Correspondence between 2D Fixed-Point Tensor Network and 3D Topological Quantum Field Theory

Yong-Shi Wu

Fudan University and University of Utah

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a collaboration with Zhu-Xi Luo and Ethan Lake

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THE UNIVERSITY OF UTAH
Department of Physics & Astronomy

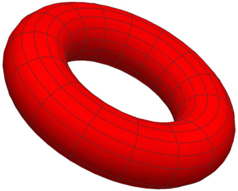
Introduction

Interdisciplinary Frontier in Theoretical Physics: Condensed Matter vs Fundamental Theory

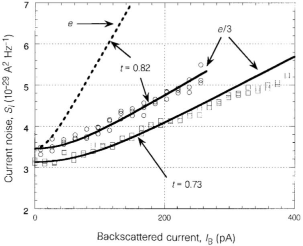
- Common Features:
  - Both deal with many-body systems.
  - Both involve emergent phenomena.
  - Both involve gauge theory.
  - Both involve quantum information.
- History:
  - Use of Quantum Field Theory
  - Use of Gauge Theory
  - Symmetry Breaking
  - Renormalization Group Flow
  - Anomalies, Dirac/Weyl Fermions, Majorana Modes, supersymmetry $\cdots$
- Some of Current Challenges in Fundamental Theory
  - Entanglement and Holography.
  - Emergent Topology/ Geometry/Gravity:

Motivation

Correspondence between 2D Fixed-Point Tensor Network and 3D Topological Quantum Field Theory

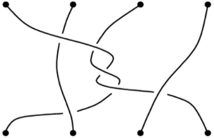


ground state degeneracy

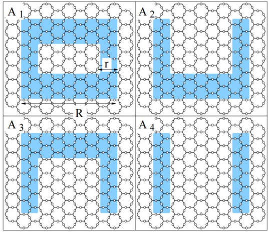


Credit: R. de-Picciotto, M. Reznikov, M. Heiblum, V. Umansky, G. Bunin and D. Mahalu (1997)

fractional quantum number



fractional/braiding statistics



Credit: M. Levin and X. G. Wen (2005)

long-range entanglement

Two-dimensional topological states of matter

Motivation

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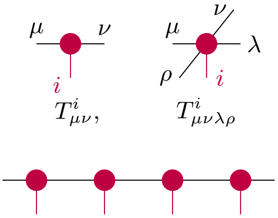
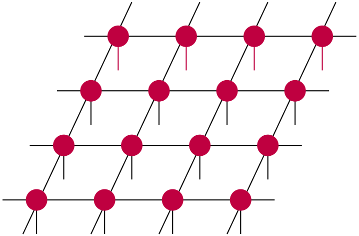


Matrix Product State



Projected Entangled Pair State

Tensor network states

- correctly gives the ground states for many topological systems, and
- encodes topological properties in a local way.

$$|\Psi\rangle = \sum_{i,\dots} \sum_{\mu,\nu,\lambda,\rho,\dots} T^i_{\mu\nu\lambda\rho} \cdots |i \dots\rangle.$$

A. Klumper, A. Schadschneider and J. Zittartz (1991), M. Fannes, B. Nachtergaele and R. F. Werner (1992)

A. Klumper, A. Schadschneider and J. Zittartz (1993), F. Verstraete and J. I. Cirac (2004)

Motivation

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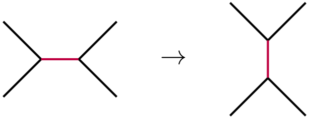
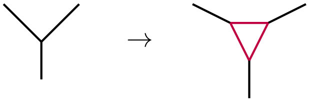
Fixed-point tensor network captures the long-range physics.

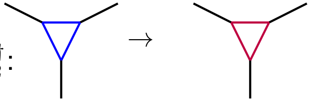
P_1:



P_2:



A special combination: $P_2 P_2^\dagger$:



The set of data that determines such a tensor network state is called a unitary fusion category $\mathcal{C}$. X. Chen, Z. C. Gu, X. G. Wen (2010); M. Levin, and X. G. Wen (2005)

Motivation

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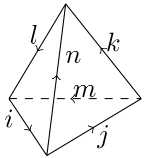
- TQFT is the low-energy effective theory for topological phases.
- State sum TQFTs mimic the lattice structure and manifests locality.
- 3D state sum TQFT is described by the same mathematical object: unitary fusion category $\mathcal{C}$.

weight: G^{ijm}_{kln}



Construction of 3D state sum TQFT

Triangulation of the manifold $\rightarrow$ consistent coloring $\rightarrow$ assigning weights $\rightarrow$ constructing the invariant $\rightarrow$ topological invariance.

V. G. Turaev and O. Y. Viro (1992); J. Barrett and B. Westbury (1996)

$$\tau_{\mathcal{C}}(\Sigma) = \sum_{labellings} \prod_{vertices} \frac{1}{D} \prod_{tetrahedron} G \prod_{edges} d$$

Preview

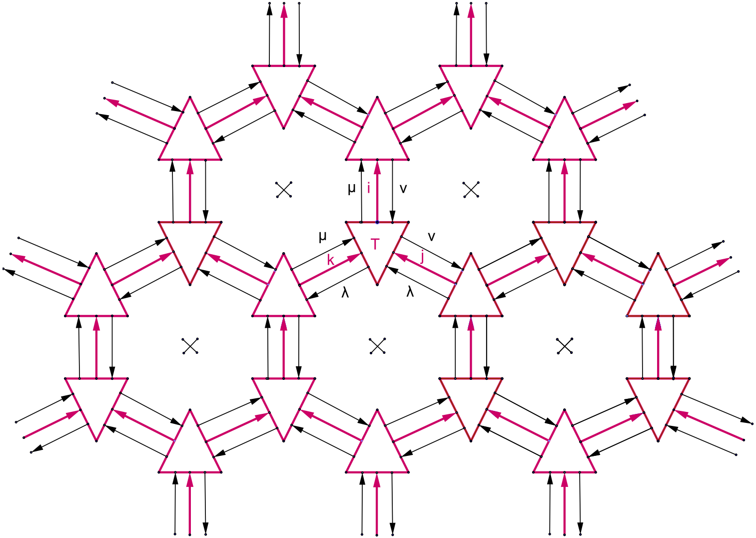
Correspondence between 2D Fixed-Point Tensor Network and 3D Topological Quantum Field Theory

- The 1D fixed-point tensor network (MPS) $\leftrightarrow$ 2D TQFT correspondence has recently been discussed. A. Kapustin, A. Turzillo and M. Y.You (2016); K. Shiozaki and S. Ryu (2016)
- The same mathematical object $\mathcal{C}$ are used to specify the data of 2D fixed-point TN and 3D TQFT.
- But the role of the additional dimension has not been made clear.

Main Result:

$$\text{2D fixed-point TN state} \quad \underset{\text{bulk-to-boundary map}}{\overset{\text{generation of pre-geometry}}{\rightleftharpoons}} \quad \text{3D TQFT}$$

2D Fixed-Point Tensor Network State



- 2D fixed-point TN are most conveniently described using a triple-line structure.

  Z. C. Gu, M. Levin, B. Swingle and X. G. Wen (2009);

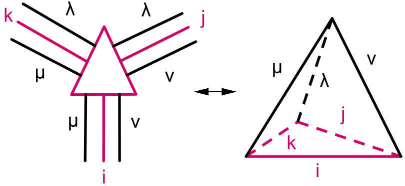
  O. Buerschaper, M. Aguado and G. Vidal (2009)

- $\{i, j, k, \cdots\}$ are physical; $\{\mu, \nu, \lambda\}$ auxiliary.
- The tensor on the central vertex is $T^{ijk}_{\mu\nu\lambda} \propto G^{ijk}_{\mu\nu\lambda}$.

$$|\Psi\rangle = \sum_{ijk\ldots} \sum_{\mu\nu\lambda\ldots} T^{ijk}_{\mu\nu\lambda} \cdots |ijk\ldots\rangle.$$

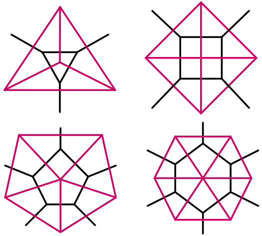
The Correspondence

Every vertex in the tensor network can be mapped to a tetrahedron, where



- the physical degrees of freedom i, j, k compose the bottom triangle;
- the auxiliary plaquette degrees of freedom μ, ν, λ stretch into the third dimension;
- The edges in every triangle satisfy the branching rules, e.g., $\mu \otimes \nu \otimes i \to \mathbf{1} + \cdots$.

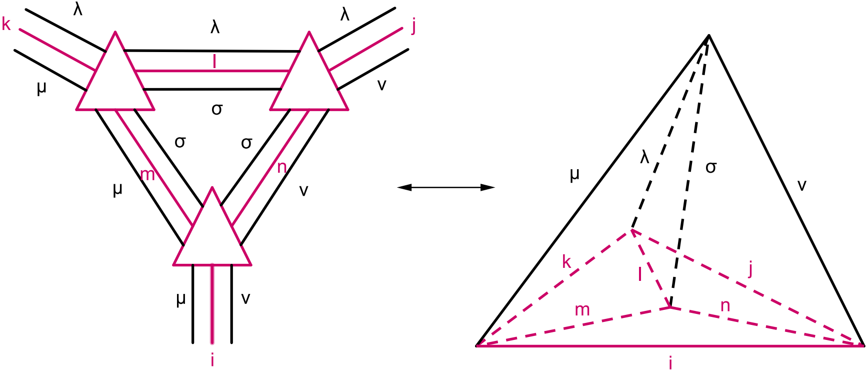
Generally, the tensor network can be in any lattice: triangular, square, honeycomb, etc..



For a plaquette surrounded by n links, it will map to n tetrahedra that are pasted together.

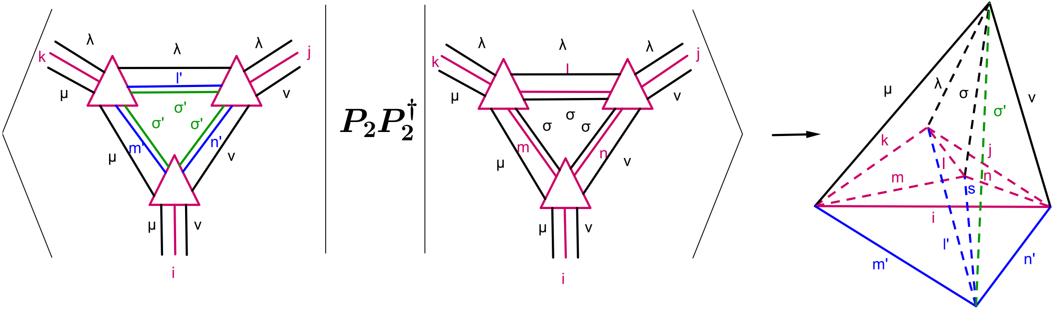
The Correspondence

For simplicity, consider a triangular plaquette labeled by σ. It is formed by three vertices and is mapped to three tetrahedra sharing a common edge σ.



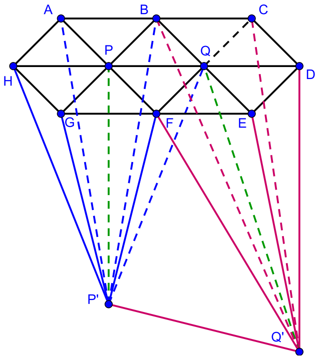
3D Pre-geometry I

Composition of the elementary moves $P_2 P_2^\dagger$ would change the inner plaquette degree of freedom and the three links surrounding that plaquette.



This operator drags l, m, n (red) down to l', m', n' (blue), generating three other tetrahedra. This is consistent with the fact that in TQFT, the cobordism between two boundaries is viewed as transition amplitude.

3D Pre-geometry II



$$\left(P_2(R)P_2(R)^\dagger\right)\circ\left(P_2(L)P_2(L)^\dagger\right)|\Psi\rangle.$$

Since the tensor network state is invariant under the action of $P_2P_2^\dagger$, one can act it on arbitrary plaquettes for arbitrary times, which leads to a three-dimensional pre-geometry or a colored triangulation.

Conversely, one can also push three-dimensional bulk back to the top layer of tetrahedra and take the duality back to the triple-line tensor network.

We have thus established the correspondence between 2D FP TN and 3D TQFT, with emphasis on the (pre-)geometric perspective.

Similar analyses have been carried out for the tensor network states that describe the symmetry enriched topological phases, making use of homotopy quantum field theory. V. G. Turaev (2000)

Outlook

- One can add a metric to the 3D pre-geometry either following the loop quantum gravity strategy C. Rovelli and L. Smolin, (1995); J. W. Barrett and L. Crane (1998) or using entanglement measures in the 2D tensor network. S. Ryu, T. Takayanagi (2006)
- One can go beyond the ground-state subspace and generate a nontrivial pre-geometry Z.-X. Luo, Y.-S. Wu, in preparation. It turns out that the long-range entanglement in the tensor network states plays a central role.

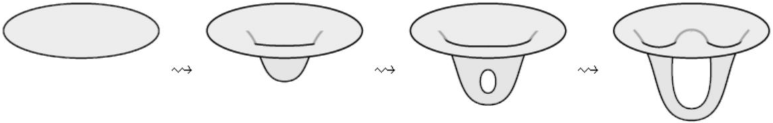
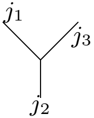
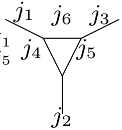


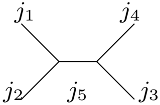
Figure credit: J. Baez (2004).

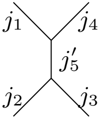
Thank You!

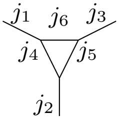
Appendix

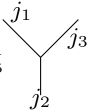
$P_1 :$ 

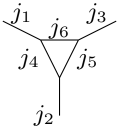
$\mapsto \sum_{j_4,j_5,j_6} \frac{v_{j_4} v_{j_5} v_{j_6}}{\sqrt{D}} G^{j_2 j_3 j_1}_{j_6 j_4 j_5}$ 

$P_3 :$ 

$\mapsto \sum_{j_5'} \frac{v_{j_5} v_{j_5'}}{\sqrt{D}} G^{j_1 j_2 j_5}_{j_3 j_4 j_5'}$ 

$P_2 :$ 

$\mapsto \frac{d_\mu}{\sqrt{D}} \frac{v_{j_4} v_{j_5} v_{j_6}}{\sqrt{D}} G^{j_2 j_3 j_1}_{j_6 j_4 j_5}$ 

$P_2 P_1 :$ 

$\mapsto$ prefactor $\times$