UNIVERSITY OF CAMBRIDGE

Dissipation Does Matter

A study on non-equilibrium phenomena in dissipative cold atom systems

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T.C.M. Group, Cavendish Laboratory, University of Cambridge

Outline

- 1. Curriculum Vitae
- 2. Publications
- 3. Main works
- 4. Future plan

Curriculum Vitae

- **Education experience**
- 2002-2006 Sun Yat-sen University, Undergraduate Student
- 2006-2012 Sun Yat-sen University, Ph.D. Student
  Supervisor: Prof. Zhibin Li

- **Work experience**
- 2012-2015 Institute for Advanced Study, Tsinghua University,
  Postdoc Researcher, Collaborator: Prof. Hui Zhai
- 2016-2018 Cavendish Laboratory, University of Cambridge,
  Postdoc Researcher, Collaborator: Prof. Nigel R. Cooper

- **Research interests**
- Theory of cold atom physics

Publications

Journal	*count*	*Contribution*	*IF*
Nature Physics	**1**	Fifth author	22.806
Physics Review Letters	**3**	First author	8.462
Physics Review A	**4**	First author (2) Corresponding –author (1) Third author (1)	2.925
Physics Review B	**2**	First author (1) Second author (1)	3.836
Journal of Physics B	**1**	First author	1.792

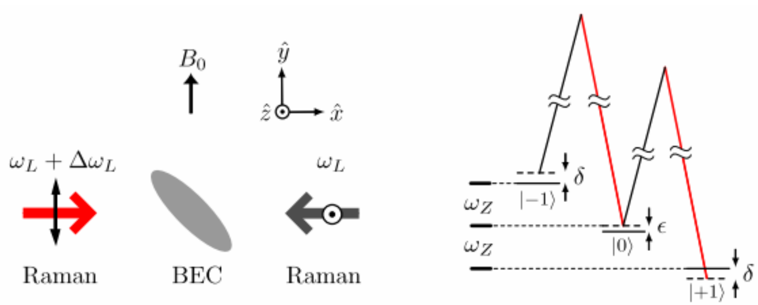
- **Total: 11, Citations:** 274
- **Two papers under submission**

Main Works

- 1. Spin-orbit coupled BEC
- 2. Driving induced topological band
- 3. Bose-Fermi superfluid mixture
- 4. Dissipation induced current
- 5. Anomalous diffusion inside cavity

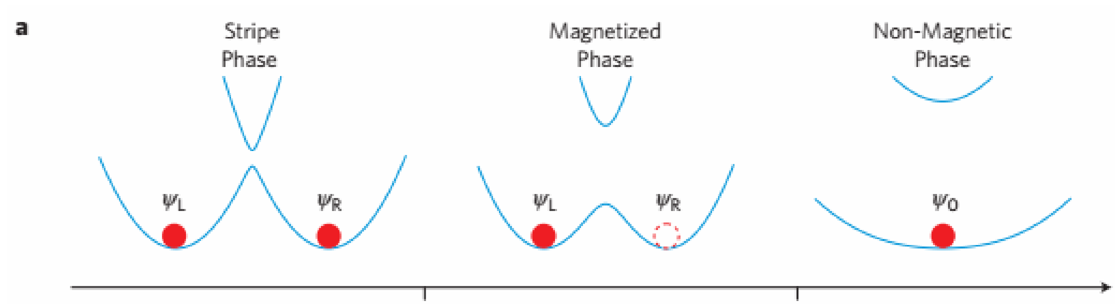
1.Spin-orbit coupled BEC

- Spin-orbital coupling can be realized by apply two counter-propagating Raman laser.



- Spin-orbit coupled BEC exhibits three phases at zero temperature.

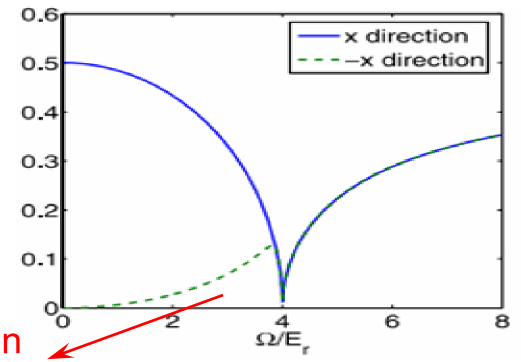
$T = 0$



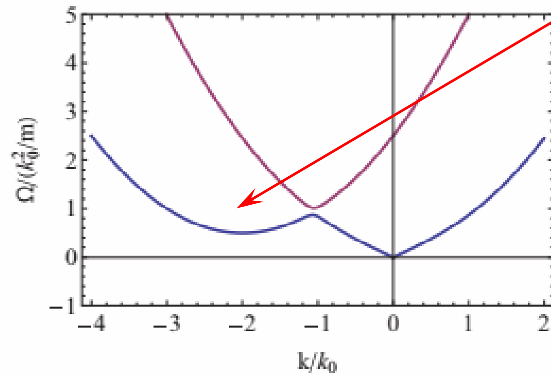
X.-J. Liu, M. F. Borunda, X. Liu & Sinova, PRL, **102**, 046402 (2009)
Y.-J. Lin, K. Jiménez-García, I. B. Spielman, Nature **83,** 471 (2011)
Zhan Wu, Long Zhang, Wei Sun, Xiao-Tian Xu, Bao-Zong Wang, Si-Cong Ji, Youjin Deng, Shuai Chen, Xiong-Jun Liu, Jian-Wei Pan, Science 354, 83-88 (2016)

1.Spin-orbit coupled BEC

- In this direction, our contribution is that we first obtained the excitation spectrum in the plane wave phase, and found the roton structure.

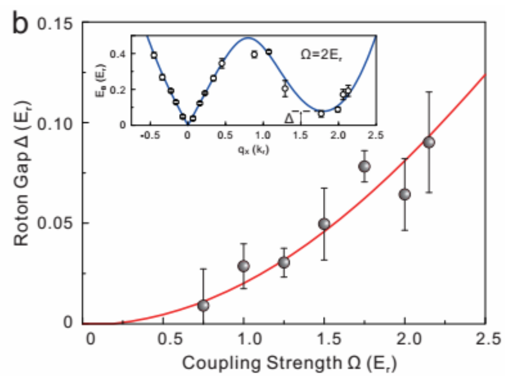


Roton Gap soften



Roton

- Our prediction of the roton is observed by USTC experiment group.

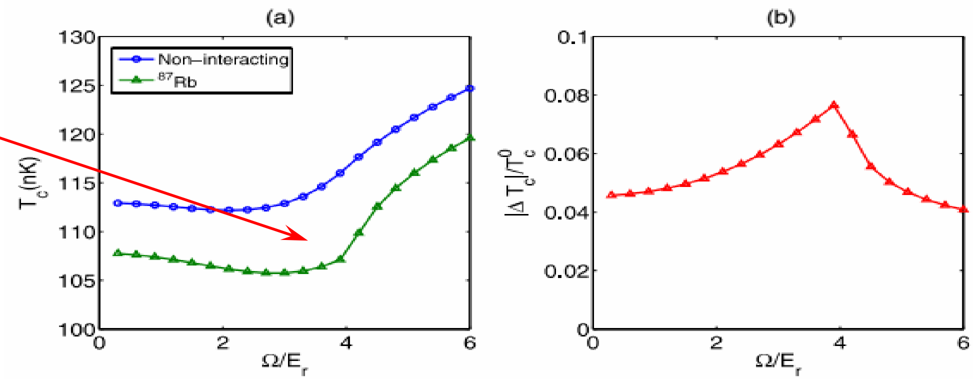


Wei Zheng and Zhibing Li, PRA **85**, 053607 (2012),
Wei Zheng, Zeng-Qiang Yu, Xiaoling Cui and Hui Zhai, JPB **46**, 134007 (2013),
Si-Cong Ji, Long Zhang, Xiao-Tian Xu, Zhan Wu, Youjin Deng, Shuai Chen, and Jian-Wei Pan, PRL **114**, 105301 (2015)

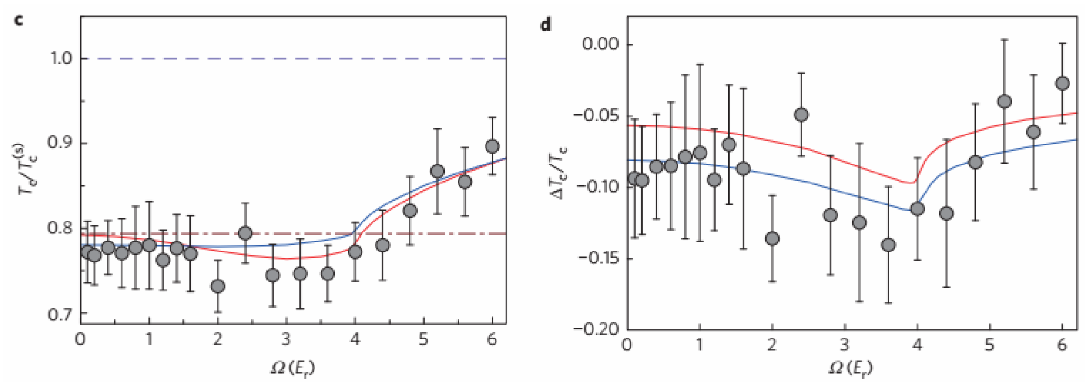
1.Spin-orbit coupled BEC

- We first study the transition temperature of the spin-orbit coupled BEC.

Low-energy DOS
Reaches its maximum



- This phenomenon was also observed by the experiment of USTC.

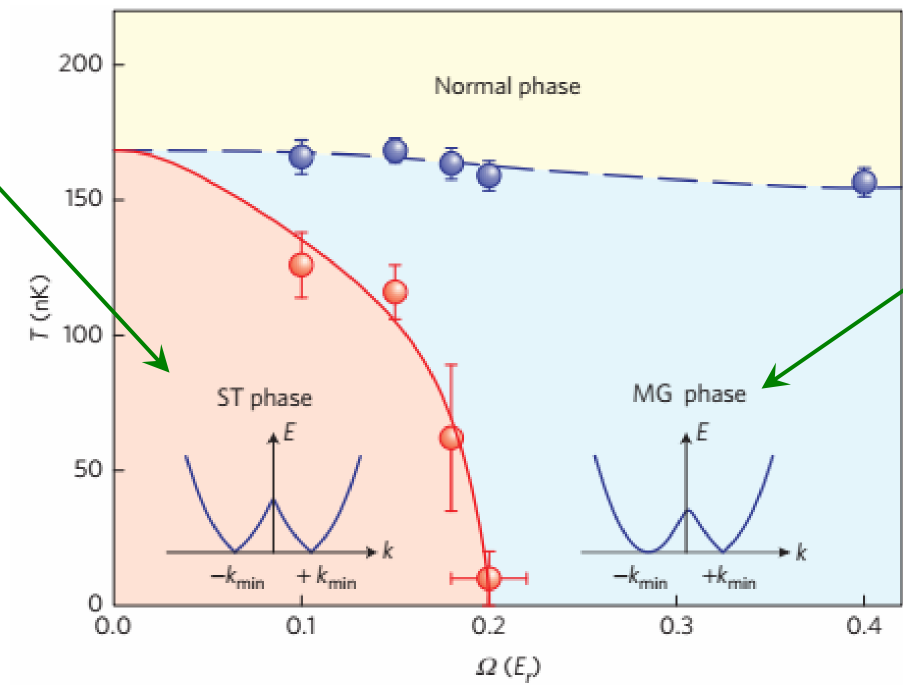


Wei Zheng, Zeng-Qiang Yu, Xiaoling Cui and Hui Zhai, JPB **46**, 134007 (2013),
Si-Cong Ji, Jin-Yi Zhang, Long Zhang, Zhi-Dong Du, Wei Zheng, You-Jin Deng, Hui Zhai, Shuai Chen, and Jian-Wei Pan, Nature Physics **10**, 1038 (2014)

1.Spin-orbit coupled BEC

- USTC experiment group determined the finite-T phase diagram of the spin-orbit coupled BEC, we help them to understand the structure of the phase diagram.

Two linear excitations



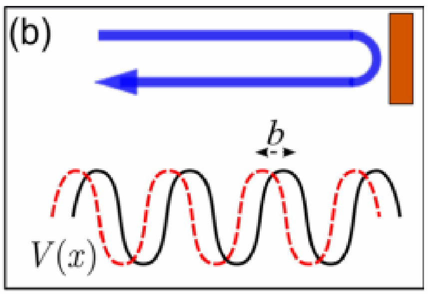
One linear
One quadratic

- The plan wave phase has larger low-energy DOS. So it has larger entropy and lower free energy at finite-T.

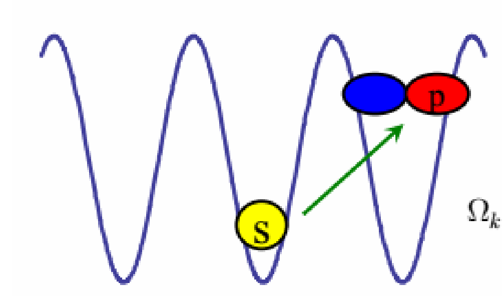
Si-Cong Ji, Jin-Yi Zhang, Long Zhang, Zhi-Dong Du, Wei Zheng, You-Jin Deng, Hui Zhai, Shuai Chen, and Jian-Wei Pan, Nature Physics **10**, 1038 (2014)

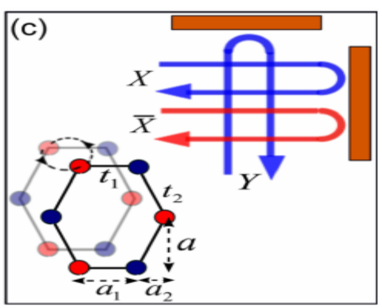
2.Dirving induced topological band

- We propose that by shaking optical lattice, one can realized the topological band in cold atom system.



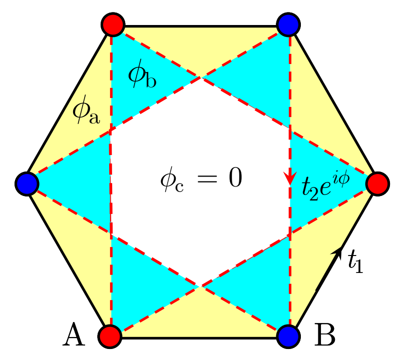
Shaking 1D lattice





Shaking 2D lattice

Haldane model

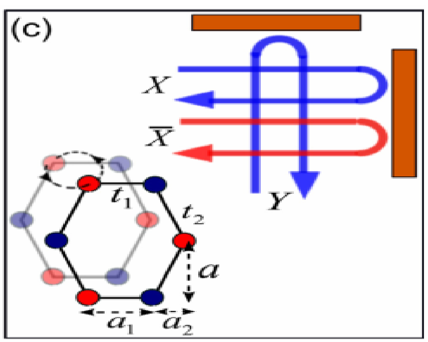


Wei Zheng and Hui Zhai, PRA **89**, 061603(R) (2014)

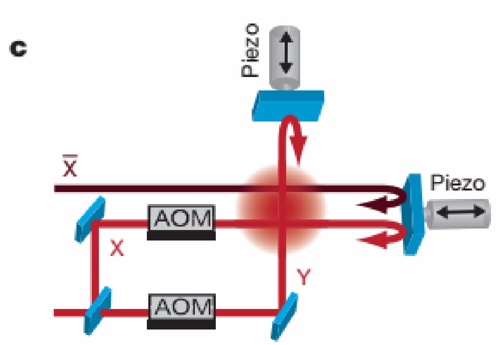
2.Dirving induced topological band

- After our paper, ETH group realized the Floquet Haldane Model by the same proposal. This is the first topological band in cold atom physics.

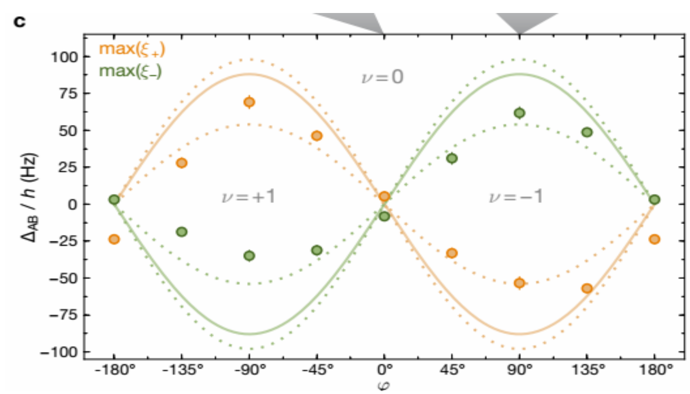
be found in references [S2–S8], and applications to circularly modulated honeycomb lattices can be found in very recent work [S5, S9, S10].



Our proposal



Experiment setup

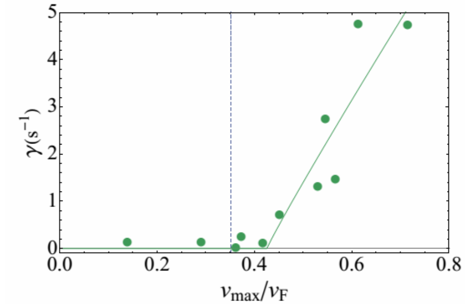


Phase diagram

G. Jotzu, M. Messer, R. Desbuquois, M. Lebrat, T. Uehlinger, D. Greif, T. Esslinger, Nature **515**, 237 (2014)

3.Bose-fermi superfluid mixture

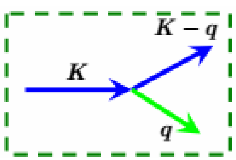
- ENS group found the damping of dipole mode of the Bose-Fermi superfluid mixture has a critical velocity.

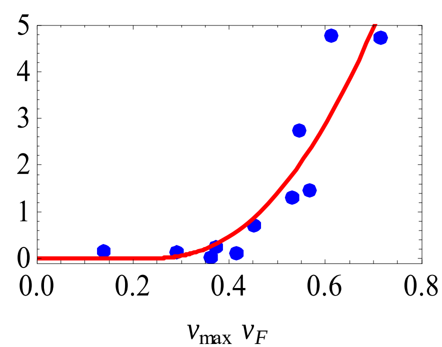


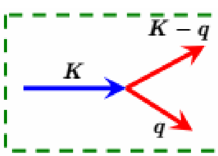
- We explain it as the Beliaev damping of the collective mode, and explain the critical velocity behavior.

BEC

BCS







I. Ferrier-Barbut, et.al. Science, **345**, 1035 (2014)
Wei Zheng and Hui Zhai, PRL. **113**, 265304 (2014),

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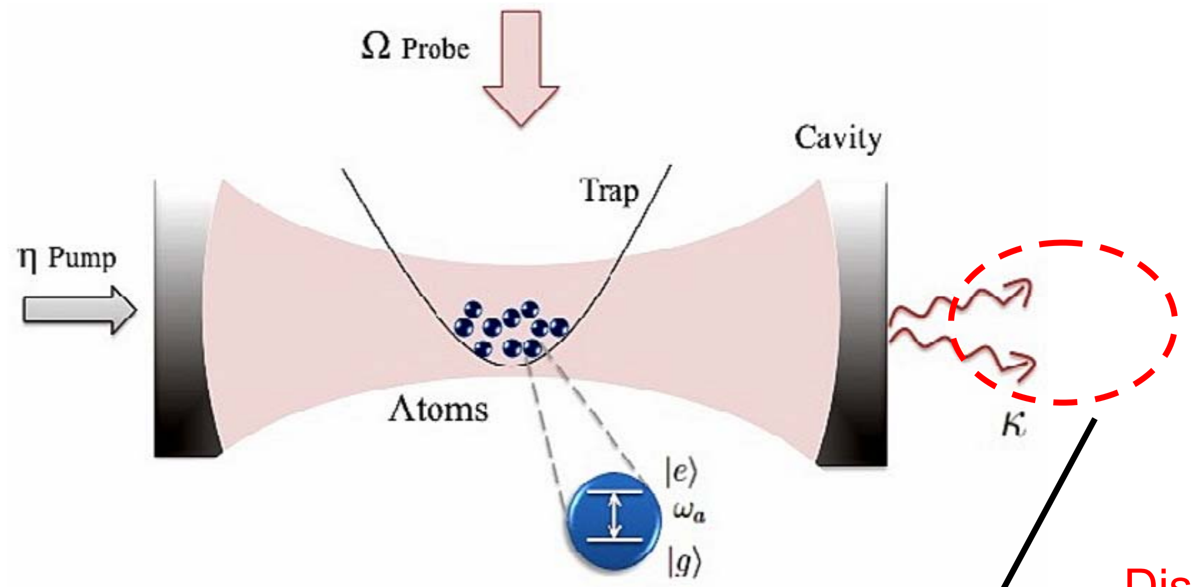
in dissipative cold atom systems

Cold atoms coupled to optical cavities

Driving:
Pump laser

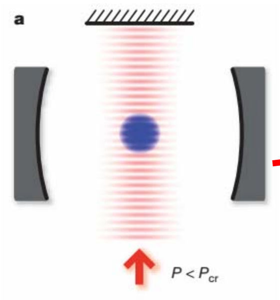


Dissipation:
Cavity Photon loss

$$\partial_t \rho = -i[H,\rho] + \kappa\left(2a\rho a^{+} - a^{+}a\rho - \rho a^{+}a\right),$$

Steady state

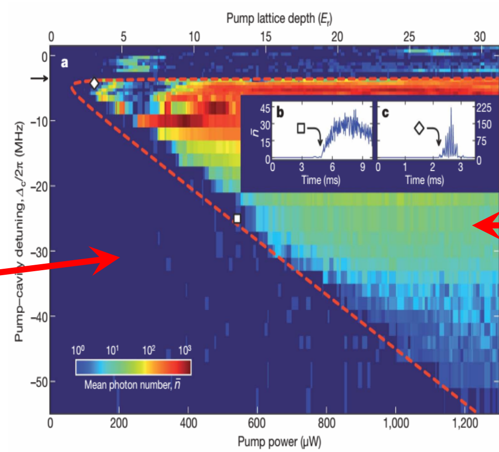
BEC in cavity：

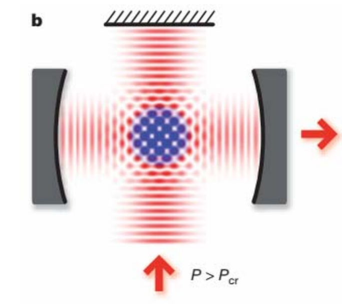


Normal phase

$$\langle a \rangle = 0$$

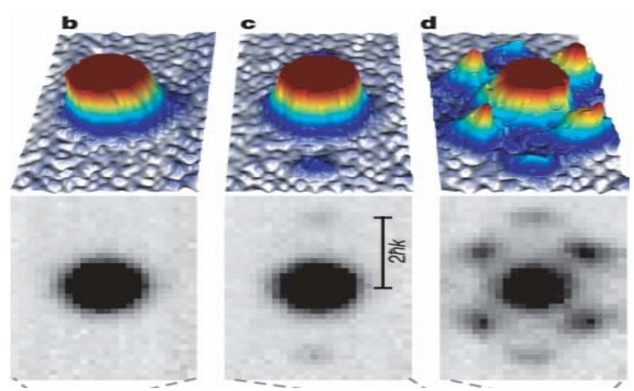
cavity：superradiant transition





Superradiant phase

$$\langle a \rangle \neq 0$$

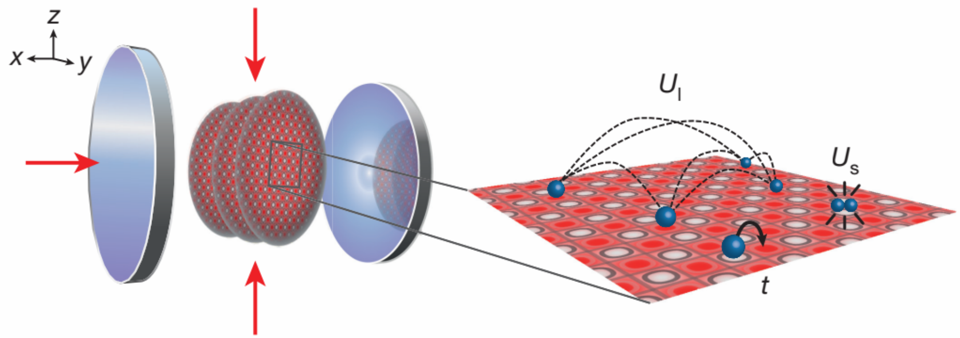


Atom：self-organized to CDW

K. Baumann, et al., Nature (London) **464**, 1301 (2010).
R. Mottl, et al., Science 336, 1570 (2012).

Cavity induced long range interaction

Bose-Hubbard Model inside cavity

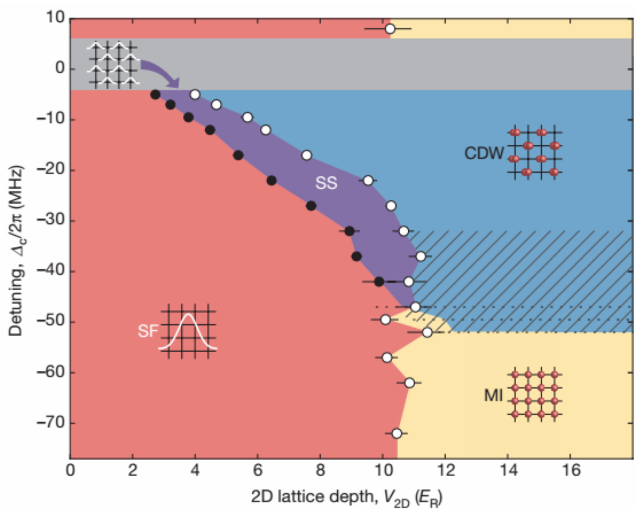


Adiabatically Eliminate the cavity field

$$\hat{H} = -t\sum_{\langle e,o\rangle}(\hat{b}_e^\dagger \hat{b}_o + \mathrm{h.c.}) + \frac{U_s}{2}\sum_{i\in e,o}\hat{n}_i(\hat{n}_i - 1)$$

$$-\frac{U_l}{K}\left(\sum_e \hat{n}_e - \sum_o \hat{n}_o\right)^2 + \sum_{i\in e,o}\mu_i \hat{n}_i$$

Infinite long range interaction!



R. Landig, Nature (London) **532**, 476 (2016).

J. Klinder, et al, PRL **115**, 230403 (2015).

M. R. Bakhtiari, et al, **PRL** 114, 123601 (2015).

Cold atoms coupled to cavities

Many works in this direction:

M. J. Bhaseen, et al., PRL **102**, 135301 (2009).

J. Keeling, et al., PRL **112**, 143002 (2014).

F. Piazza, et al., PRL **112**, 143003 (2014).

Y. Chen, et al., PRL **112**, 143004 (2014).

Y. Deng, et al., PRL **112**, 143007 (2014).

L. Dong, et al., PRA **89**, 011602 (2014)

Y. Chen, et al., PRA **91**, 021602 (2015).

J.-S. Pan, et al., PRL **115**, 045303 (2015).

G. Szirmai, et al., PRA **91**, 023601 (2015).

C. Kollath, et al., PRL **116**, 060401 (2016).

Y. Chen, et al., PRA **93**, 041601 (2016).

A. Sheikhan, et al., PRA **93**, 043609 (2016).

N. Dogra, et al., PRA **94**, 023632 (2016).

G. Konya, et al., PRA **89**, 051601(R) (2014).

S. Wolff, et al., PRA **94**, 043609 (2016).

Y. Chen, et al, arXiv:1711.01382.

Z. Wu, et al., arXiv: arXiv:1707.05579.

Including:

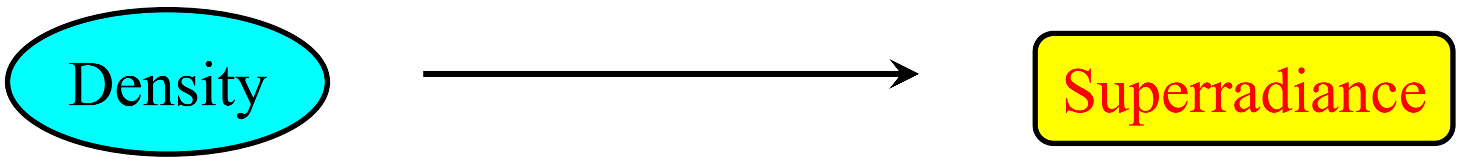
Degenerate Fermi gas in cavity

BEC coupled to multi-cavity mode

Today

1. Cavity is coupled to the density of the atom cloud:

$$H_{\text{coupling}} = \int d\boldsymbol{r} \left\{ \lambda(\boldsymbol{r}) \left(a + a^{+} \right) + U(\boldsymbol{r}) a^{+} a \right\} n(\boldsymbol{r}),$$

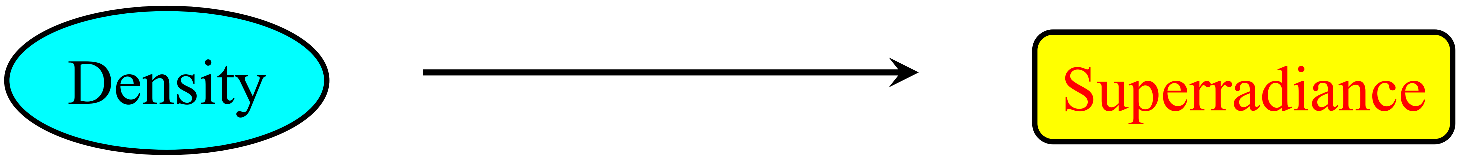


Other degree of freedom of atoms?

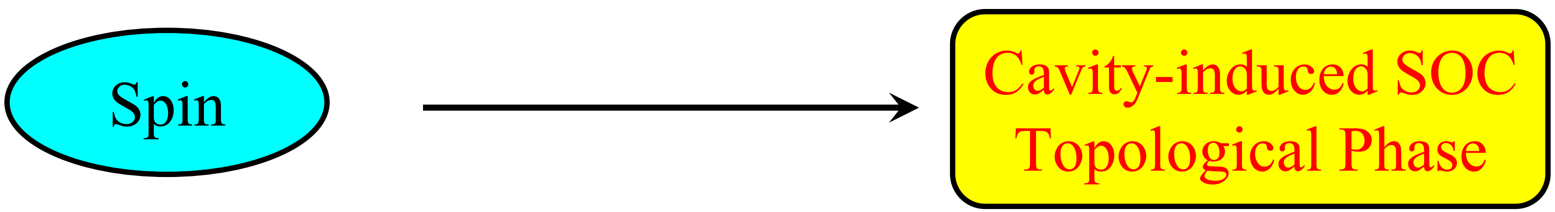
Today

1. Cavity is coupled to the density of the atom cloud:

$$H_{\text{coupling}} = \int d\boldsymbol{r} \{ \lambda(\boldsymbol{r})(a + a^{+}) + U(\boldsymbol{r}) a^{+} a \} n(\boldsymbol{r}),$$



Other degree of freedom of atoms?



J.-S. Pan, X.-J. Liu, W. Zhang, W. Yi, G.-C. Guo, PRL **115**, 045303 (2015).
Y. Deng, J. Cheng, H. Jing, S. Yi, PRL **112**, 143007 (2014).
L. Dong, et al., L. Zhou, B. Wu, B. Ramachandhran, H. Pu PRA **89**, 011602 (2014)

Today

Density → Superradiance

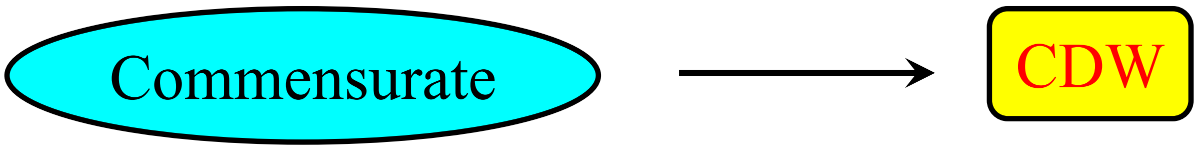
Spin → Cavity-induced SOC
Topological Phase

J.-S. Pan, X.-J. Liu, W. Zhang, W. Yi, G.-C. Guo, PRL **115**, 045303 (2015).
Y. Deng, J. Cheng, H. Jing, S. Yi, PRL **112**, 143007 (2014).
L. Dong, et al., L. Zhou, B. Wu, B. Ramachandhran, H. Pu PRA **89**, 011602 (2014)

Current → ? superradiance?

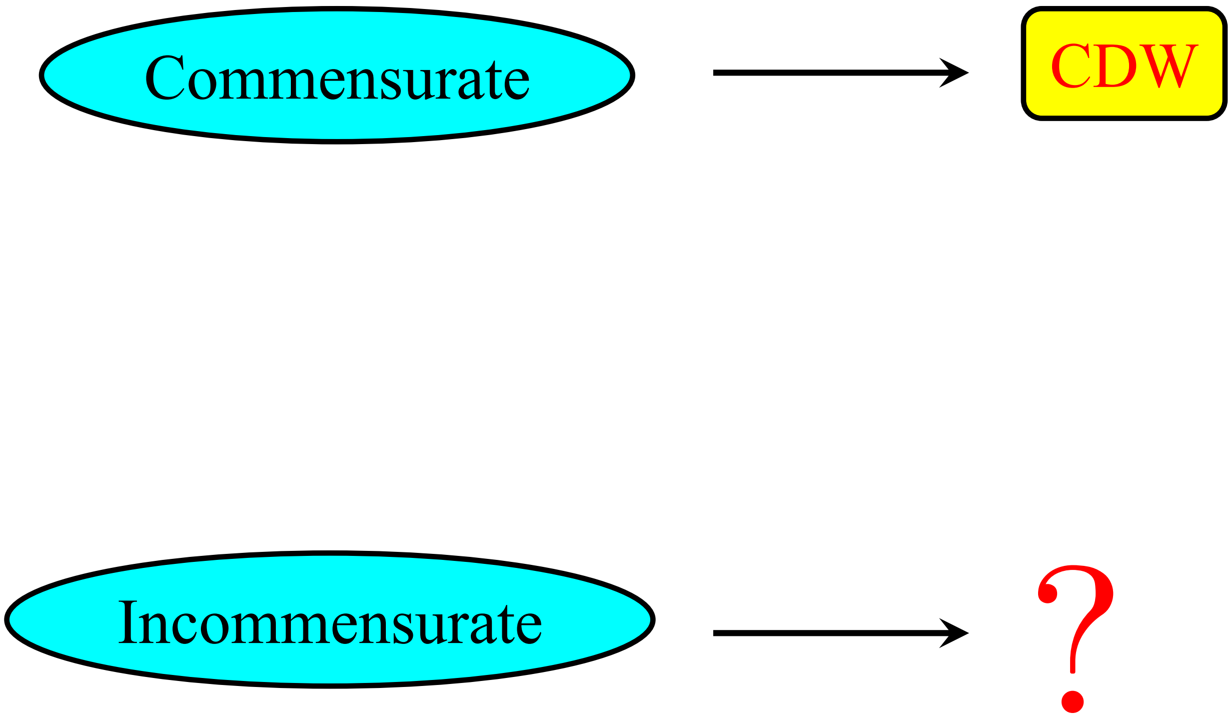
Today

2. The cavity generated optical lattice is commensurate with the underline static lattice.



Today

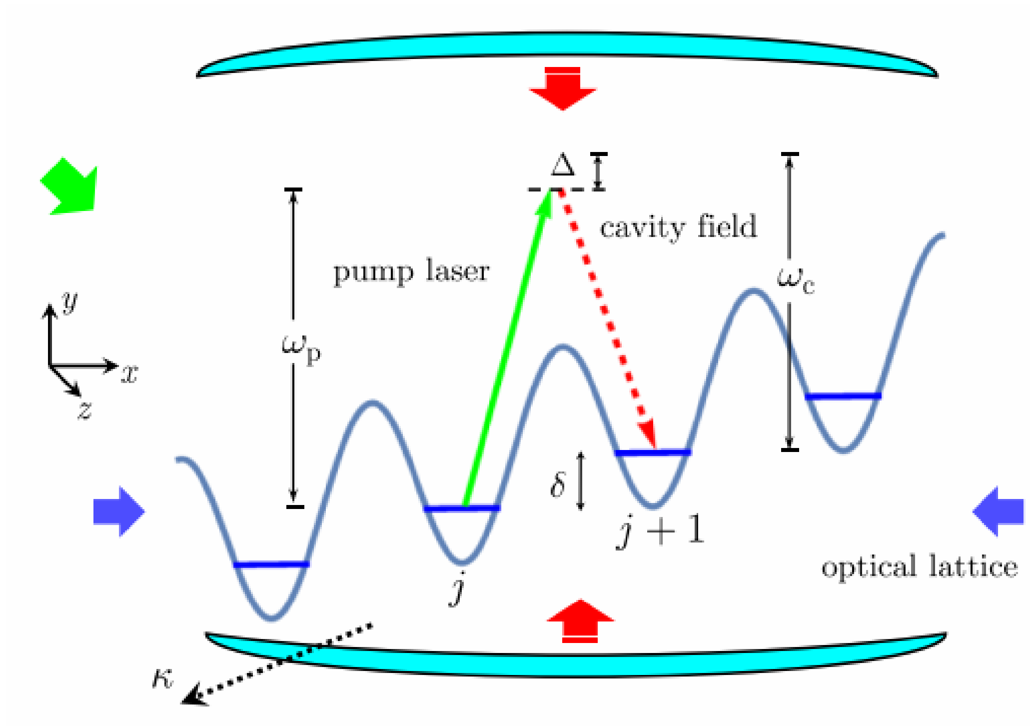
2. The cavity generated optical lattice is commensurate with the underline static lattice.



4. Dissipation induced current

Wei Zheng and Nigel R. Cooper, PRL **117**, 175302 (2016)

Cavity-assisted-hopping lattice



1. Energy gradient suppresses the direct hopping.
2. Atoms can only hop by a cavity-assisted Raman process.

Effective Hamiltonian

Considering the particle number conservation, Hamiltonian can be simplified into:

$$H = \Delta_c' a^+ a - \lambda \sum_j \left(a^+ c_{j+1}^+ c_j + a c_j^+ c_{j+1} \right),$$

$$\Delta_c' = \Delta_c - N\varepsilon_c,$$

$$N = \sum_j c_j^+ c_j,$$

As we know the current on the lattice reads:

$$J_j \propto i\left(c_{j+1}^+ c_j - c_j^+ c_{j+1} \right),$$

The cavity field is coupled to the current of the atoms.

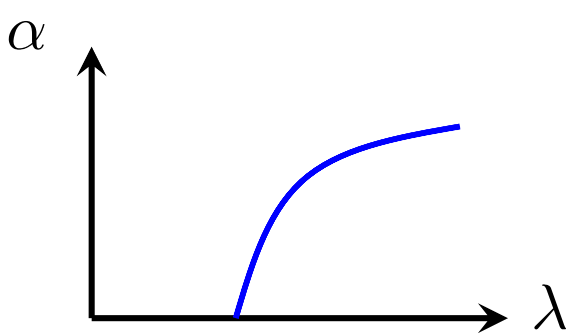
Mean field steady state: Periodic Boundary Condition

In lattice with periodic boundary condition (or in an infinite long lattice), we can make Fourier transformation:

$$\alpha = \frac{\lambda}{\Delta_c' - i\kappa}\langle K \rangle$$

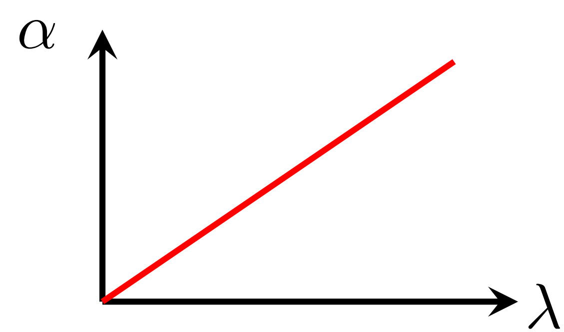
$$\alpha = \langle a \rangle$$
$$n_k = c_k^+ c_k$$
$$K = \sum_{j=1}^{L-1} c_{j+1}^+ c_j$$

$$[\, n_k, H \,] = 0, \quad [\, K, H \,] = 0,$$



Dicke type coupling

Initial $\langle K \rangle \neq 0$



No threshold

Dynamical gauge field

In the superradiance phase,

$\alpha = |\alpha| e^{i\theta}$

$$H_{\mathrm{MF}}(\alpha) = \sum_k E(k) n_k,$$

$$E(k) = -\lambda\,|\,\alpha\,|\cos(k+\theta) \approx \frac{1}{2m^*}(k+\theta)^2,$$

Phase of the cavity $\theta(t)$ ⟷ Vector Potential $A(t)$

*A route to the dynamical gauge field in cold atom system?

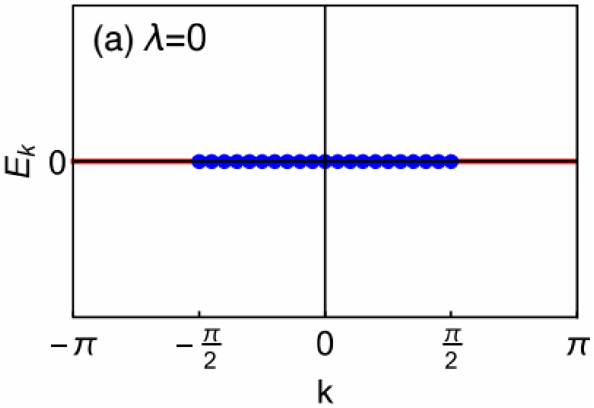
$*$Single-cavity : global gauge field

Multi-cavity : local gauge field

Superradiance induced Current

Cosider half filling $n_k = \Theta(|k| - \pi/2), \quad \Rightarrow \quad \langle K \rangle = L/\pi,$

No pumping



$\alpha_{\rm ss} = 0$

no superradiance

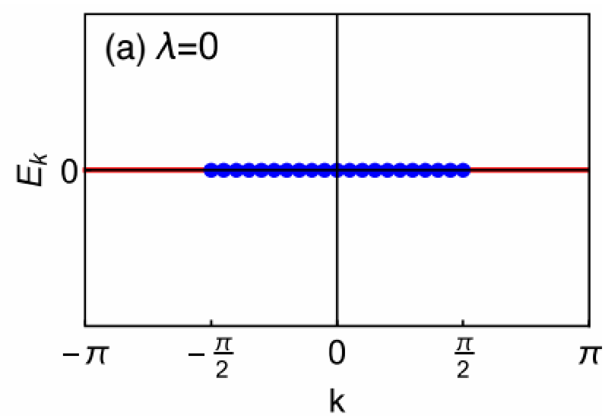
flat band

no current

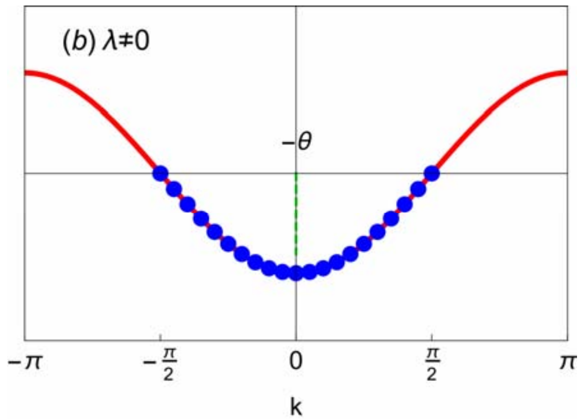
Superradiance induced Current

Cosider half filling $n_k = \Theta(|k| - \pi/2), \quad \Rightarrow \quad \langle K \rangle = L/\pi,$

No pumping



$\kappa = 0$



$\alpha_{\text{ss}} = 0$

no superradiance

flat band

no current

$\alpha_{\text{ss}} = \lambda L / \pi \Delta_c'$

dispersive band

$\theta = 0$

no current

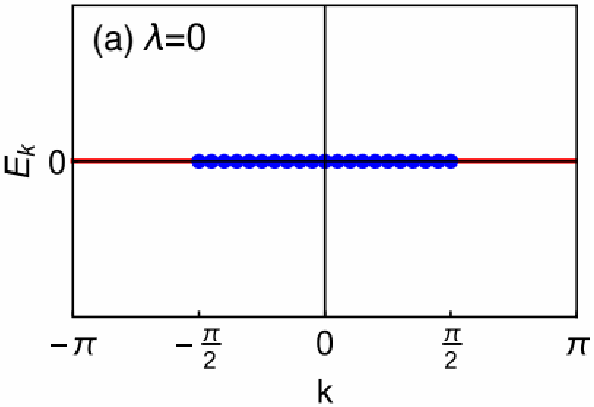
Superradiance induced Current

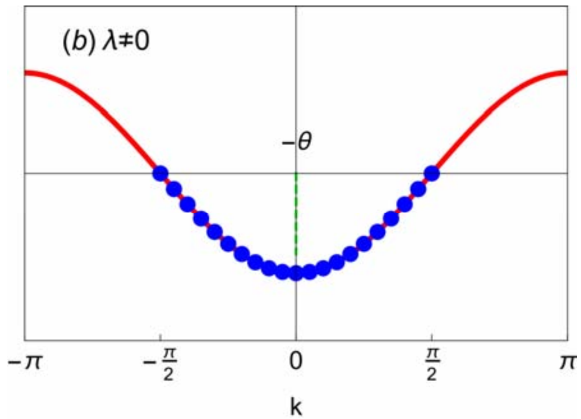
Cosider half filling $n_k = \Theta(|k| - \pi/2), \quad \Rightarrow \quad \langle K \rangle = L/\pi,$

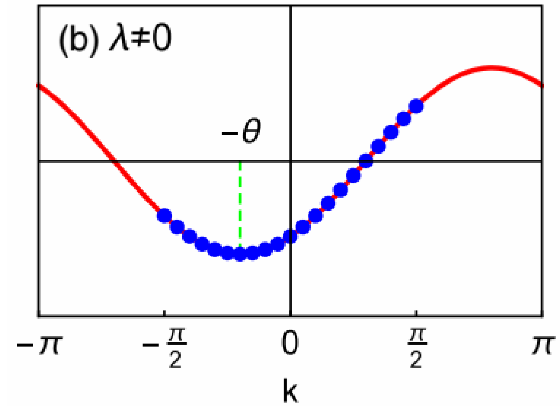
No pumping

$\kappa = 0$

$\kappa \neq 0$







$\alpha_{ss} = 0$

no superradiance

flat band

no current

$\alpha_{ss} = \lambda L/\pi\Delta_c'$

dispersive band

$\theta = 0$

no current

$\alpha_{ss} = \lambda L/\pi(\Delta_c' - i\kappa)$

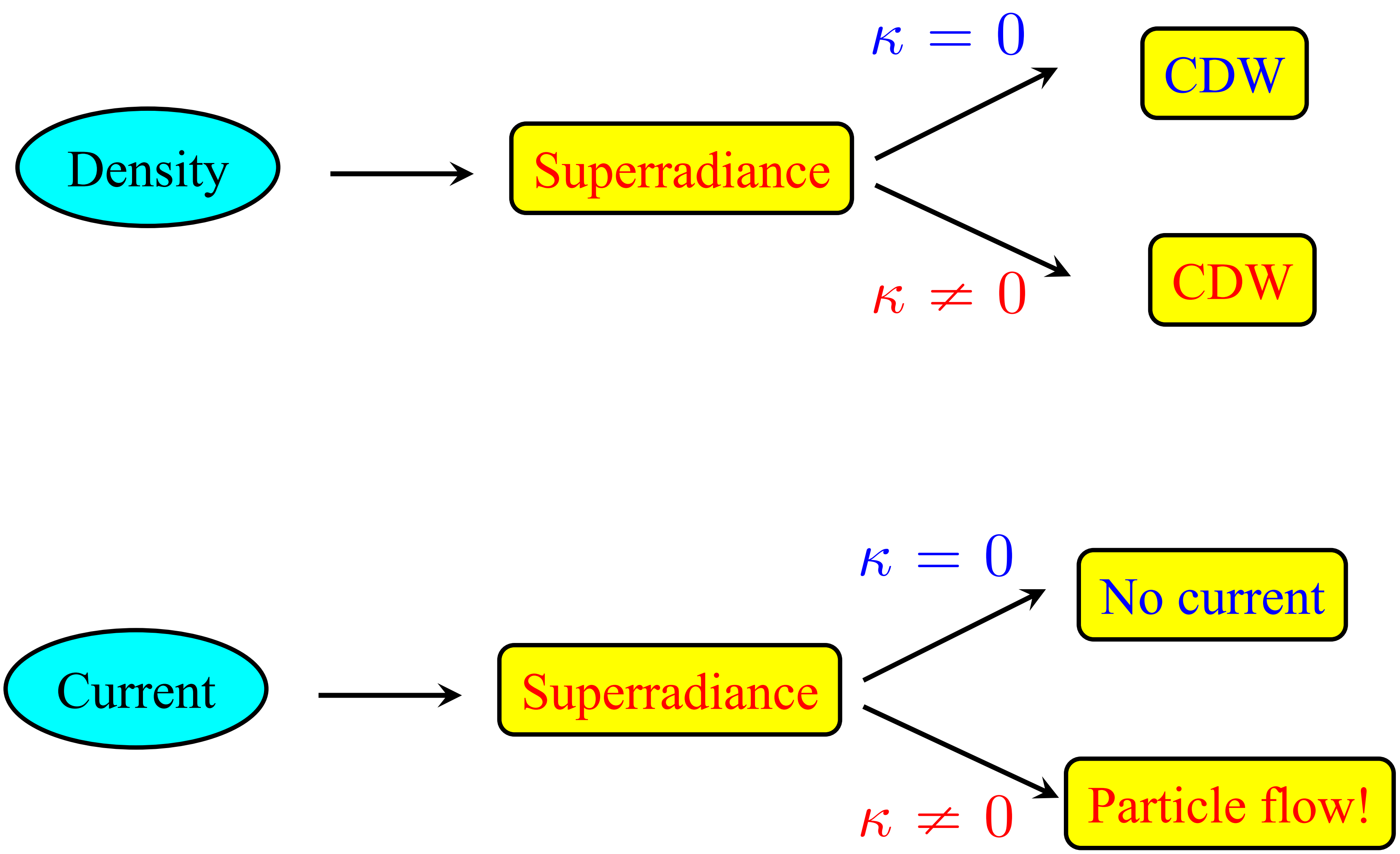
dispersive band

$\theta \neq 0$

$J = \kappa|\alpha_{ss}|^2$

Dissipation Does Matter

Dissipation Does Matter



Consider the fluctuation

superradiance induced current

Local current reads

$$J_i^{\mathrm{sr}} = -\lambda \, \mathrm{Im}\left(\alpha^* \rho_{i+1,i}\right),$$

Consider the fluctuation

Local current reads

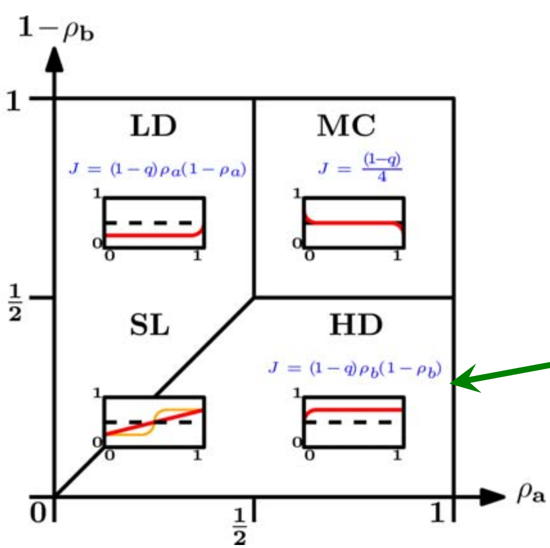
superradiance induced current

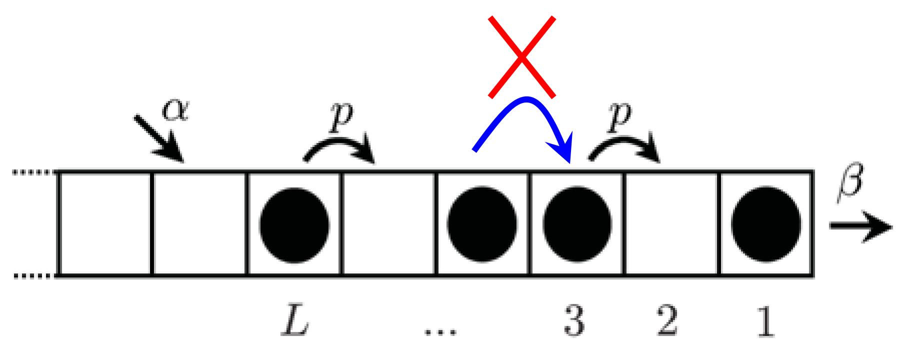
$$J_i^{\rm sr} = -\lambda\, {\rm Im}\left(\alpha^* \rho_{i+1,i}\right),$$

$$f_i = \rho_{ii}$$

$$J_i^{\rm cl} = \frac{2\kappa\lambda^2}{\Delta_{\rm c}'^2 + \kappa^2}\left(1 - f_{i+1}\right) f_i,$$

Asymmetric exclusion process (ASEP)





*Steady State is depend onthe boundary condition

Consider the fluctuation

Local current reads

superradiance induced current

$$J_i^{\mathrm{sr}} = -\lambda\,\mathrm{Im}\big(\alpha^* \rho_{i+1,i}\big),$$

$$J_i^{\mathrm{cl}} = \frac{2\kappa\lambda^2}{\Delta_{\mathrm{c}}'^2+\kappa^2}(1-f_{i+1})f_i,$$

$f_i = \rho_{ii}$

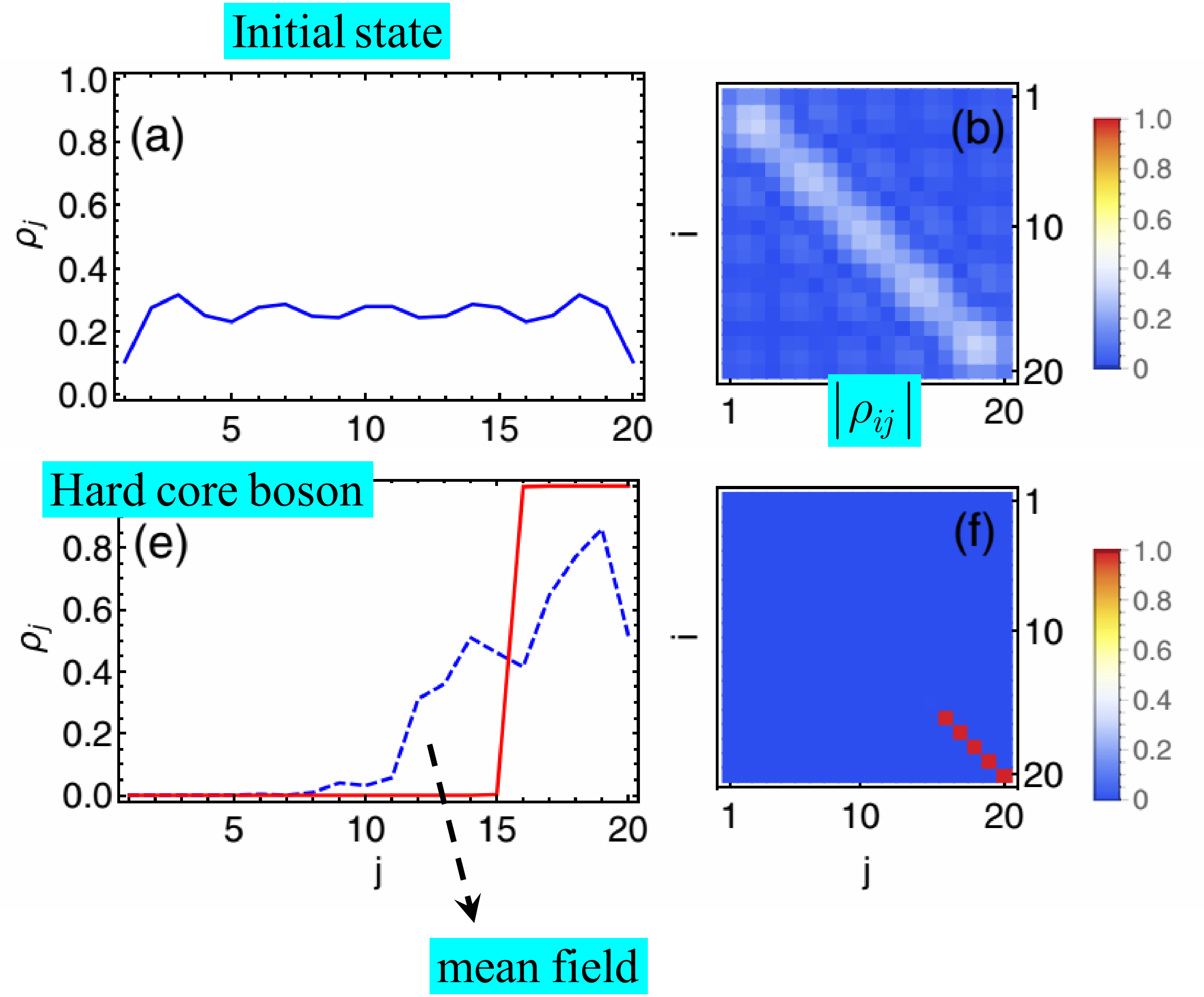
$$J_i^{\mathrm{qu}} = -\frac{2\kappa\lambda^2}{\Delta_{\mathrm{c}}'^2+\kappa^2}\sum_{j\neq i}\mathrm{Re}\big(\rho_{i+1,j+1}\rho_{ji}\big),$$

Quantum correction

*Even when $\alpha = 0$, the fluctuation will induce a current $J_i^{\mathrm{cl}} + J_i^{\mathrm{qu}}$.

Steady state

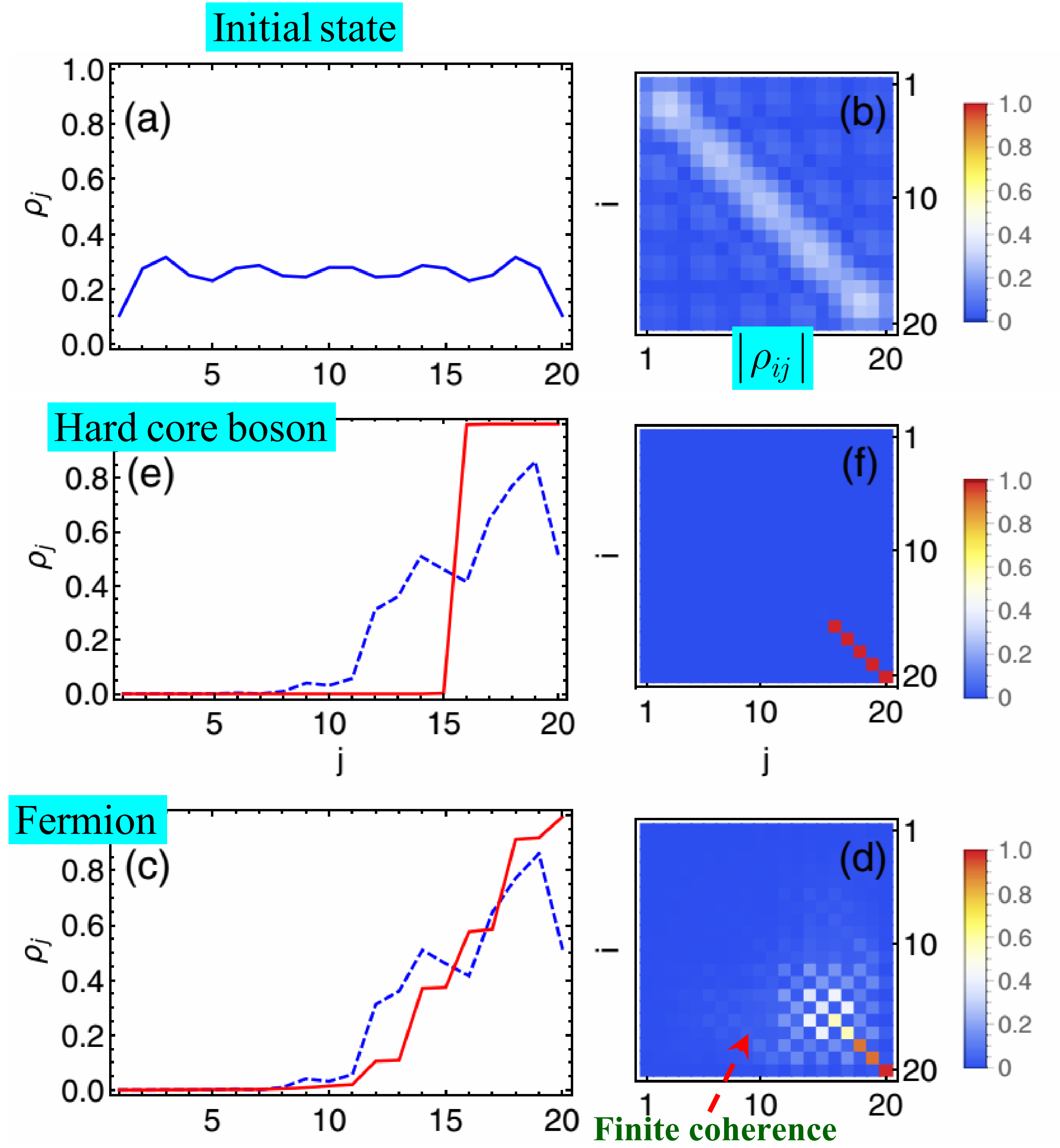
$$J_i^{\mathrm{cl}} = J_i^{\mathrm{qu}} = 0,$$



Steady state

$$J_i^{\rm cl} = J_i^{\rm qu} = 0,$$

$$J_i^{\rm cl} = -J_i^{\rm qu} \neq 0,$$



How the understand the steady state?

Adiabatically eliminate the cavity field when $\kappa \gg \Delta_c', \lambda$,

$$\partial_t \rho_f = -i[H_{\text{eff}}, \rho_f] + \kappa\left(2L_{\text{eff}}\rho_f L_{\text{eff}}^+ - L_{\text{eff}}^+ L_{\text{eff}}\rho_f - \rho_f L_{\text{eff}}^+ L_{\text{eff}}\right),$$

$$H_{\text{eff}} = -\frac{\lambda^2\kappa}{\Delta^2+\kappa^2}K^+K,$$
$$L_{\text{eff}} = K,$$

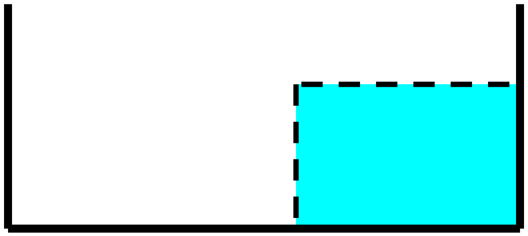
For a pure state $|D\rangle$

$$\text{If}\begin{cases} H_{\text{eff}}|D\rangle = E|D\rangle, \\ L_{\text{eff}}|D\rangle = 0, \end{cases} \Rightarrow |D\rangle \text{ is a steady state,}$$

$$K|D\rangle = 0,$$

Possible steady states

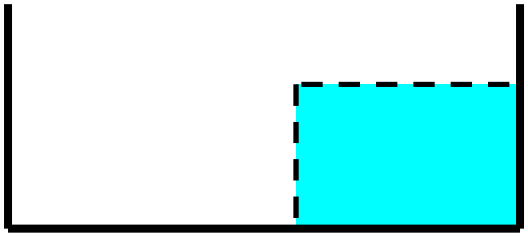
$$|\mathrm{FS}\rangle = \prod_{j=1}^{N} c_{L-N+j}^{+}|0\rangle, \longrightarrow$$ Fermi sea in real space



$$K|\mathrm{FS}\rangle = 0,$$

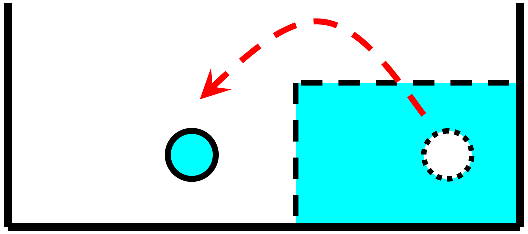
Possible steady states

$$|\mathrm{FS}\rangle = \prod_{j=1}^{N} c^{+}_{L-N+j}|0\rangle, \rightarrow \text{ Fermi sea in real space}$$



$$K|\mathrm{FS}\rangle = 0,$$

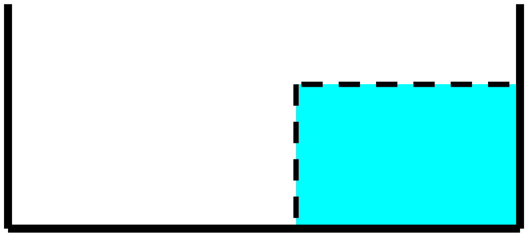
$$b_s^{+} = \sum_{j=s+1}^{L} c^{+}_{j-s} c_j \rightarrow \text{ bosonic excitation in Luttinger theory}$$



$$K b_s^{+}|\mathrm{FS}\rangle = 0, \quad s \neq 1,$$

Possible steady states

$$|\mathrm{FS}\rangle = \prod_{j=1}^{N} c_{L-N+j}^{+} |0\rangle, \rightarrow \text{ Fermi sea in real space}$$



$$K|\mathrm{FS}\rangle = 0,$$

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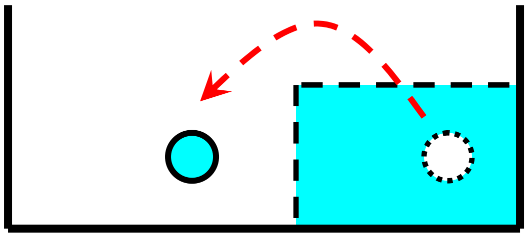
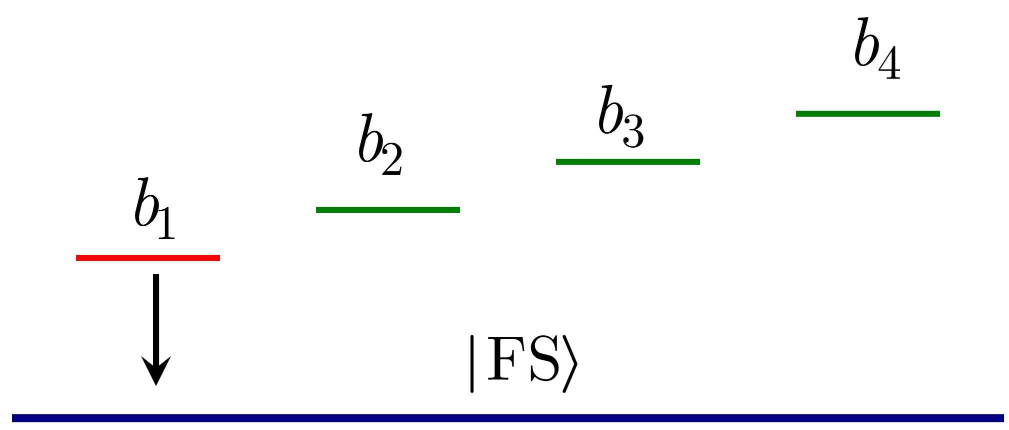
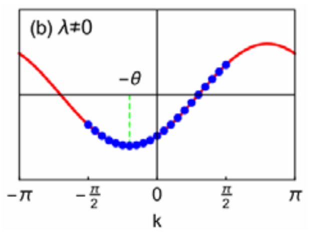


$$K b_s^{+} |\mathrm{FS}\rangle = 0, \quad s \neq 1,$$

*Only $b_1 = K$ is couple to the cavity field, and can be damped.



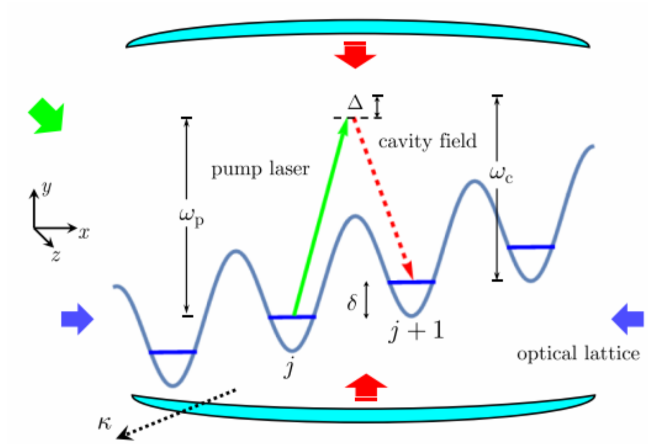
Summary



Dissipation induced flow

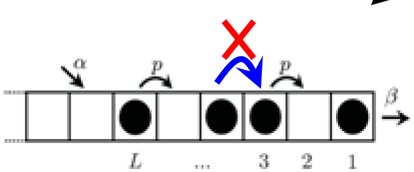
$$E(k) \approx \frac{1}{2m^*}(k+\theta)^2 \, ,$$

Dynamical gauge field

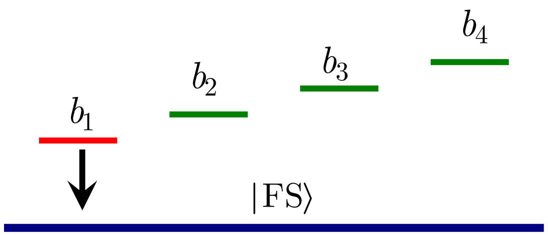


Cavity-assisted-hopping-lattice

ASEP



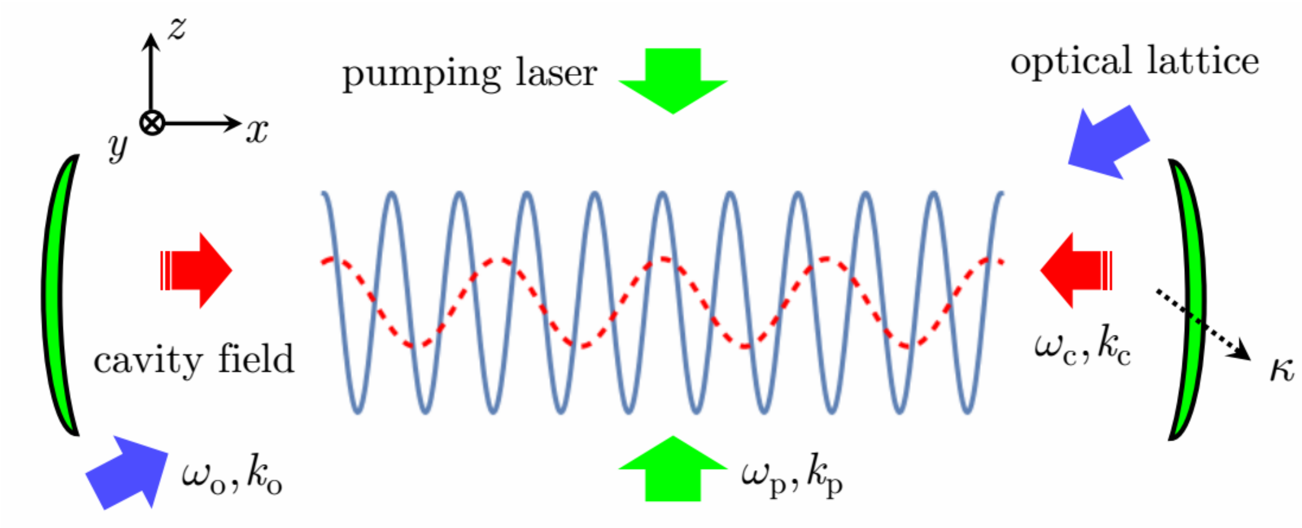
Multiple steady states



5. Anomalous diffusion inside cavity

Wei Zheng and Nigel R. Cooper arXiv: 1709.03916

Experiment setup



The tight-binding Hamiltonian,

$$H = \Delta_c a^+ a - J \sum_j \left(c_{j+1}^+ c_j + h.c. \right) - \lambda \left(a + a^+ \right) \sum_j \cos\left(2\pi\beta j + \phi \right) c_j^+ c_j$$

$\beta = k_c / 2k_o,$

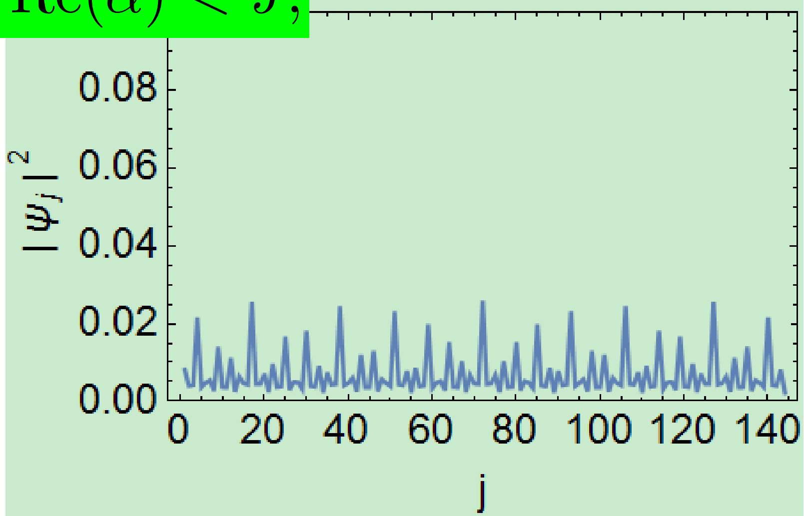
A dynamical Aubry-André (AA) model

Considering the mean field Hamiltonian for atoms, and setting β to be an irrational number

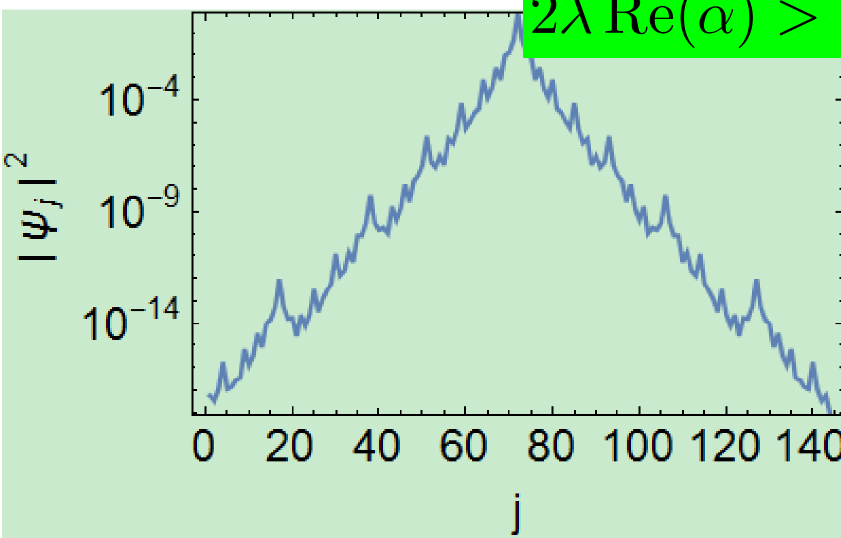
$$H_{\rm MF}(\alpha) = -J\sum_j \left(c_{j+1}^{+}c_j + h.c.\right) - 2\lambda\,{\rm Re}(\alpha)\sum_j \cos\left(2\pi\beta + \phi\right)c_j^{+}c_j,$$

Aubry-André (AA) model: delocalization-localization transition

$2\lambda\,{\rm Re}(\alpha) < J,$

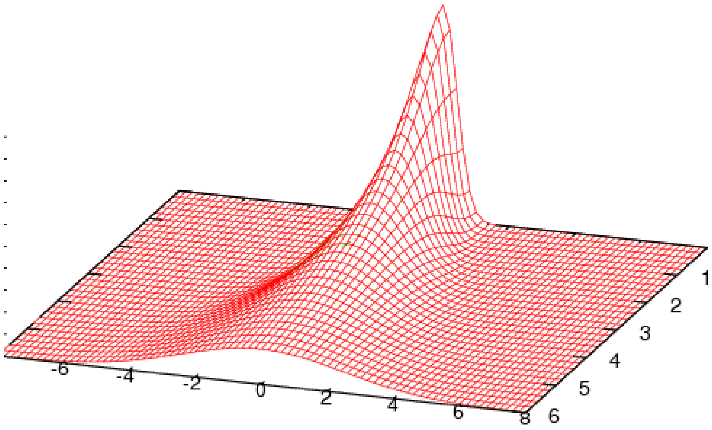


$2\lambda\,{\rm Re}(\alpha) > J,$



How is the particle transport?

Considering the wave packet spreading dynamics



width of wave packet

$$\sigma(t) = \sqrt{\langle X^2 \rangle - \langle X \rangle^2}$$

$$X(t) = \sum_j j |\langle j | \psi \rangle|^2$$

In general, long time behavior is a power law

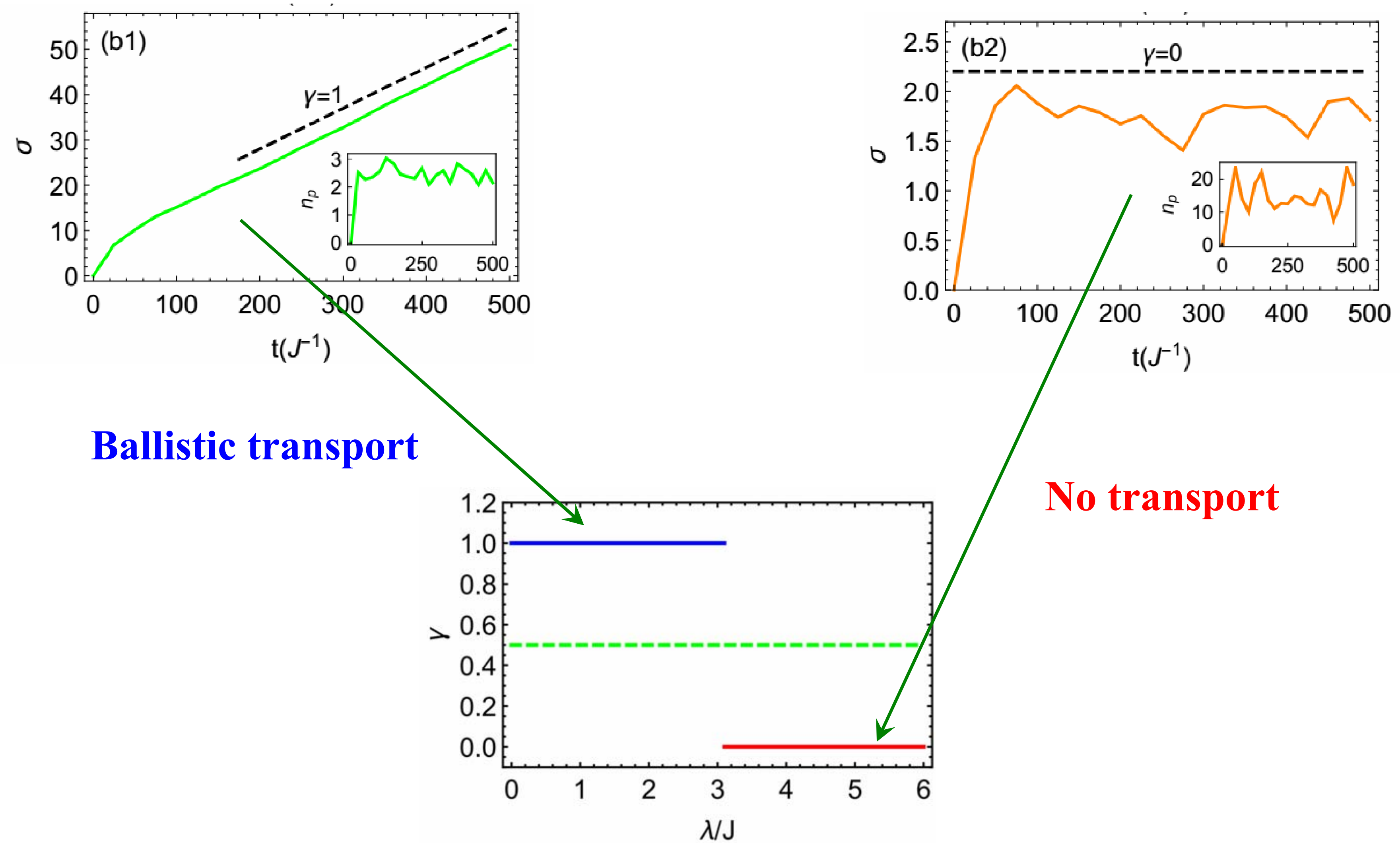
$$\sigma(t) \sim t^{\gamma}$$

$\gamma = 0, \quad \rightarrow$ localization

$\gamma = 1/2, \rightarrow$ diffusive

$\gamma = 1, \quad \rightarrow$ ballistic

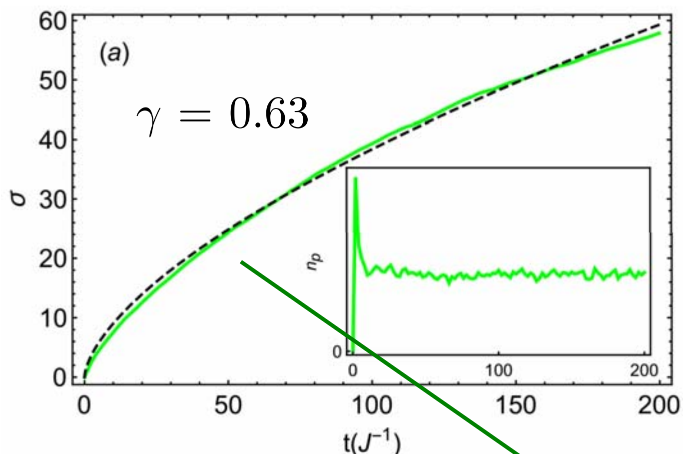
Dissipationless limit $\kappa = 0$

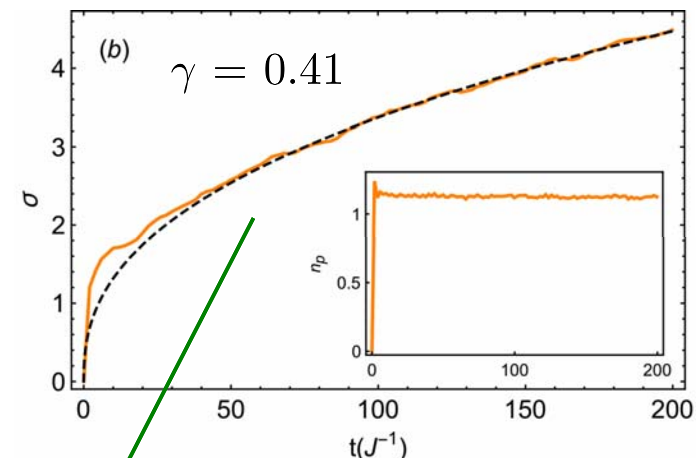


Adding Dissipation

We found anormalous diffusion behavior：

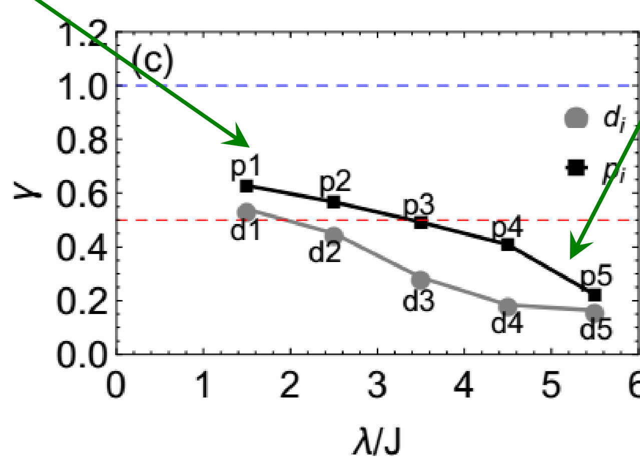
$$\sigma(t) \sim t^{\gamma}, \quad 0 < \gamma < 1,$$





Super-diffusion

$$1/2 < \gamma < 1$$

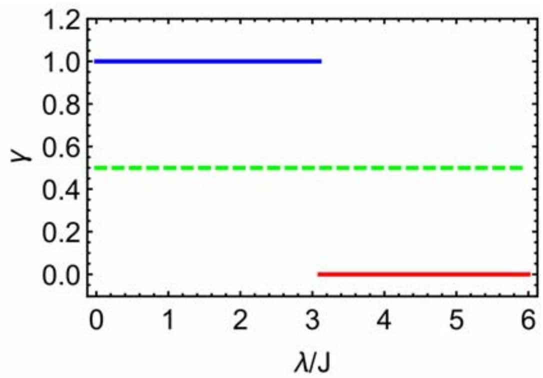


Sub-diffusion

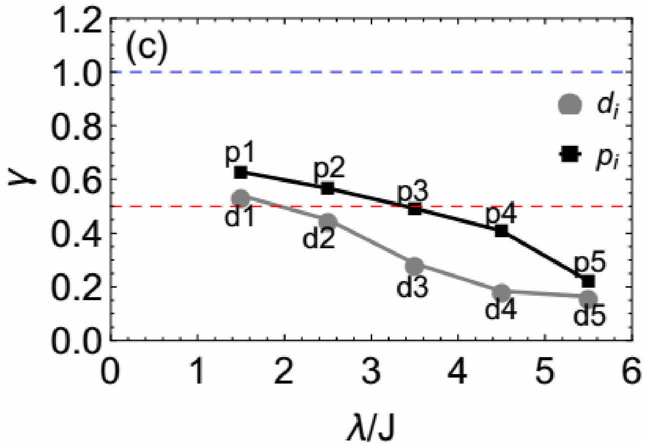
$$0 < \gamma < 1/2$$

Dissipation Does Matter

No dissipation, $\kappa = 0$



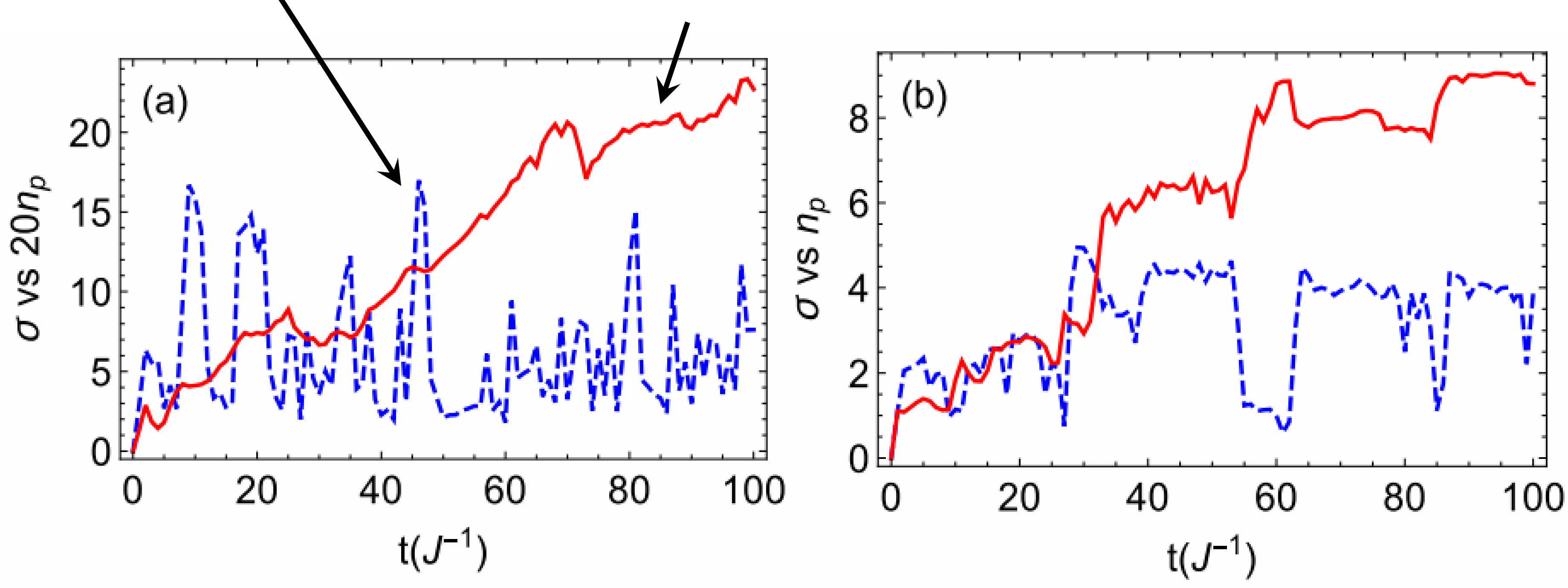
With dissipation, $\kappa \neq 0$



How to understand this anomalous diffusion?

Check the single trajectory

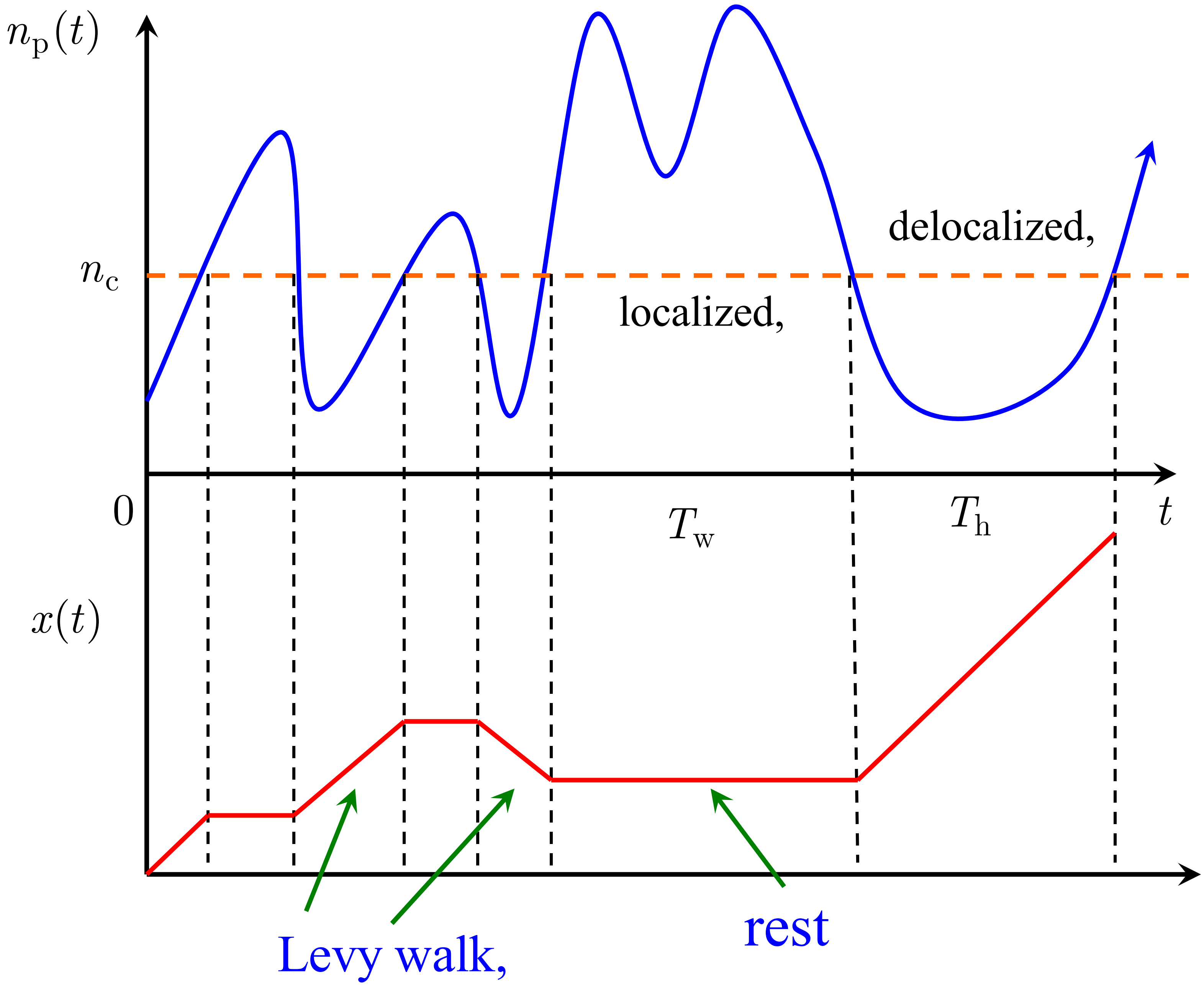
photon number wave packet width



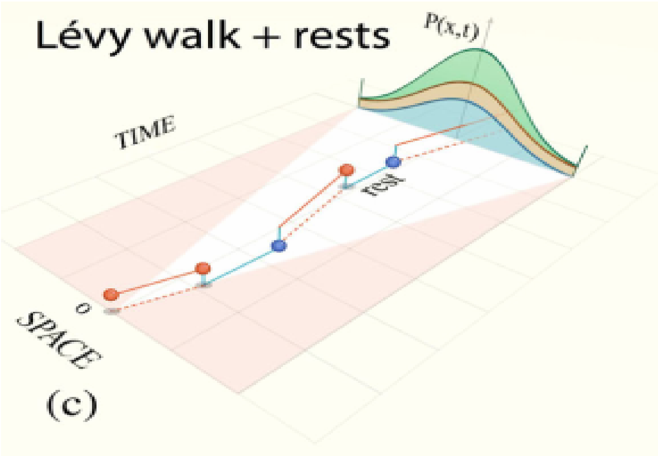
Large photon number → Large effective Potential → Localized States → No transport

Small photon number → Small effective Potential → Delocalized states → Ballistic transport

Map to Levy walk with rest



Levy walk with rest



$$\begin{cases} \langle T_{\rm w} \rangle \to \text{finite} \\ \langle T_{\rm h} \rangle \to \text{finite} \end{cases}, \quad \text{diffusion}, \gamma = 1/2$$

$\langle T_{\rm h} \rangle \to$ diverging

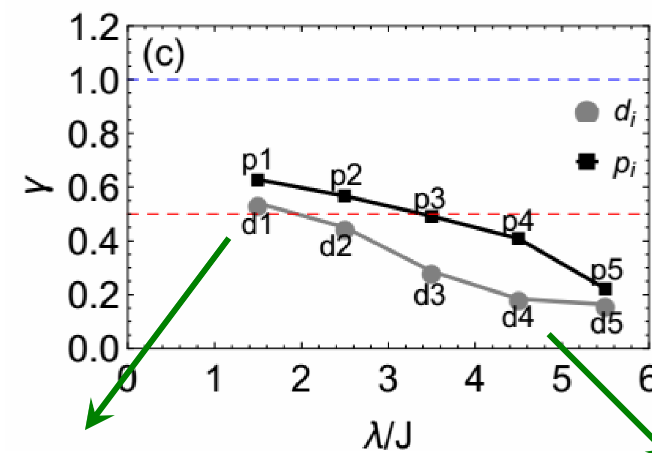
superdiffusion, $\gamma > 1/2$

$\langle T_{\rm w} \rangle \to$ diverging

subdiffusion, $\gamma < 1/2$

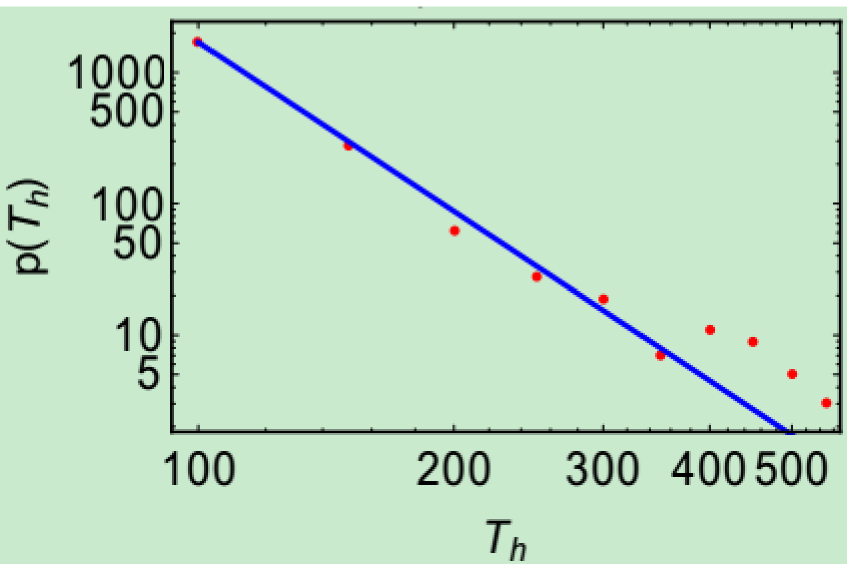
Map to Levy walk with rest

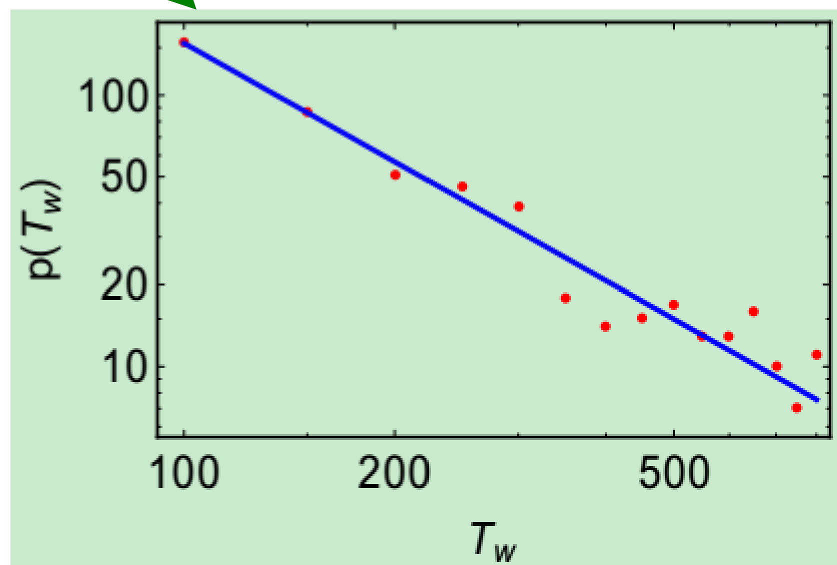
estimate an n_c



superdiffusion

subdiffusion





$p(T_\mathrm{h})$ has long tail

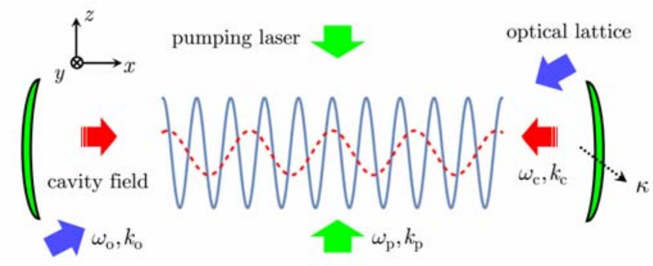
$\langle T_\mathrm{h} \rangle \rightarrow$ diverging

$p(T_\mathrm{w})$ has long tail

$\langle T_\mathrm{w} \rangle \rightarrow$ diverging

Summary

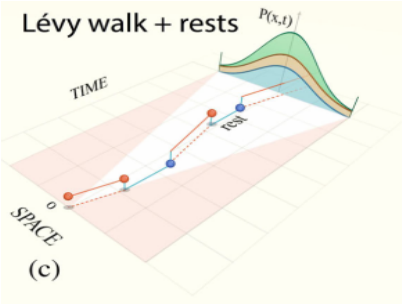
Cavity AA model



Localization-delocalization Transition

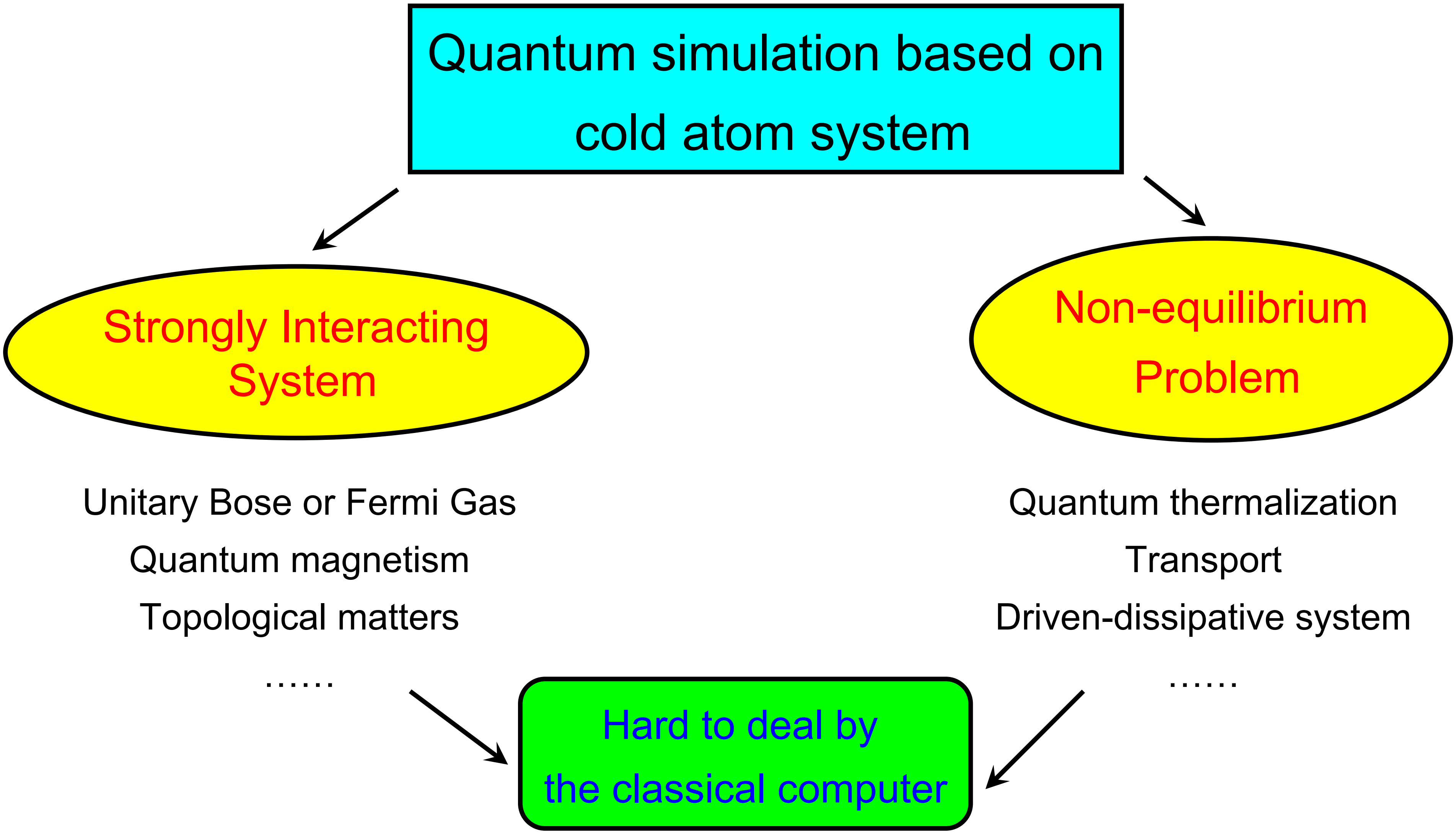
Quantum noise

Levy walk with rest



Anomalous diffusion

Future Plan


Quantum simulation based on cold atom system
Strongly Interacting System
Non-equilibrium Problem
Unitary Bose or Fermi Gas
Quantum magnetism
Topological matters
……
Quantum thermalization
Transport
Driven-dissipative system
……
Hard to deal by the classical computer

Thank you for your attention!

Thanks to my cooperator:
Prof. Nigel R. Cooper