Bosonic Topological Insulator and Self-dual QCP: Theory, numerics, and Experimental Platform

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Bosonic Topological Insulator and Self-dual QCP: Theory, numerics, and Experimental Platform

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Wang, Nahum, Metlitski, Senthil

Topological Insulator

Topological Insulator:

A novel class of quantum (ground) states that have the following properties:

(1) a gapped (incompressible) d-dimensional bulk, but gapless (compressible) (d-1)-dimensional boundary:

(2) its gapless (compressible) boundary states cannot be realized as a (d-1) dimensional system itself.

Topological Insulator

Free fermion TI: Full classification: S. Ryu, et.al. 2009, A. Kitaev 2009;

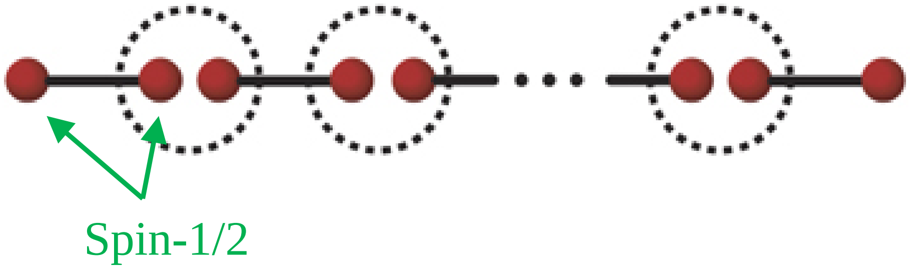
Phase transition between free fermion TI and trivial state, or between different free fermion TIs:
Gapless Dirac/Majorana fermion in the d-dimensional bulk.
The critical point between the 2d Chern insulator with different Chern numbers: massless 2d Dirac fermion.
The critical point between the 3d TI and trivial insulator is one single 3d Dirac fermion.

In this talk we will see that with interaction the situation can be very different.

Bosonic Symmetry Protected Topological States

Symmetry Protected Topological States: Generalization of TI and TSC, i.e. the bulk is gapped and nondegenerate, with gapless boundary.

Bosonic SPT states (Bosonic TI):
simplest example; 1d Haldane phase:



Each boundary has two fold spin-1/2 degeneracy.

Higher dimensional bosonic SPT states, much more complicated, can be classified mathematically: Chen, Gu, Liu, Wen 2011

Goal: To look for generic models and realistic systems that realize/mimics the physics of bosonic SPT states in 2d.

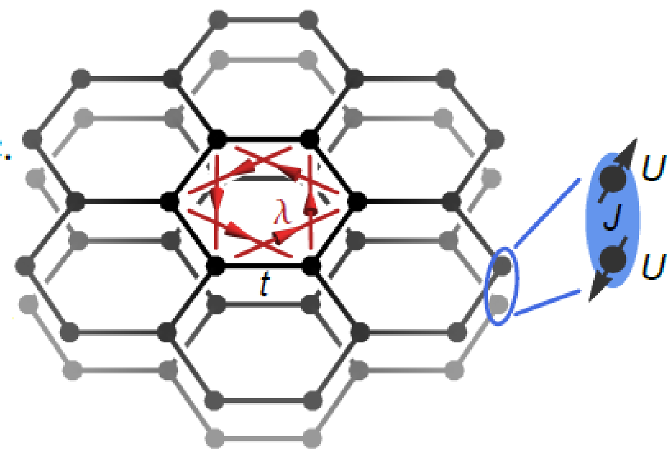
Sign problem free lattice model

We want to design a lattice model with the key physics of bosonic SPT state, and "easy" to study numerically:

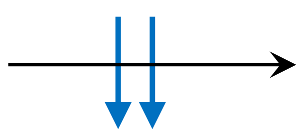
$$H = H_{\rm band} + H_{\rm int},$$

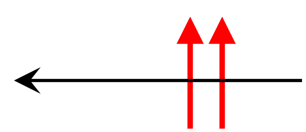
$$H_{\rm band} = -t \sum_{\langle ij \rangle, \ell} c^\dagger_{i\ell} c_{j\ell} + \sum_{\langle\langle ij \rangle\rangle, \ell} \mathrm{i}\lambda_{ij} c^\dagger_{i\ell} \sigma^z c_{j\ell} + H.c.$$

$$H_{\rm int} = \ + J \sum_i \Big[\boldsymbol{S}_{i1} \cdot \boldsymbol{S}_{i2} + \frac{1}{4}(n_{i1}-1)(n_{i2}-1)$$



Simple limits of this model:
(1) Noninteracting: bilayer quantum spin Hall, boundary has two channels of gapless fermion modes





Sign problem free lattice model

We want to design a similar lattice model with all the key physics, and "easy" to study numerically:

$$H = H_{\mathrm{band}} + H_{\mathrm{int}},$$

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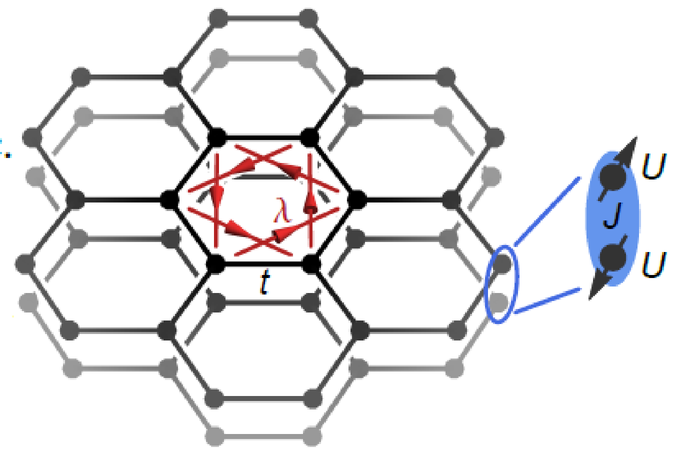


Simple limits of this model:
(2) Strong *J*-interacting limit: trivial Mott insulator, with inter-layer spin singlet one every site. $|\Psi\rangle = \prod |\mathrm{Singlet}\rangle$

What happens at intermediate *J* ?

Sign problem free lattice model

Apparently, this model has at least $U(1)_{spin}$ x $U(1)_{charge}$ symmetry. At relatively weak interaction J, we can directly bosonize the edge states:

$$H_0 = \int dx \sum_{l=1}^{2} \psi_{l,L} i v \partial_x \psi_{l,L} - \psi_{l,R} i v \partial_x \psi_{l,R}$$

$$\Downarrow$$

$$H_0 = \int dx \sum_{l=1}^{2} \frac{v}{2K} (\partial_x \theta_l)^2 + \frac{vK}{2} (\partial_x \phi_l)^2$$

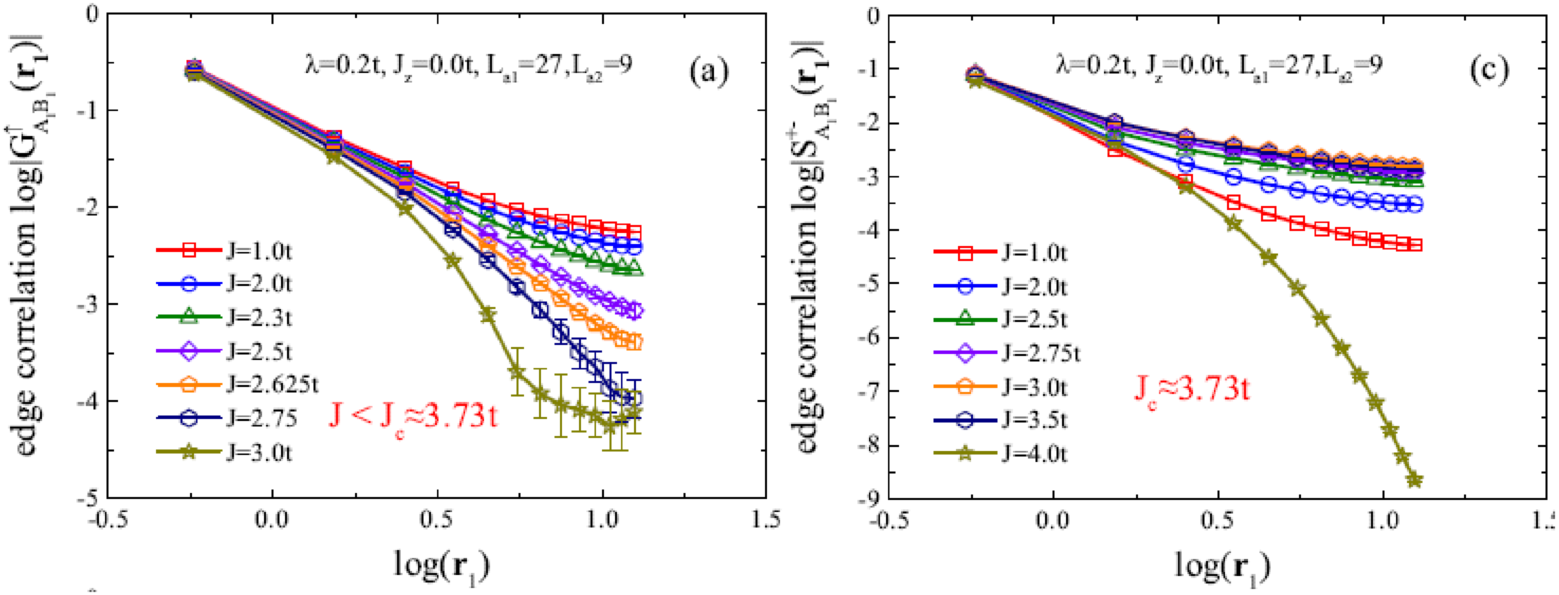
$\Downarrow$ interaction $H_v \sim \alpha \cos(2\pi \phi_1 - 2\pi \phi_2)$

$$\tilde{H} = \int dx \, \frac{\tilde{v}}{2\tilde{K}} (\partial_x \theta)^2 + \frac{\tilde{v}\tilde{K}}{2} (\partial_x \phi)^2$$

When H_v is relevant, all the fermion modes are gapped at the boundary, but bosonic modes are gapless, and protected by symmetry.

Sign problem free lattice model

Determinant QMC data for edge states:

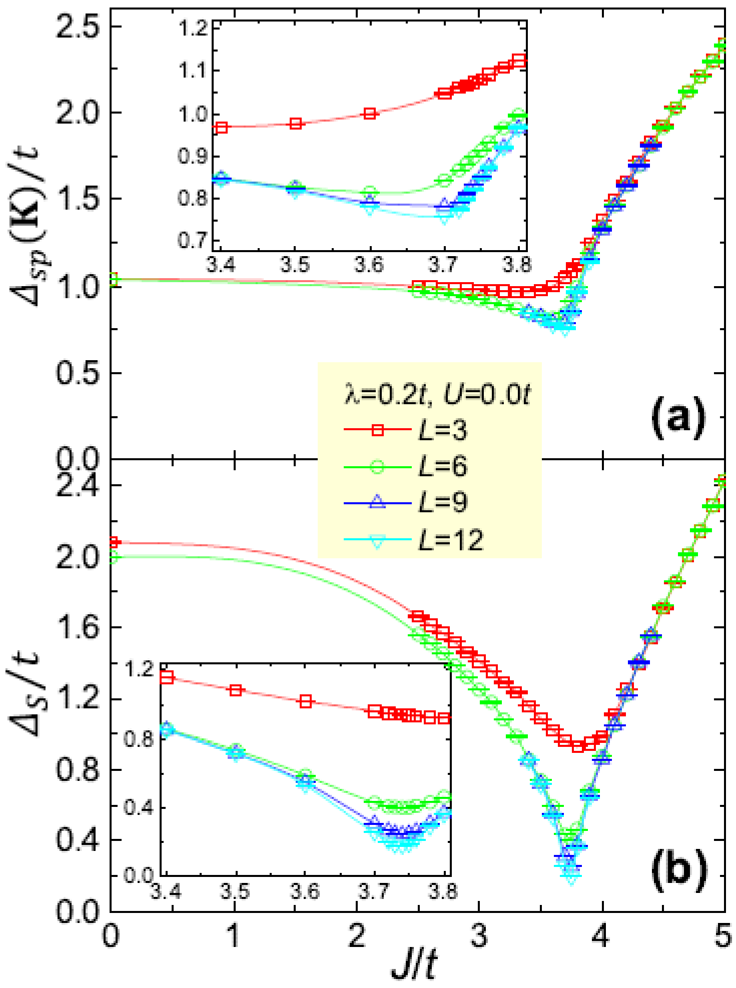


When fermion Greens function already decays exponentially at the boundary, Bosonic modes still have quasi-long range order, until the system hits the bulk quantum phase transition into the trivial insulator.

Sign problem free lattice model

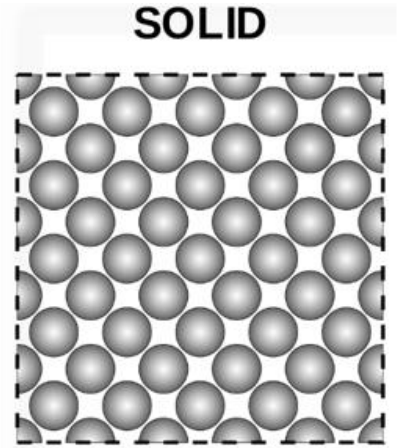
Determinant QMC data for bulk: (arXiv:1508.06389) we saw that the fermion gap is always finite, but bosonic modes, both spin and charge, becomes gapless at the SPT-trivial Quantum critical point. This is fundamentally different from free fermion topological transition.

Because the fermionic degrees of freedom never show up at low energy at either the boundary or the bulk quantum transition, the whole system can be viewed as a bosonic system.

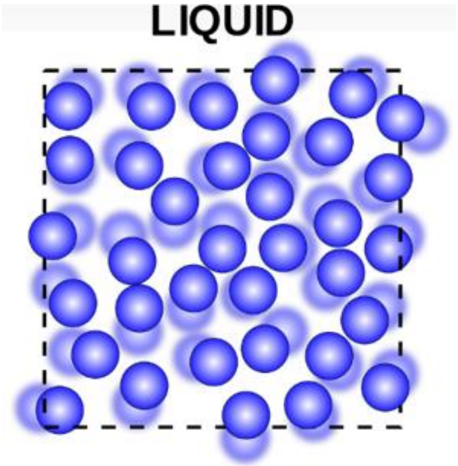


Bulk bosonic SPT-trivial phase transition

The bulk quantum phase transition between SPT and trivial state, is of great interest, because it is different from the usual “order-disorder” phase transition, such as the solid-liquid transition:



order



disorder

In our system, both phases around the transition are equally disordered, they are only distinct in topology.

Bulk bosonic SPT-trivial phase transition

Here is a proposed field theory for the bulk bosonic SPT-trivial phase transition (Grover, Vishwanath 2012, Lu, Lee 2012):

$$\mathcal{L}_{\mathrm{QED}} = \bar{\psi}\gamma \cdot (\partial - \mathrm{i}a - \mathrm{i}A_{\mathrm{A}}\sigma^3)\psi + \frac{\mathrm{i}}{2\pi} a \wedge \mathrm{d}A_{\mathrm{B}} + \mathrm{CS}_1[A_{\mathrm{A}}] + \mathrm{CS}_{-1}[A_{\mathrm{B}}]$$

$$+ \, m\bar{\psi}\psi + M\bar{\psi}\sigma^3\psi$$

1. *m* is the tuning parameter of the trivial-SPT transition (*M*=0). The CS terms of the back ground gauge fields change from 2 and -2 to 0 by changing sign of *m*.

2. This theory (in the infrared) has the desired SO(4) ~ SU(2) x SU(2) symmetry, because it is self-dual (Xu, You, 2015, Karch, Tong, 2016, Hsin, Seiberg, 2016)

Bulk bosonic SPT-trivial phase transition

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$$+\ m\bar{\psi}\psi + M\bar{\psi}\sigma^3\psi$$

3. Numerics suggest that this theory at m=M=0 is indeed a CFT: (Karthik, Narayanan 2016)

4. If all background gauge fields are ignored, the point m=M=0 (if it is a CFT) has an O(4) symmetry.
Prediction: in 2+1d there is an O(4) invariant CFT, if O(4) is preserved there is no relevant perturbation; breaking O(4) down to SO(4) allows one relevant perturbation m.

More duality of N=2 QED3

Dual to easy plane NCCP[1] (N=2 bosonic QED): m=0, M is the tuning parameter (Potter et.al. 1609.08618, Wang, et.al. 1703.02426)

$$\mathcal{L} = \sum_j |(\partial_\mu - ib_\mu)z_j|^2 + r|z_j|^2 + g|z_j|^4$$

$r < 0$ SF order (AF) SF order (VBS) $r > 0$

$M < 0$ SF order 1 SF order 2 $M > 0$

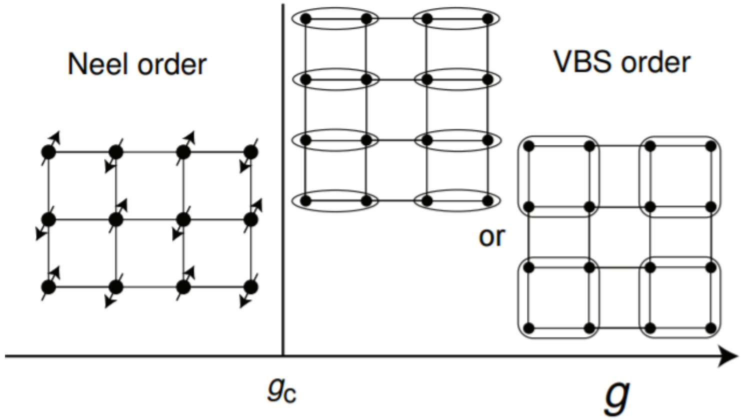
$$\mathcal{L}_{\mathrm{QED}} = \bar{\psi}\gamma \cdot (\partial - \mathrm{i}a - \mathrm{i}A_{\mathrm{A}}\sigma^3)\psi + \frac{\mathrm{i}}{2\pi} a \wedge \mathrm{d}A_{\mathrm{B}} + M\bar{\psi}\sigma^3\psi$$

The easy plan NCCP[1] model is the theory for the deconfined quantum phase transition between easy plane Neel order and VBS order.

More duality of N=2 QED3

Some of (many) concrete predictions:

$$\mathcal{L} = \sum_{j=1}^{2} \bar{\psi}_j \gamma_\mu (\partial_\mu - i a_\mu) \psi_j$$



$$\eta[\sum_{j=1}^{2} \bar{\psi}_j \psi_j] = \eta[S^z]$$

$$\Delta[\bar{\psi} \sigma^z \psi] = 3 - \frac{1}{\nu}$$

Wang, Nahum, Metlitski, Xu, Senthil, 1703.02426

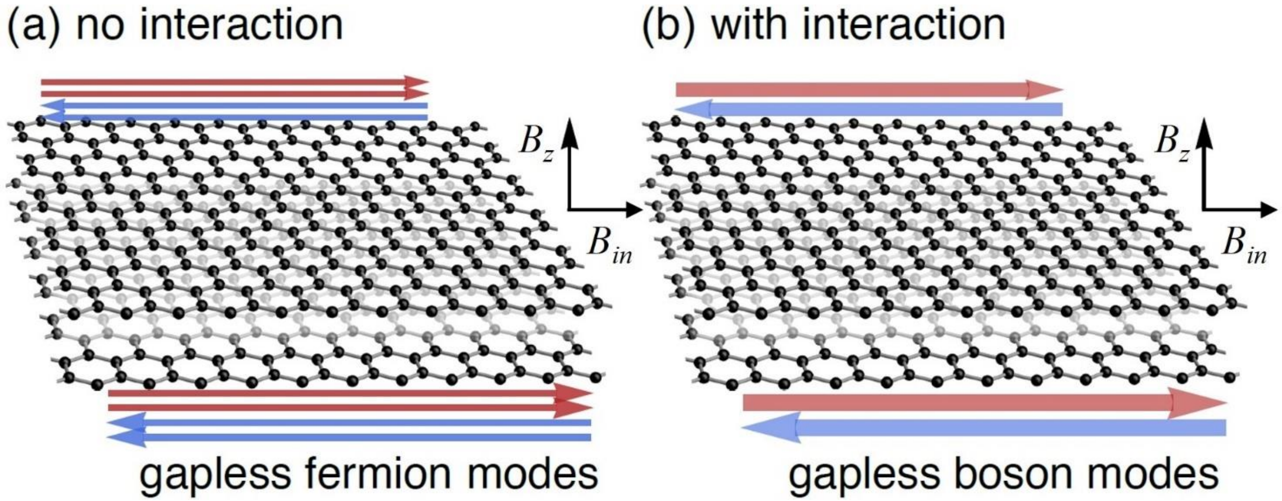
Realize bosonic SPT states in bilayer Graphene

Claim:

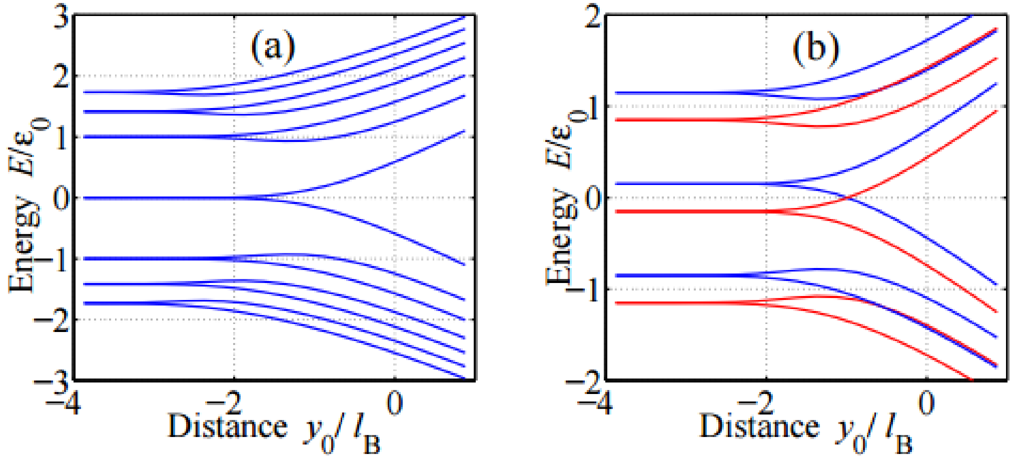
Bilayer graphene under (strong) magnetic field (with both *z* and inplane components), will be driven into a "bosonic" SPT state with U(1)xU(1) symmetry by Coulomb interaction.

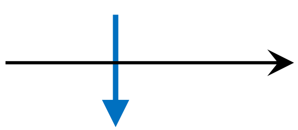
Meaning:

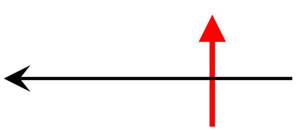
boundary states must remain gapless with U(1)xU(1) symmetry, but, only protected gapless bosonic modes, no gapless fermion modes, under Coulomb interaction.



Boundary of mono layer graphene under (strong) magnetic field, without interaction at boundary: single channel of spin-filtered counter-propagating electrons: (Abanin, Lee, Levitov, 2006)

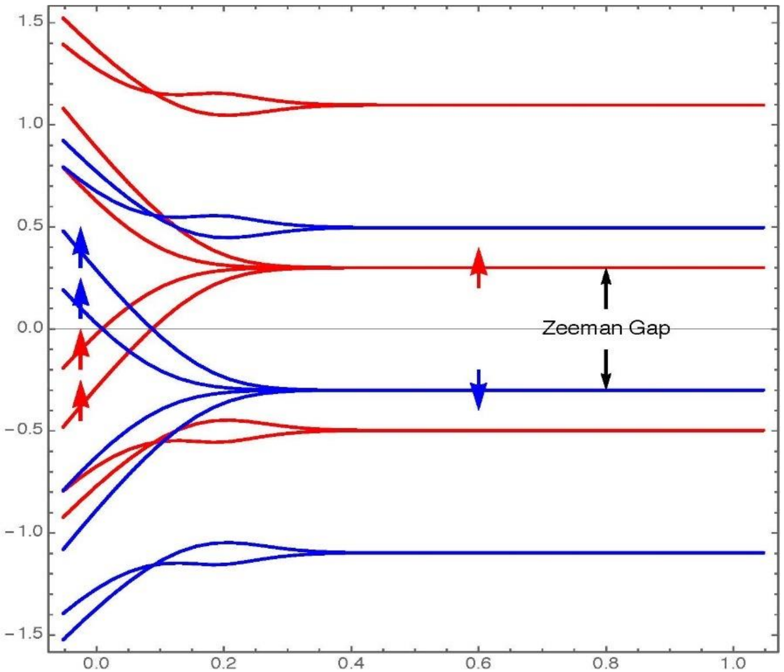






Boundary analysis

Boundary of bilayer graphene under magnetic field, without interaction: two channels of counter-propagating spin-filtered electron states.



Boundary analysis

Boundary of bilayer graphene under magnetic field, without interaction: two channels of counter-propagating spin-filtered electron states.

Direct bosonization: Coulomb interaction gaps out all the fermion modes, and drive the boundary into a c=1 CFT, with only one gapless boson mode (and its dual mode):

$$H_0 = \int dx \sum_{l=1}^{2} \psi_{l,L} i v \partial_x \psi_{l,L} - \psi_{l,R} i v \partial_x \psi_{l,R}$$

$$\Downarrow$$

$$H_0 = \int dx \sum_{l=1}^{2} \frac{v}{2K} (\partial_x \theta_l)^2 + \frac{vK}{2} (\partial_x \phi_l)^2$$

$\Downarrow$ Coulomb $H_v \sim \alpha \cos(2\pi \phi_1 - 2\pi \phi_2)$

$$\tilde{H} = \int dx \frac{\tilde{v}}{2\tilde{K}} (\partial_x \theta)^2 + \frac{\tilde{v}\tilde{K}}{2} (\partial_x \phi)^2$$

Boundary analysis

Boundary states must remain gapless with U(1)xU(1) symmetry, but, only gapless protected bosonic modes, no gapless fermion modes, under Coulomb interaction.

$$\tilde{H} = \int dx \; \frac{\tilde{v}}{2\tilde{K}}(\partial_x \theta)^2 + \frac{\tilde{v}\tilde{K}}{2}(\partial_x \phi)^2$$

$$\begin{aligned} n_1 + in_2 \quad & \sim \epsilon_{\alpha\beta}\psi_{1,\alpha}\psi_{2,\beta} \quad\quad \sim e^{i\theta}, \\ n_3 + in_4 \quad & \sim \sum_l (-1)^l \psi_l^\dagger \sigma^+ \psi_l \sim e^{i2\pi\phi}. \end{aligned}$$

Naïve picture for why the boundary must be gapless: the topological defect of the magnetic order at the boundary carries charge, likewise, a charge defect carries spin.

This purely bosonic gapless boundary cannot occur with only one layer of quantum spin Hall insulator: (Wu, Bernevig, Zhang 2005, Xu, Moore 2005)

Bulk wave function for Bosons

Following Read&Moore 1992, the 2d bulk ground state wave function can be represented as 1+1d correlation functions at the boundary:

$$\Psi(z_1, z_2 \cdots w_1, w_2 \cdots) \sim \langle \prod_j e^{i\theta(z_j)} \prod_k e^{2\pi i\phi(w_k)} \mathcal{O}_{bg} \rangle$$

z and *w* are the coordinates of the charge and spin bosons in the bulk;

$$\Psi(z_1, z_2 \cdots w_1, w_2 \cdots) \sim \mathrm{Norm}(z_j, w_k) \prod_{j,k} (z_j - w_k)$$

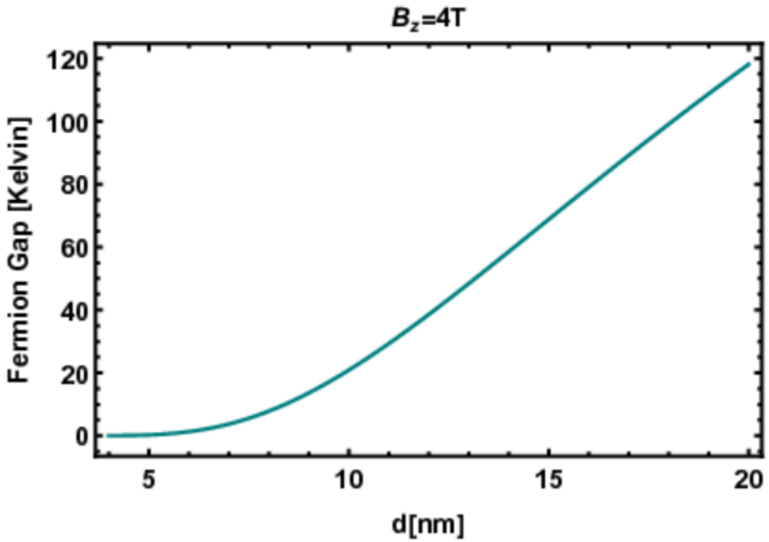
The same wave function constructed by Senthil&Levin 2012 using flux attachment pictures for bosons.
Spin and charge view each other as a 2pi flux, key of the bosonic SPT state.

Possible Experiments

Predictions:

Main prediction, the boundary of bilayer graphene under B field is a conductor with a single particle gap;

Single particle gap comes from interaction, which is tunable by tuning the distance to the metallic gate (screening):



Tunneling from a normal tip would see this gap, but a superconductor tip would see zero gap.

Possible Experiments

Predictions:

The competition between interaction and the Zeeman energy can lead to a canted AF state in graphene under magnetic field:

CAF breaks $U(1)_{spin}$ symmetry, opens up a local gap (backscattering) for the edge states. Current noise measurement.

$$\tilde{S}(\omega = 0) = 2e^* \langle I \rangle$$