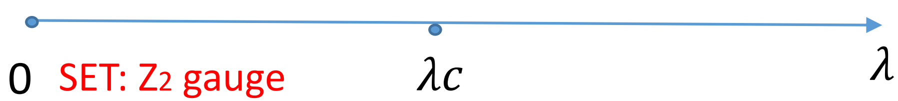
$$t \ll V_1 = V_2 = V_3 = V$$

In this regime, $H_{U(1)}$ is in a deconfined Z2 spin liquid phase:
e-particle carries ½ U(1)-charge.
(Balents,Fisher,Girvin 2001)

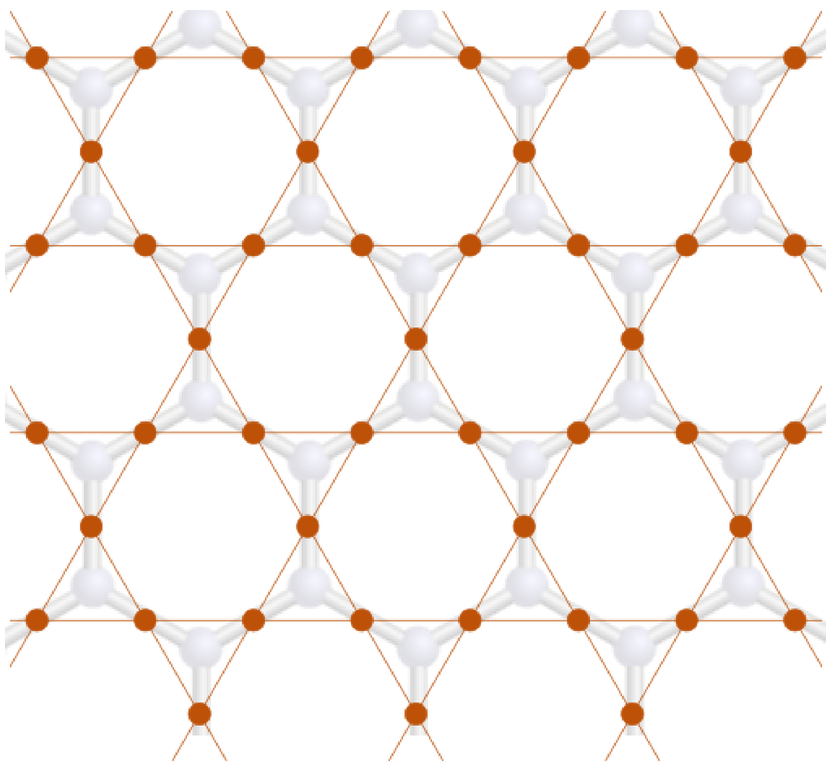
A somewhat simple model realizing SPT

Global symmetry: U(1) x Ising

$$H = H_{U(1)} + H_{Ising} + \lambda \cdot W$$



SET: Z_2 gauge
e carry ½ U(1) charge



U(1)-layer: Half-filled hard-core bosons on the kagome lattice

$$H_{U(1)} = -t\Sigma b_i^+ b_j + V_1 \Sigma n_i n_j + V_2 \Sigma n_i n_j + V_3 \Sigma n_i n_j$$

$$t \ll V_1 = V_2 = V_3 = V$$

Ising-layer: transverse field Ising spins on the honeycomb lattice

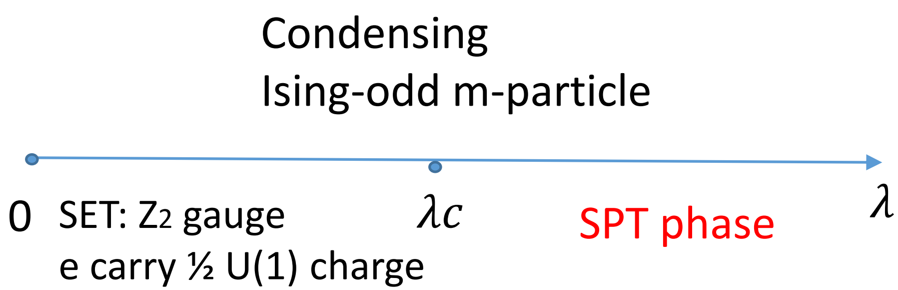
$$H_{Ising} = h \cdot \Sigma \sigma^x$$

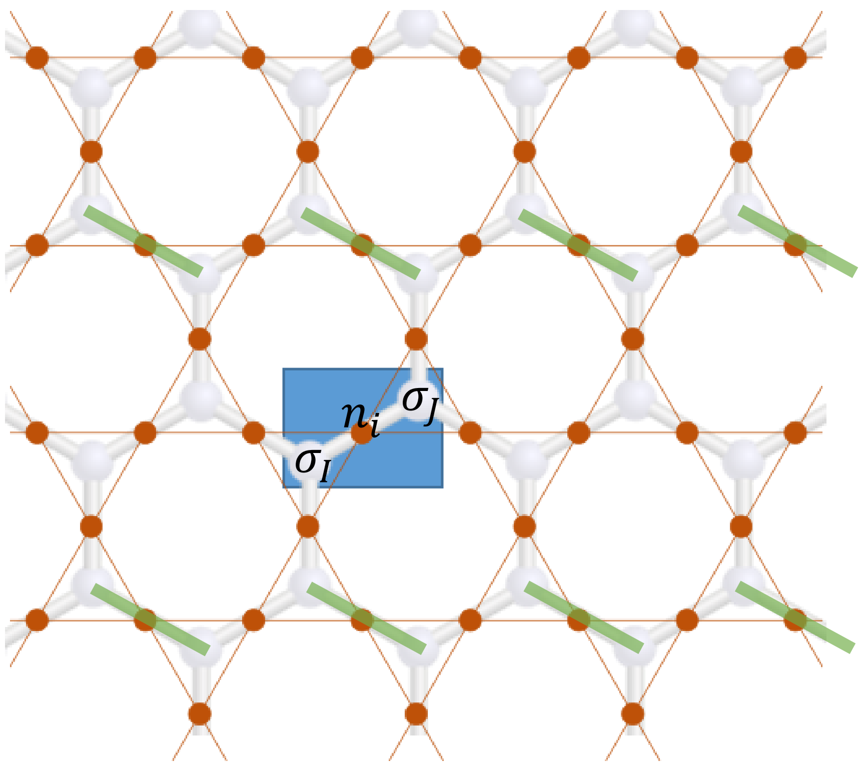
$$h \ll V$$

A somewhat simple model realizing SPT

Global symmetry: U(1) x Ising

$$H = H_{U(1)} + H_{Ising} + {\color{red}\lambda \cdot W}$$





U(1)-layer: Half-filled hard-core bosons on the kagome lattice

$$H_{U(1)} = -t\Sigma b_i^+ b_j + V_1 \Sigma n_i n_j + V_2 \Sigma n_i n_j + V_3 \Sigma n_i n_j$$

$$t \ll V_1 = V_2 = V_3 = V$$

Ising-layer: transverse field Ising spins on the honeycomb lattice

$$H_{Ising} = h \cdot \Sigma \sigma^x$$

$$h \ll V$$

${\color{red}\lambda \cdot W}$: 3-spin interaction coupling two layers

$$\lambda \cdot W = \lambda \cdot \sum (n_i - 1/2) \cdot (s_{IJ} \sigma_I^z \sigma_J^z)$$

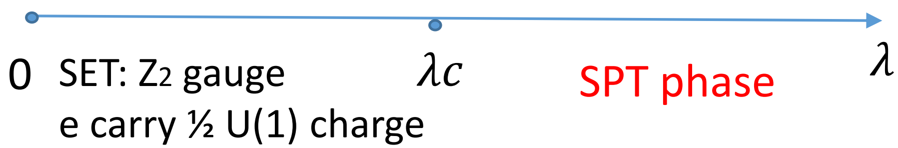
s_{IJ}=-1 on green bonds, s_{IJ}=+1 otherwise

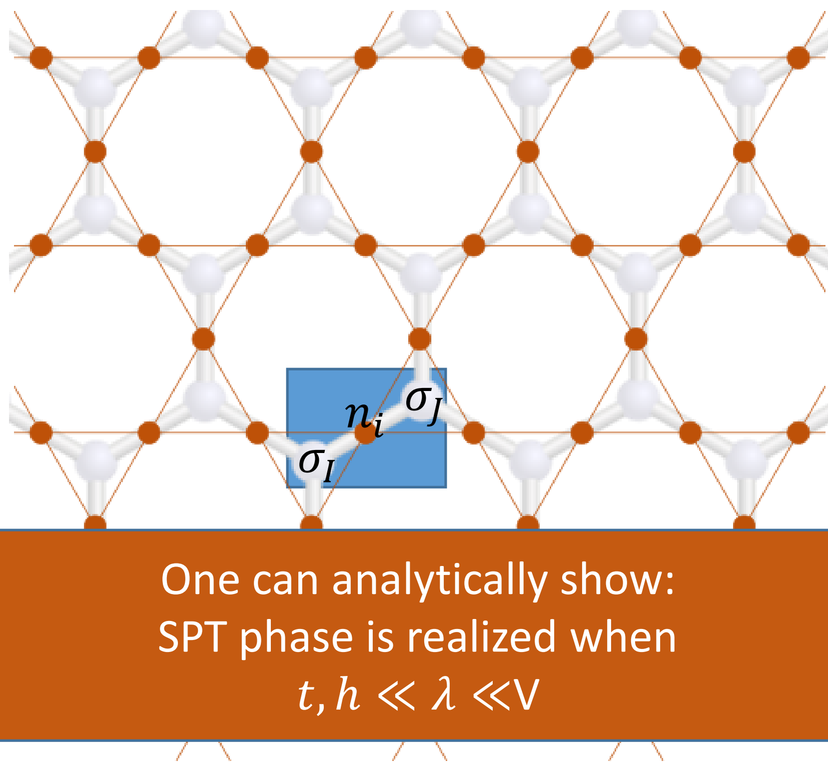
A somewhat simple model realizing SPT

Global symmetry: U(1) x Ising

$$H = H_{U(1)} + H_{Ising} + \lambda \cdot W$$

Condensing
Ising-odd m-particle





One can analytically show:
SPT phase is realized when
$t, h \ll \lambda \ll$V

U(1)-layer: Half-filled hard-core bosons on the kagome lattice

$$H_{U(1)} = -t\Sigma b_i^{+} b_j + V_1 \Sigma n_i n_j + V_2 \Sigma n_i n_j + V_3 \Sigma n_i n_j$$

$$t \ll V_1 = V_2 = V_3 = V$$

Ising-layer: transverse field Ising spins on the honeycomb lattice

$$H_{Ising} = h \cdot \Sigma \sigma^x$$

$$h \ll V$$

$\lambda \cdot W$: 3-spin interaction coupling two layers

$$\lambda \cdot W = \lambda \cdot \sum (n_i - 1/2) \cdot (s_{IJ} \sigma_I^z \sigma_J^z)$$

s_{IJ}=-1 on green bonds, s_{IJ}=+1 otherwise

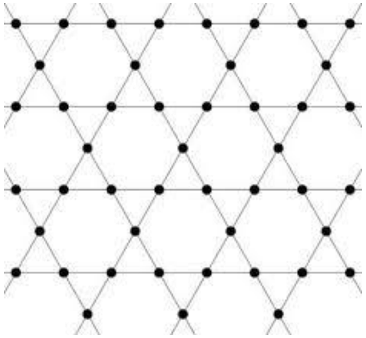
Summary

SPT: partial classification

$H^{d+1}[SG, U(1)]$

SG: on-site & spatial symmetries

T and P antiunitary



Numerical simulation

Input: symmetries of the model

Running Machinery

Output: SET/SPT classes and generic wavefunctions for every classes

Thank you!

Fractionalized phases $\xrightarrow{\text{anyon condensation}}$ SPT