Symmetry-enforced Genons

Meng Cheng

Yale University

Collaborators:
Jong Yeon Lee (Harvard)
Matteo Ippoliti, Michael Zaletel (Princeton)

Workshop on Topological Matter and Quantum Computing, KITS, Beijing

Search for non-Abelian Excitations

Non-Abelian topological phases

NONABELIONS IN THE FRACTIONAL QUANTUM HALL EFFECT

Gregory MOORE

Department of Physics, Yale University, New Haven, CT 06511, USA

Nicholas READ

Departments of Applied Physics and Physics, Yale University, New Haven, CT 06520, USA

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Non-Abelian Statistics in the Fractional Quantum Hall States

X. G. Wen

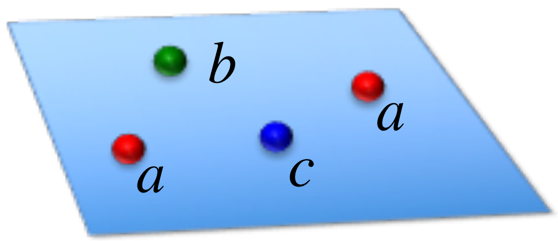
School of Natural Sciences, Institute of Advanced Study, Princeton, New Jersey 08540

(Received 5 October 1990)

The fractional quantum Hall states with non-Abelian statistics are studied. Those states are shown to be characterized by non-Abelian topological orders and are identified with some of the Jain states. The gapless edge states are found to be described by non-Abelian Kac-Moody algebras. It is argued that the topological orders and the associated properties are robust against any kind of small perturbations.

PACS numbers: 73.20.Dx, 05.30.–d

Non-Abelian Anyons



Finite-energy topological quasiparticle excitations

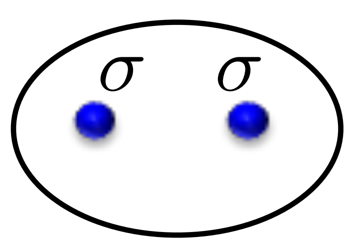
Quasiparticle types *a, b, c, …*

Degenerate ground states in the presence of multiple anyons

They can not be distinguished locally
(thus good qubits)

Example: Ising anyons or Majorana zero modes

n Ising anyons has $2^{n/2-1}$ GSD

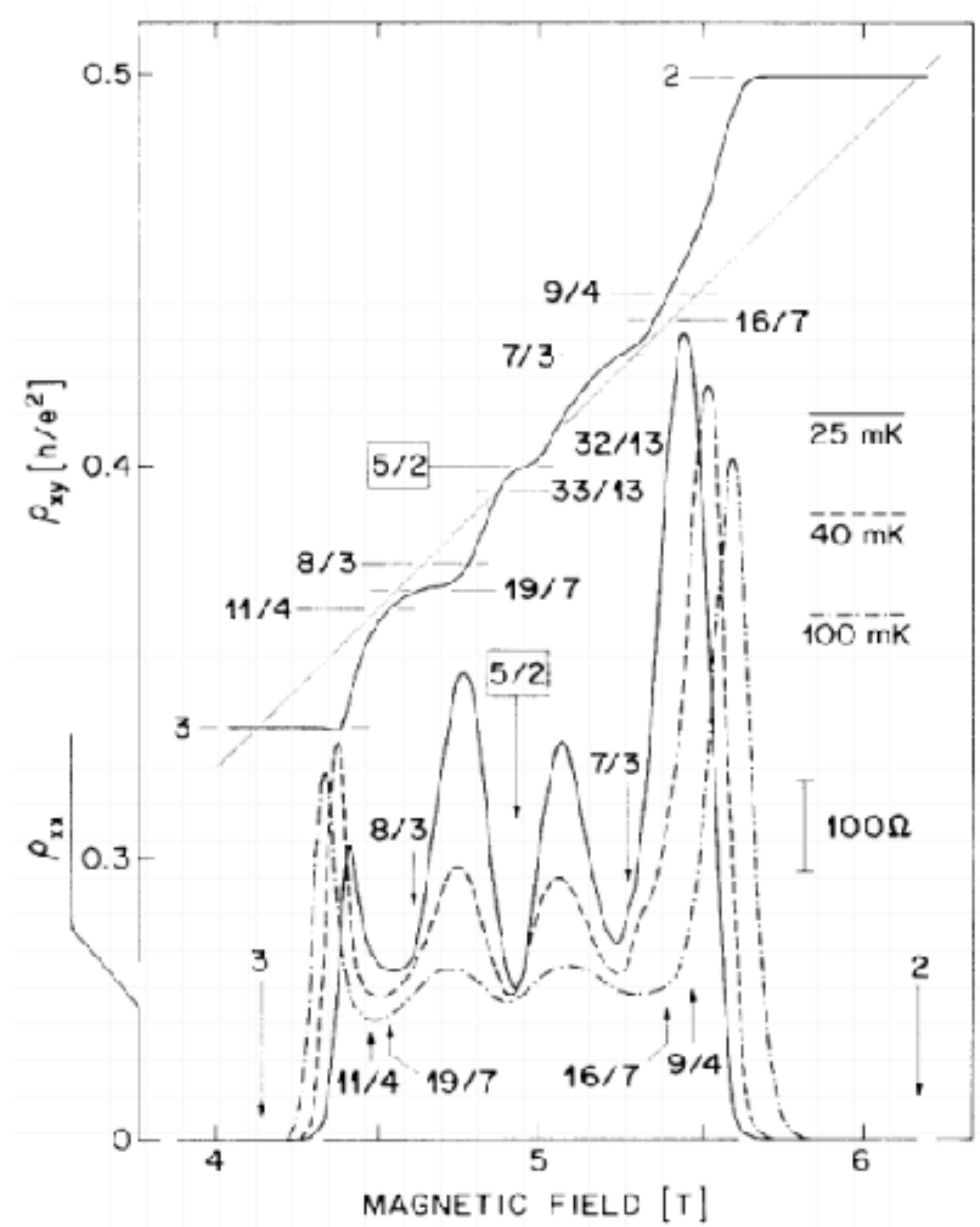


= vacuum or a fermion

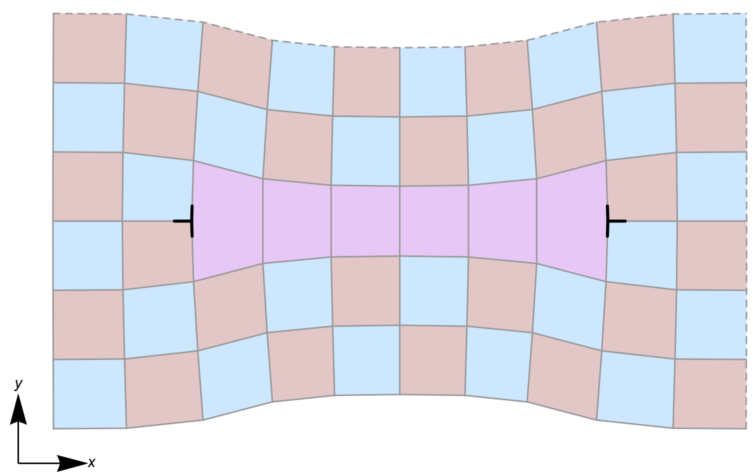
$$\sigma \times \sigma = 1 + \psi$$

Read and Green; Kiatev

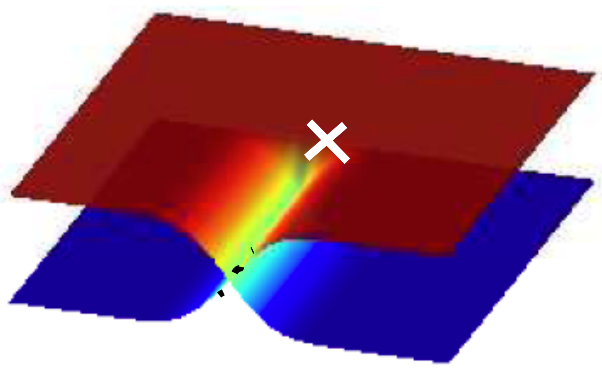
Candidate: 5/2 FQH?



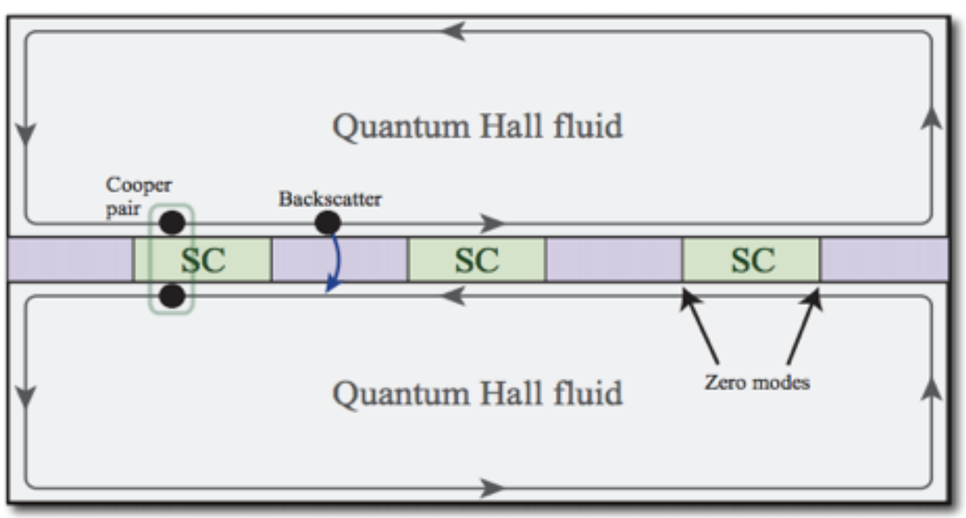
Non-Abelian Defects



You and Wen; Bombin



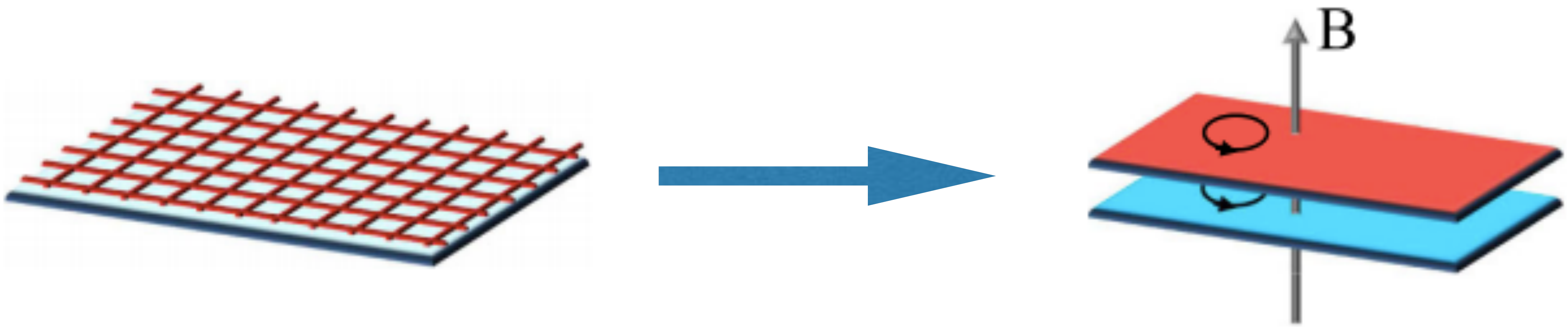
Barkeshli and Qi



Cheng; Clarke et al; Lindner et al;

Defects in Abelian topological phases can harbor non-Abelian zero modes

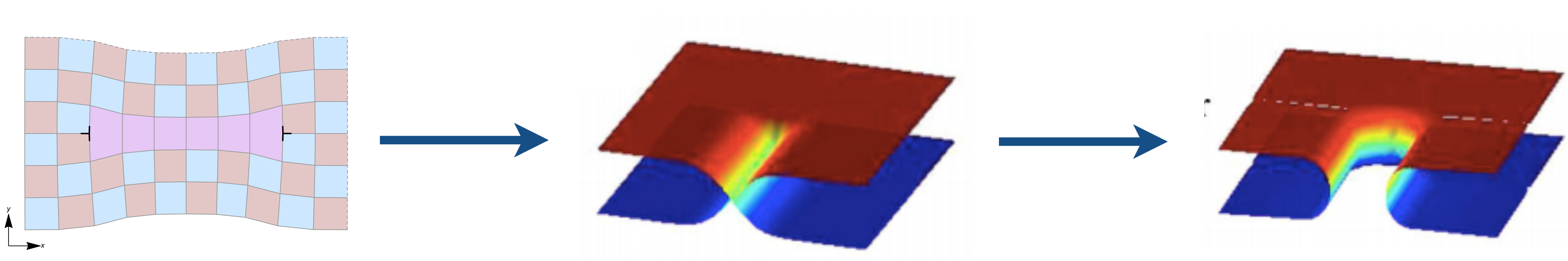
Genons = Non-Abelian Dislocations



C=2 band

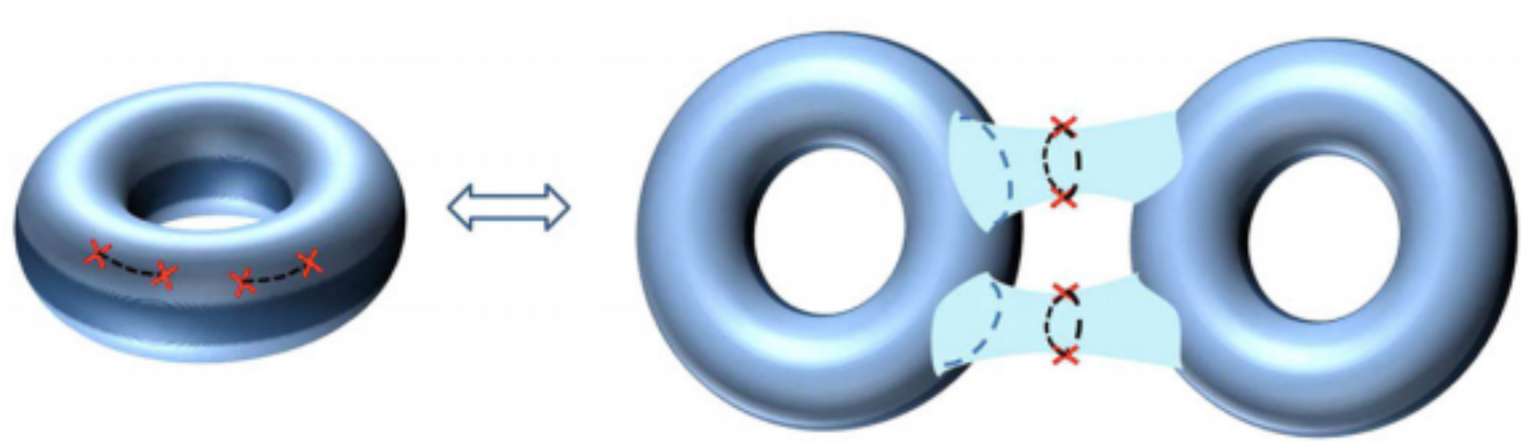
Bilayer QH

Unit translation = layer exchange



Figures from Barkeshli and Qi, PRX 2012

Non-Abelian Degeneracy



Figures from Barkeshli and Qi, PRX 2012

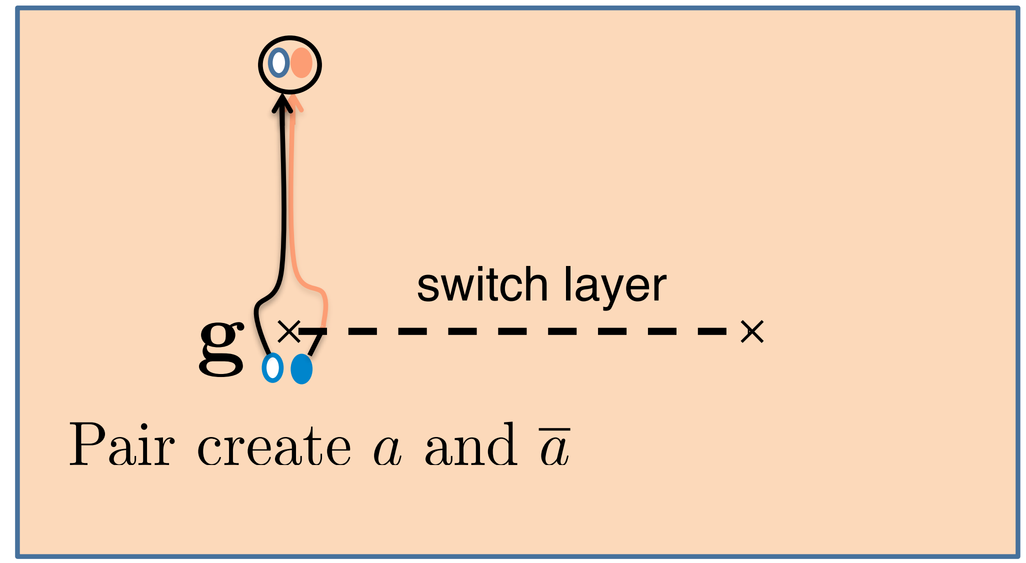
Each pair of dislocations adds a “wormhole”

n pairs of dislocations = (n+1)-genus surface for a single layer

Topological Symmetry

$$X \times X = \sum_a (a, \bar{a})$$

X “absorbs” $(a, \bar{a})$



Genons = translation symmetries change anyon types

(symmetry-enriched topological phases)

Barkeshli et al, 2014

Part I: Genons from Lieb-Schultz-Mattis

based on Cheng, arXiv:1804.10122

Spinless Fermions with Particle-Hole Symmetry

Square or triangular lattices (one fermion mode per site)

$$H = -\sum_{\langle ij \rangle} (t_{ij} c_i^\dagger c_j + \mathrm{h.c.}) + \sum_{ij} V_{ij} \left(n_i - \frac{1}{2}\right)\left(n_j - \frac{1}{2}\right) + \ldots$$

Particle-hole (PH) symmetry: $c_i \to c_i^\dagger$

$$\text{e.g. } t_{ij} = \pm i$$

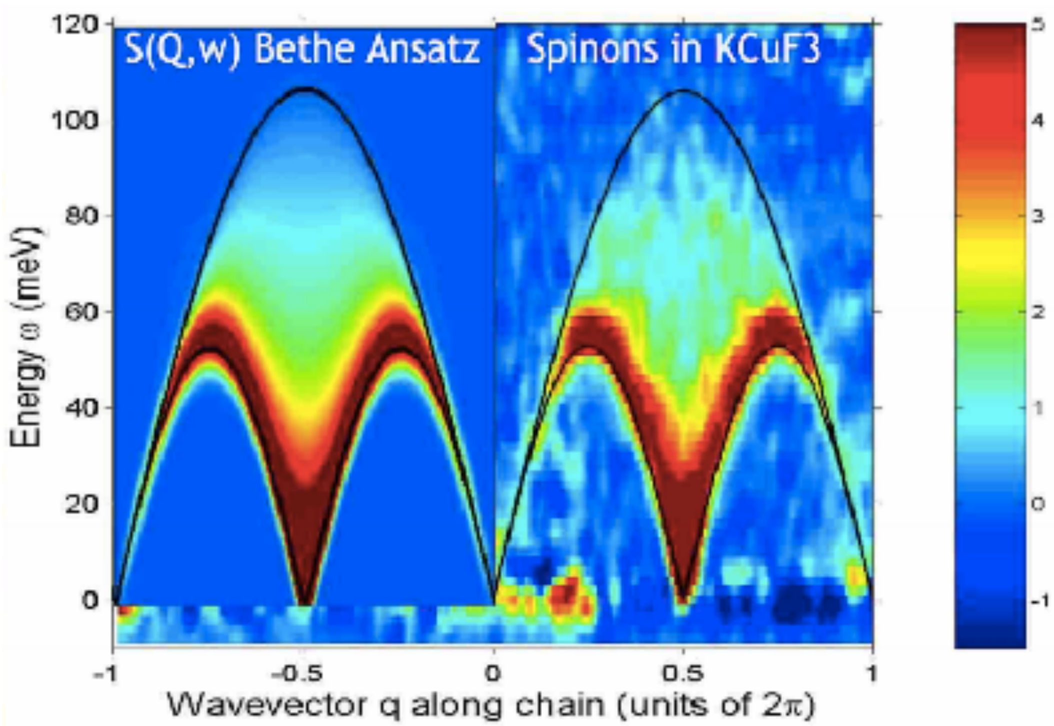
With charge conservation, PH implies half-filling. Not necessary

Symmetry: lattice translations and PH

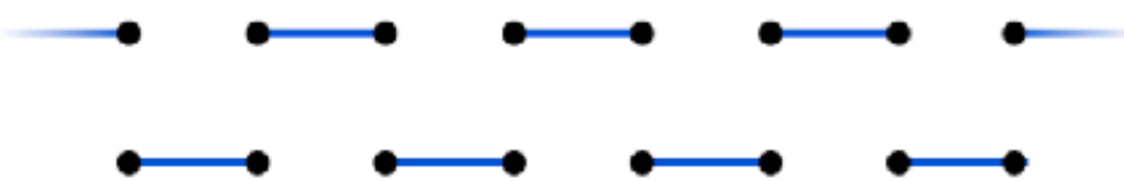
Lieb-Schultz-Mattis Theorem

S=1/2 spin chains have either degenerate GS's or vanishing gap

Assumption: short-ranged *H* with SO(3) and translation symmetries



Gapless spinons

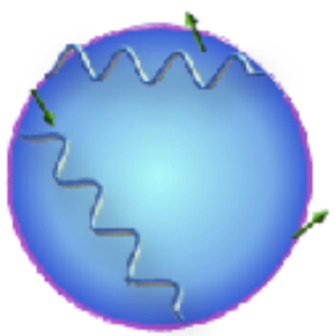


Spontaneous dimerization

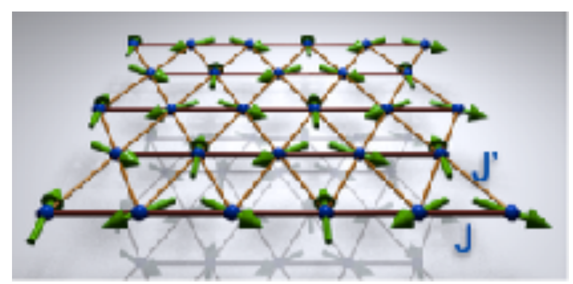
Lieb-Schultz-Mattis-Oshikawa-Hastings

Odd # of spin-1/2's per unit cell

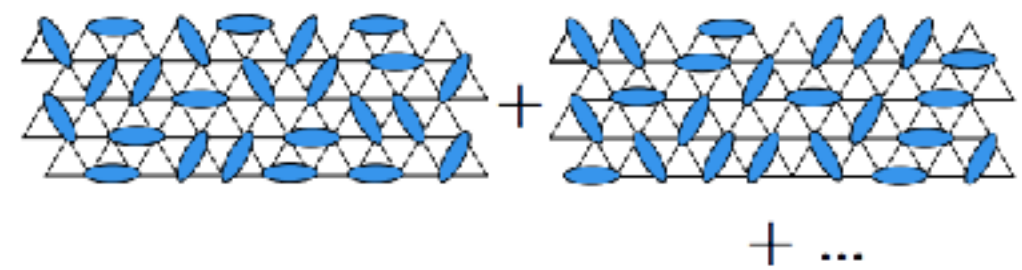
Translation and spin SO(3)



Gapless
(Dirac, spinon FS, etc.)



Symmetry breaking
(Neel, VBS, etc.)



Symmetric gapped (spin liquid)

Further generalizations: Spin-1/2 $\longrightarrow$ Kramers doublets

Translations $\longrightarrow$ space group

Watanabe et al; Po et al; …

Images from Leon Balents

Fermionic LSMHO Theorem

Spinless fermions with PHS

Gapless (e.g. free fermions)

SSB (e.g. CDW or SC)

$$V \sum_{\langle ij \rangle} (n_i - 1/2)(n_j - 1/2)$$

Symmetric gapped phases

?

Proof of Fermionic LSMHO

PHS generated by $U = \prod_i (c_i + c_i^\dagger)$

$$U c_i U^\dagger = c_i^\dagger$$

Put the system on a L_x x L_y torus with even # of sites

$$c_{x+L_x,y} = c_{x,y+L_y} = c_{x,y}$$

$$T_x U = (-1)^{L_y(L_x-1)} U T_x$$

(L_x, L_y)	Algebra	GSD
even, even	$T_x U = U T_x$	no implication
even, odd	$T_x U = -U T_x$	at least 2-fold

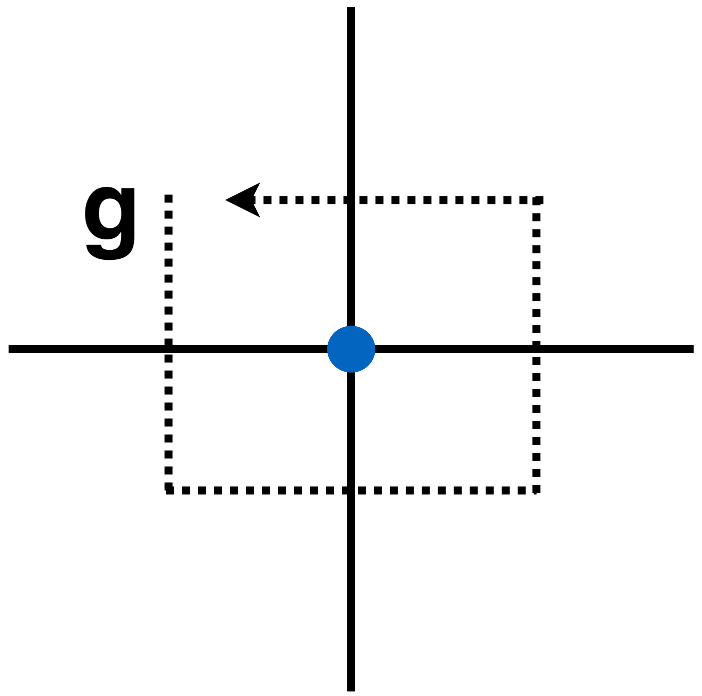
Why are there Genons?

"Flux insertion":

Drag a Z_2 flux around a unit cell

$\sim$ locally apply PHS to the unit cell

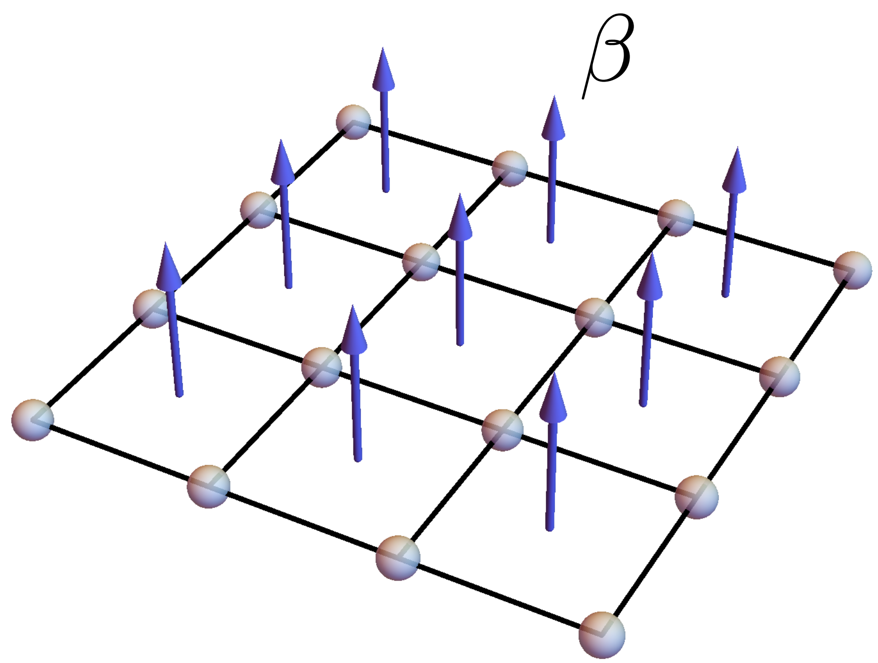
$$c_i \rightarrow c_i^\dagger, n_i \rightarrow 1 - n_i$$

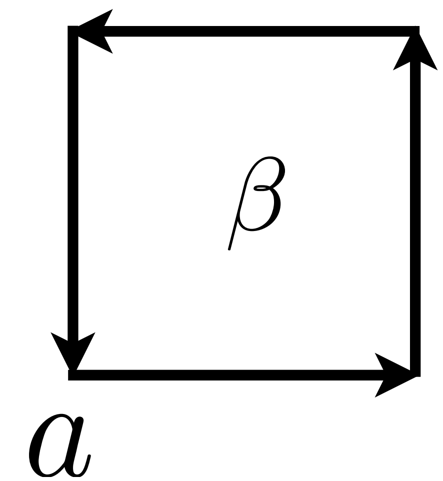


$\boxed{\text{An additional fermion appears after a } Z_2 \text{ flux moves around a unit cell}}$

Where can this fermion come from
in the low-energy effective topological theory?

Fractionalization of Translation Symmetry

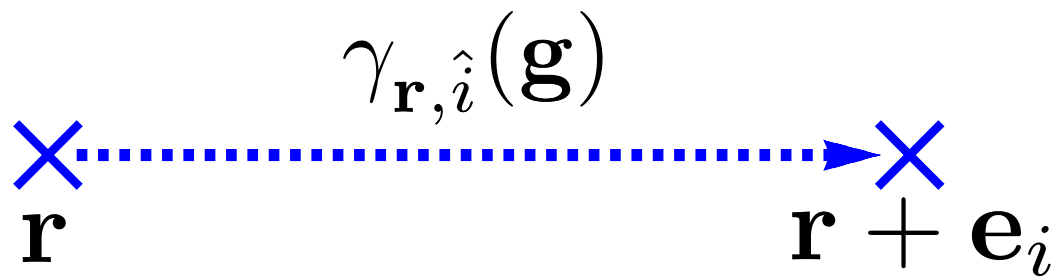




When anyons move on the lattice
they see a background (Abelian) flux

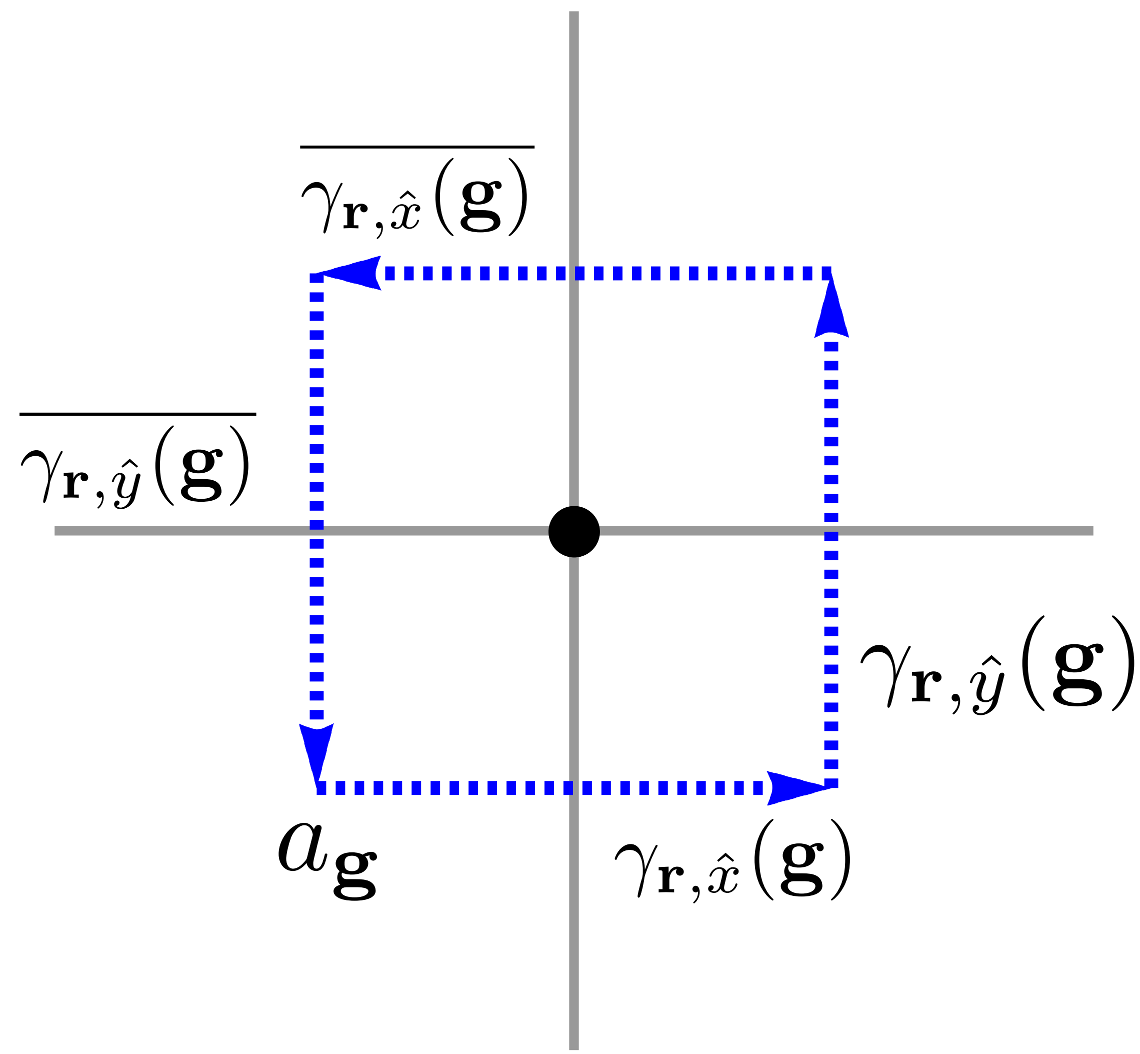
Anyonic “Spin-Orbit” Coupling

What happens to the Z_2 flux when it moves?



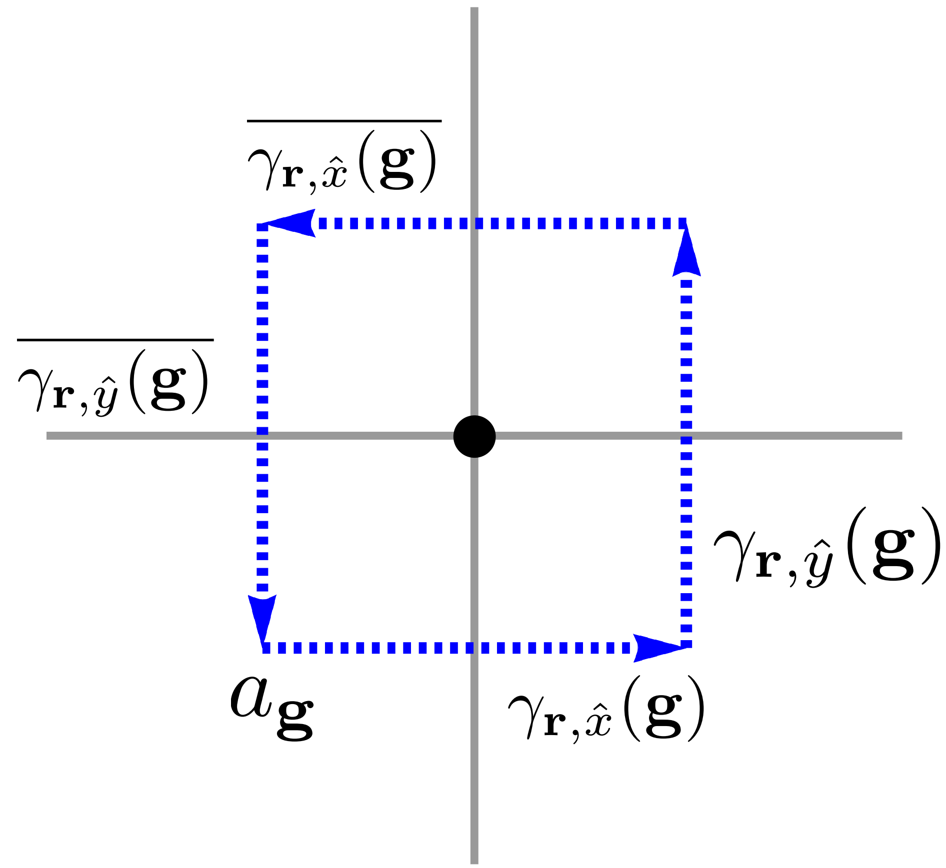
Under T_i, an Abelian anyon is attached on the flux

If anyons are **not** permuted by translations:



Necessity of Having Genons

If anyons are **not** permuted by translations:



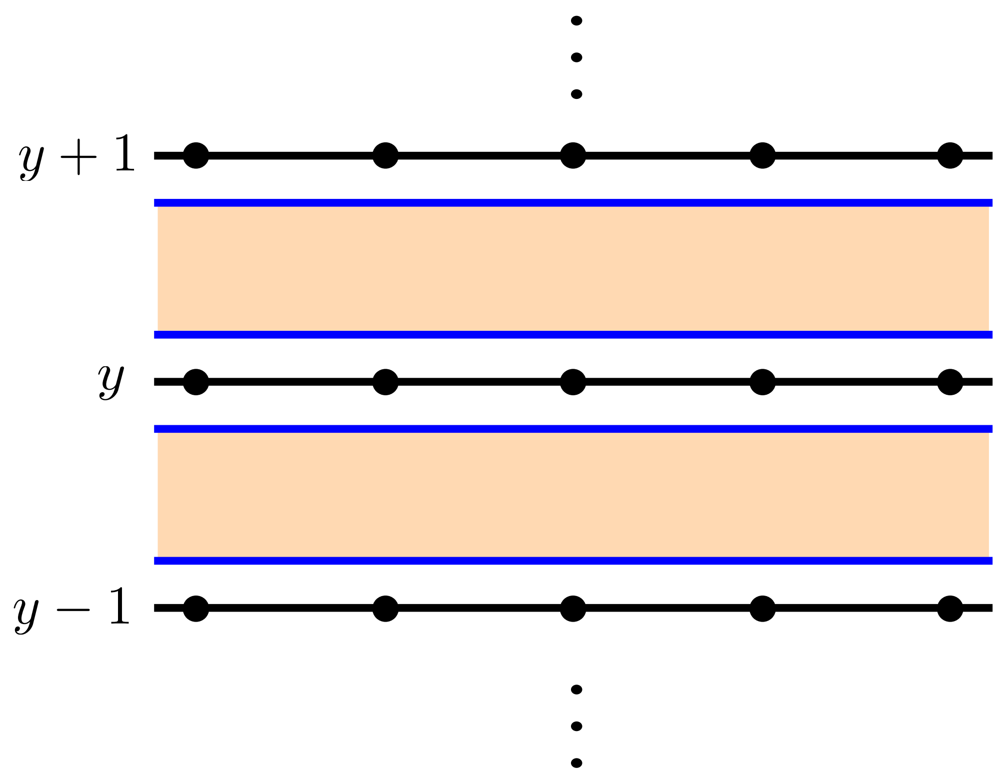
An additional fermion appears after a Z_2 flux moves around a unit cell

But the background anyon can be transformed by **g**

$$\mathbf{g}(\beta) = \beta \times f$$

Impossible because $\theta_{\mathbf{g}(\beta)} = \theta_\beta, \theta_{\beta\times f} = -\theta_\beta$

Example: Z_4 topological order



$$H_0 = \sum_y \int dx\, [\psi^\dagger_{Ry}(-i\partial_x)\psi_{Ry} + \psi^\dagger_{Ly} i\partial_x \psi_{Ly}].$$

$$\begin{aligned} H_{\text{int}} = -\int dx \Big[& U_1 \left[(\psi_R\psi_L)^2 e^{4i(\phi_+ + \phi_-)} + \text{h.c.} \right] \\ & + U_1 \left[(\psi_R^\dagger \psi_L)^2 e^{4i(\theta_+ - \theta_-)} + \text{h.c.} \right] \\ & + U_2 \cos 4(\phi_+ - \phi_-) \Big] \end{aligned}$$

Z_4 topological order generated by charge e and flux m

$$\mathbf{g} : e \to e^3, m \to m^3$$

$$T_y : e \to e, m \to m e^2 f$$

Ising dislocations

Side Note: U(1) LSMHO for Fermions

More "traditional" setting: fractional filling

Bosons at half-filling
Spins with $S^z=0$

Z_2 topological order
(e.g. Z_2 spin liquid, chiral spin liquid)

Fermions at half-filling

At least Z_4 topological order

Take-home message: topological order of spinless fermions
can be interesting!

Bultinck and Cheng

Part I: Genons in Fractional Chern Insulators

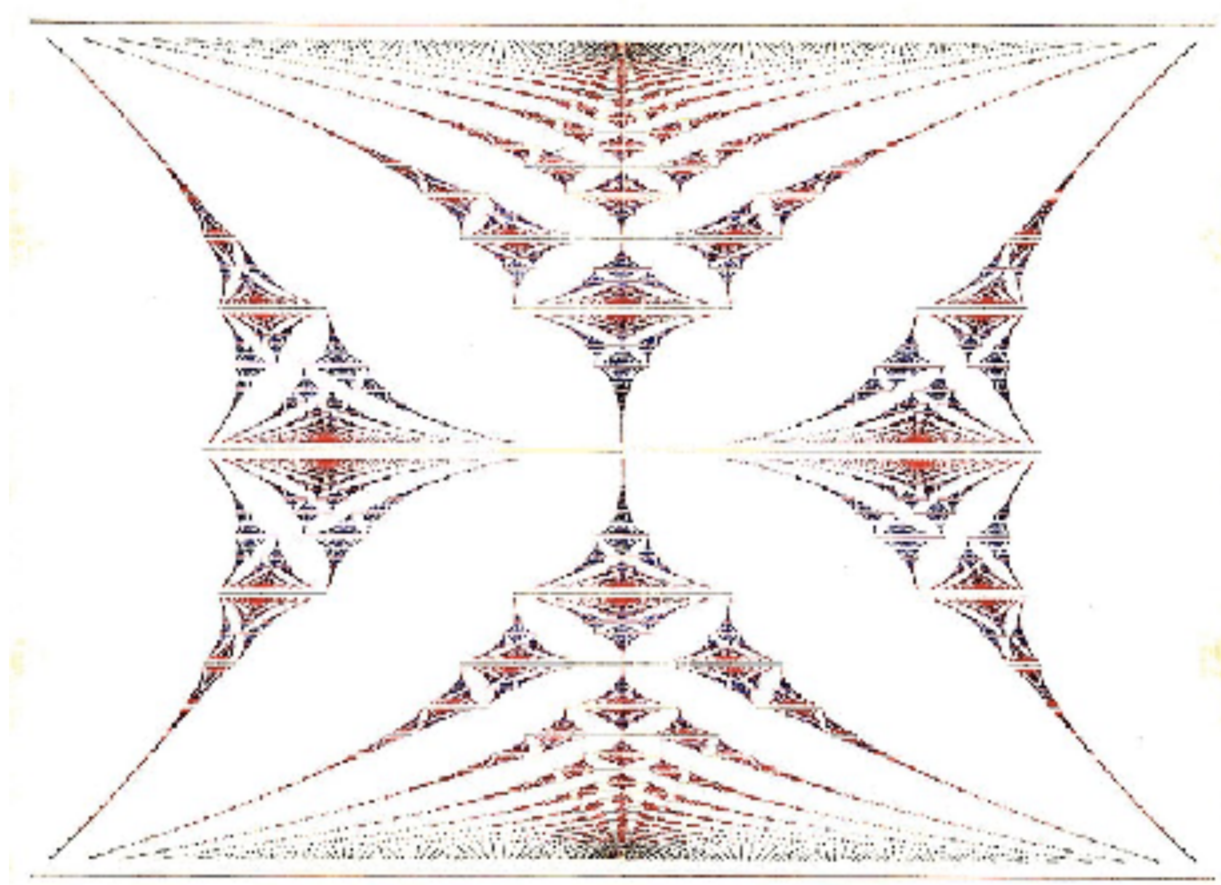
J. Lee, M. Ippoliti, MC, M. Zaletel, in preparation

Hofstadter’s Butterfly

Tight-binding electrons in a magnetic field

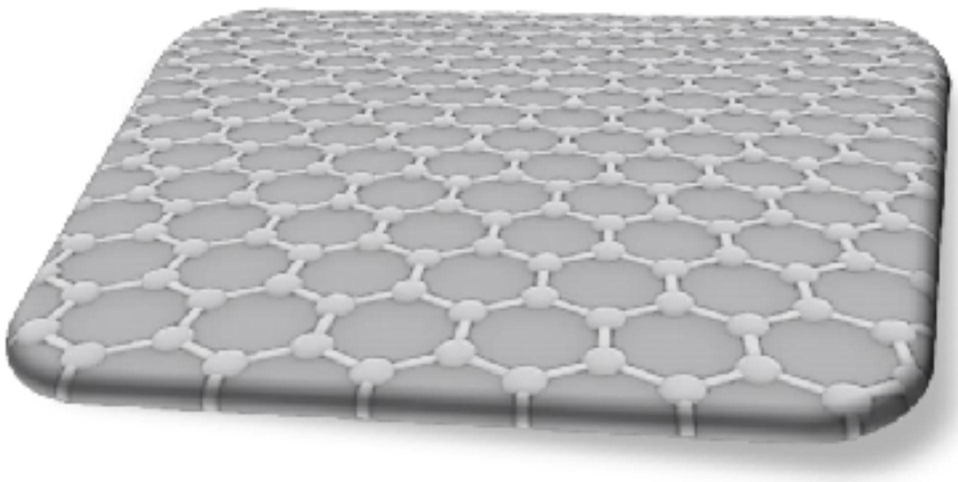
or

Landau level in a weak periodic potential

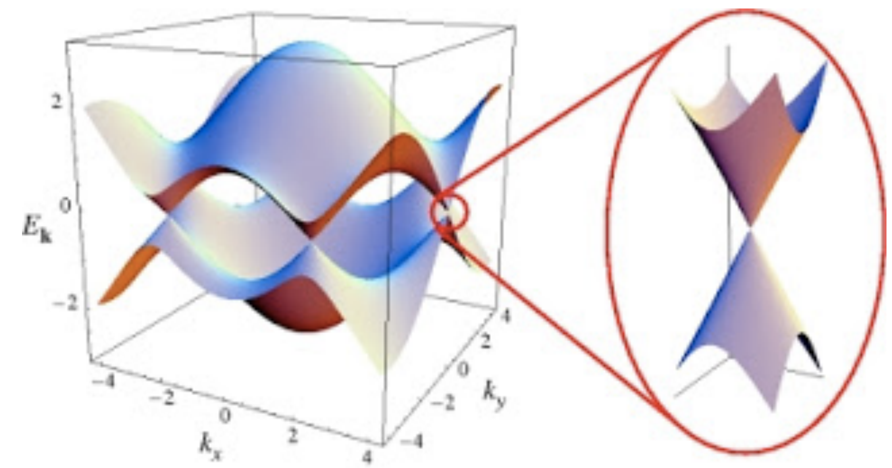


But usually unit cells are too small.

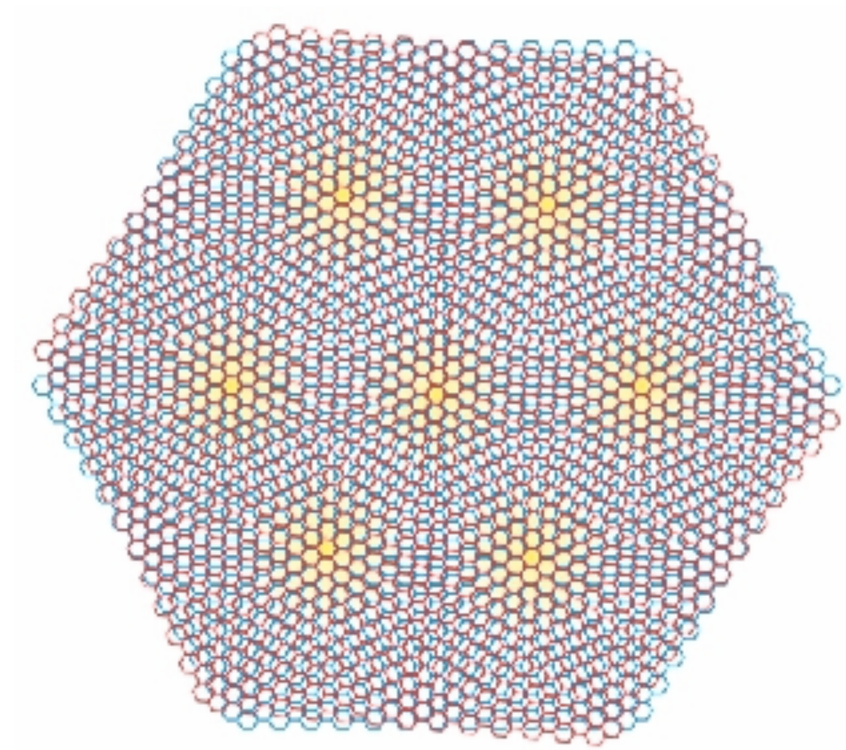
Moire Pattern in Twisted Bilayer Graphene



Honeycomb lattice



Dirac electrons



Moire pattern allows for realization of Hofstadter physics!

E. Spanton et al, arXiv:1706.06116

Quantum Hall States in Hofstadter Bands

Two filling factors: $\left\{ \begin{array}{l} \text{Electrons per flux quanta} \\ \\ \text{Electrons per site } s \end{array} \right.$

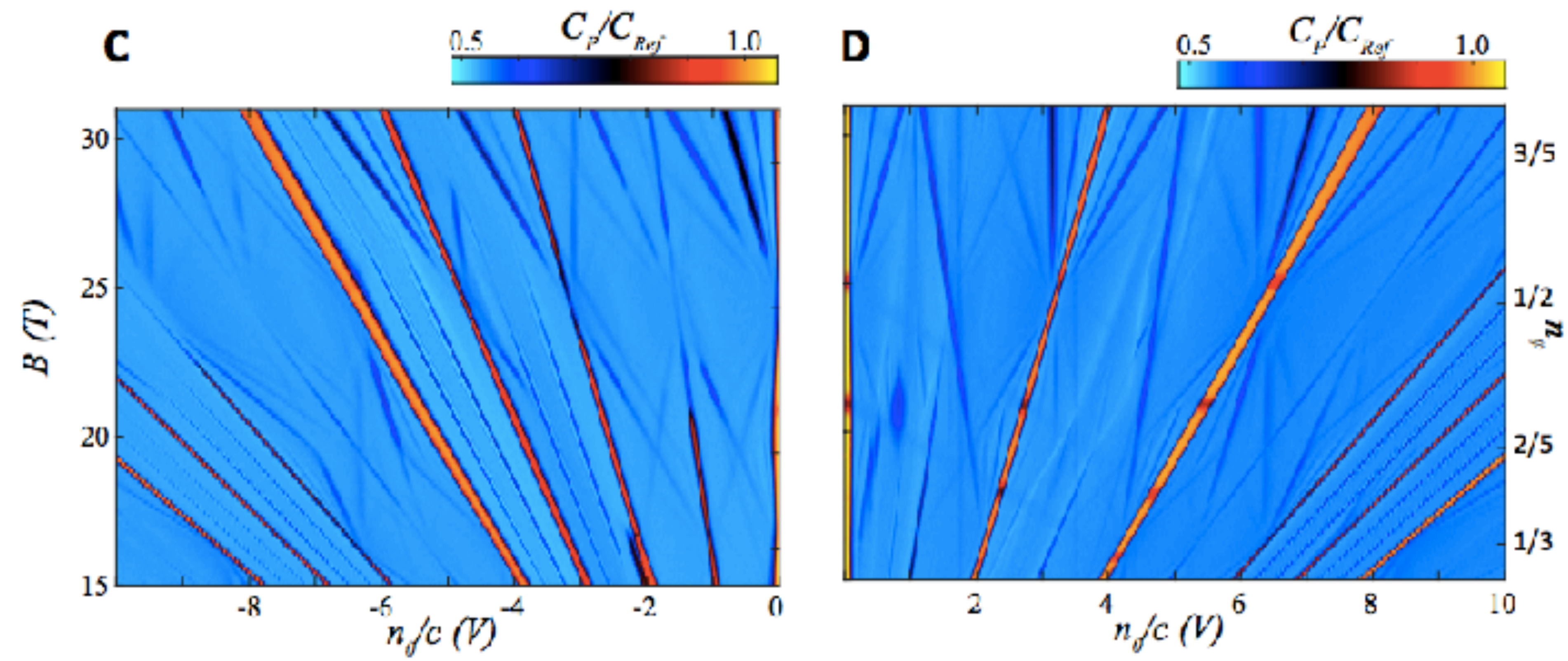
$$n_e = t\phi + s$$

Streda's formula: $t = \dfrac{\partial n_e}{\partial \phi} = \dfrac{h}{e^2}\sigma_{xy}$

Nonzero s indicates strong lattice effect

"IQH"-like states: t, s are integers

Wannier Plot

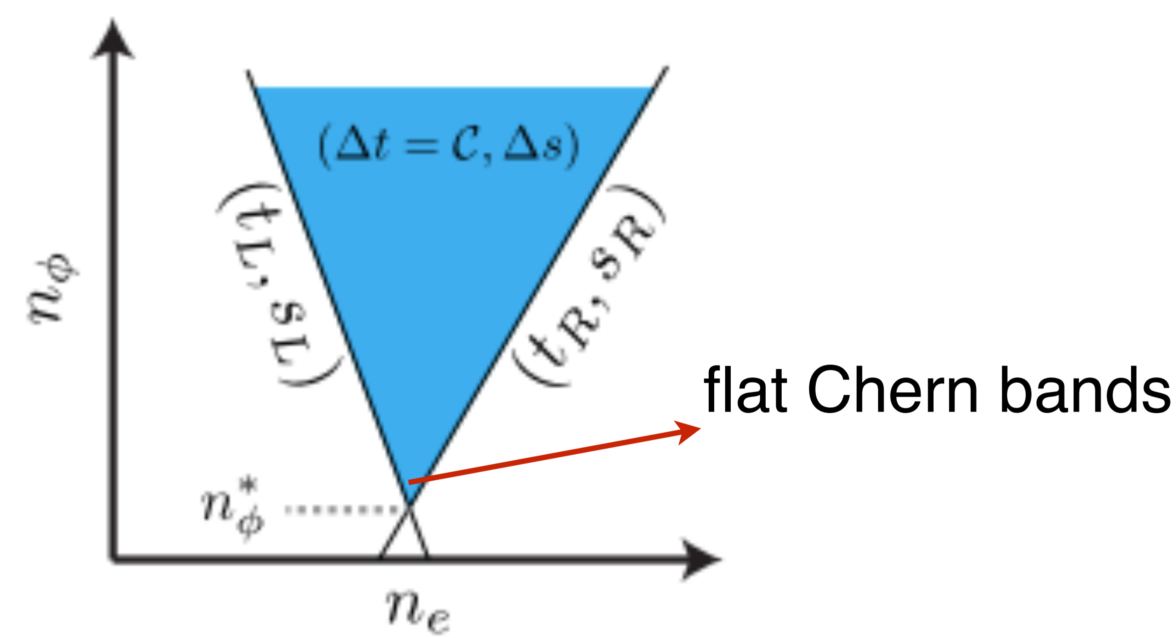


E. Spanton et al, arXiv:1706.06116

Incompressible states as lines on the *n, B* plot

CI, SBCI, FCI

What it means to
"fractionally fill" a Chern band:



Flat Chern Bands and Emergent Symmetries

We can find a (*C*, *S*) band when

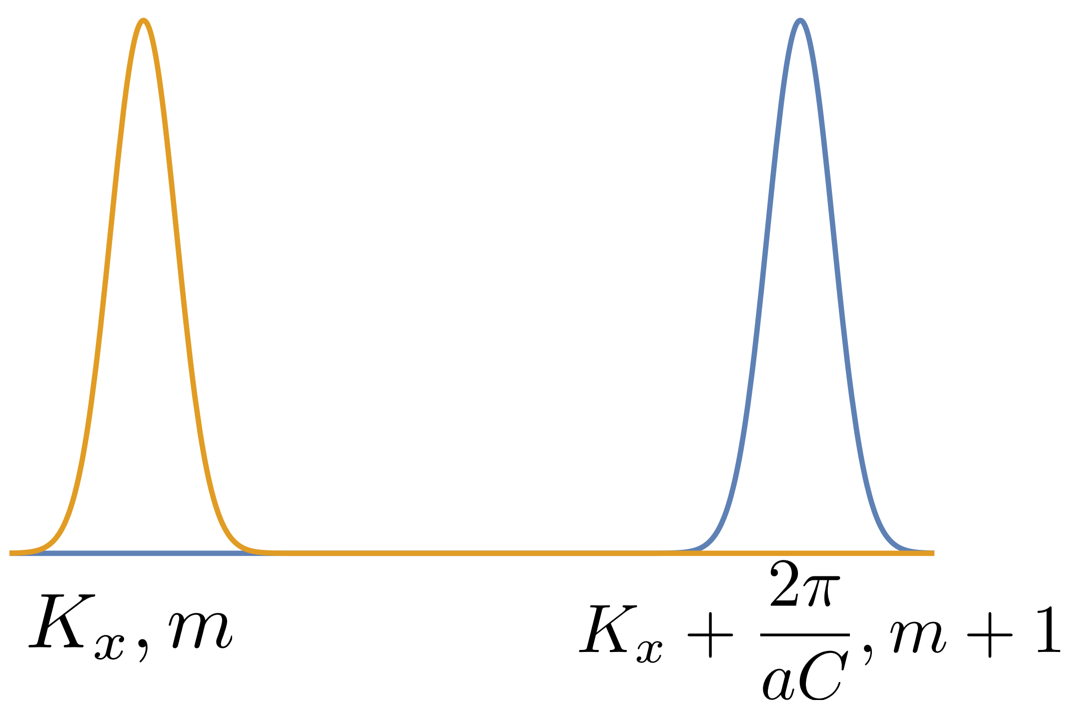
$$\phi = \frac{p}{q} = \phi_* + \bar{\phi}, \bar{\phi} = \frac{1}{qC}$$

Interested in the small $\bar{\phi}$ limit

Chern bands $\approx$ ICI continuum LLs

Form Wannier orbitals $|K_x, m\rangle$

Tunneling suppressed exponentially in the $\bar{\phi} \to 0$ limit



Flat Chern Bands and Emergent Symmetries

Magnetic translation symmetry:

$$T_x|K_x, m\rangle_{\text{shift}} = e^{iaK_x - \frac{2\pi i m}{C}}|K_x, m\rangle_{\text{shift}},$$
$$T_y|K_x, m\rangle_{\text{shift}} = e|K_x + \bar{\phi}G_x, m + S\rangle_{\text{shift}}$$

$$T_i = \tau_i T_i(a)$$

$T_i(a)$ acts on the LLs as continuum translations
τ_i acts on the layer index

They become independent in the $\bar{\phi} \to 0$ limit

$$\mathbb{Z} \times \mathbb{Z} \longrightarrow \mathbb{Z}_C \times \mathbb{Z}_C \times \mathbb{R} \times \mathbb{R}$$

$$\tau_x^C = \tau_y^C = 1, \tau_x \tau_y = e^{-\frac{2\pi i S}{C}} \tau_y \tau_x$$

Symmetry-Enriched Structure of FCIs

Two kinds of "background charge"

"vison" *v*

created by adiabatically inserting a flux quanta

"anyon per site" *a*

created by inserting one more site

$$a\tau_y(a)\tau_y^2(a)\cdots\tau_y^{C-1}(a) = v^S$$

Emergent constraint

Example: C=2 Band

Natural guess: generalized Halperin 331 states at 1/4 filling

$$\prod_{i<j}(z_i - z_j)^3 \prod_{i<j}(w_i - w_j)^3 \prod_{i,j}(z_i - w_j)$$

$$K = \begin{pmatrix} 3 & 1 \\ 1 & 3 \end{pmatrix}$$

But does the (331) have to have genons? Yes

Proof: Suppose there are no genons. Then

$$v^{-1} = a^2$$

But *v* is uniquely fixed by Hall conductance!

No solution for *a*

Summary

LSMHO theorem for spinless fermions with particle-hole symmetry

Fermionic LSMHO requires genons to exist in symmetric gapped phases

Mapping from Chern bands to multiple-layer QH

SET constraints in Hofstadter Chern bands