Characterizing Quantum Phase Transitions of Symmetry-Protected Topological Phases with Surface Critical Behavior

Long Zhang (张龙)

Kavli Institute for Theoretical Sciences, UCAS

Collaborators: Fa Wang, Chengxiang Ding, and Wenan Guo

Outline

- Introduction
  - SPT and AKLT phase
- 2D AKLT phase model
  - Model and phase diagram
  - Bulk and surface critical behavior
- Columnar model
- Summary

Symmetry-protected topological phase

- SPT is distinct from the vacuum only in the presence of certain symmetry. Gu and Wen, PRB (2009)
- Properties
  - Gapped bulk state without anyon excitations
  - Gapless or degenerate surface state
  - Surface states transform projectively under the symmetry

Haldane phase

- Spin-1 Heisenberg chain Haldane, PRL (1983)

$$H = \sum_i \mathbf{S}_i \cdot \mathbf{S}_{i+1}$$

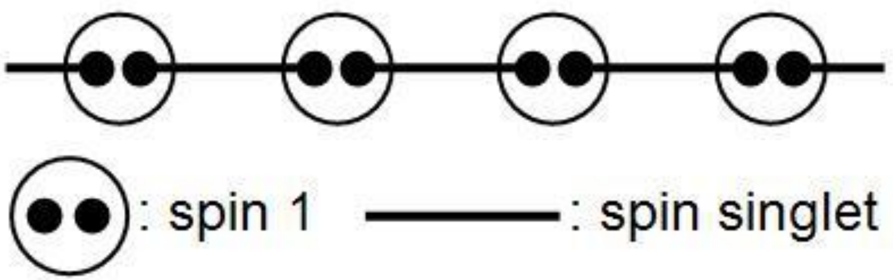
Haldane phase

- Spin-1 AKLT chain Affleck, Kennedy, Lieb, and Tasaki, PRL (1987)

$$H = \sum_i \mathbf{S}_i \cdot \mathbf{S}_{i+1} + \frac{1}{3}(\mathbf{S}_i \cdot \mathbf{S}_{i+1})^2$$

Haldane phase

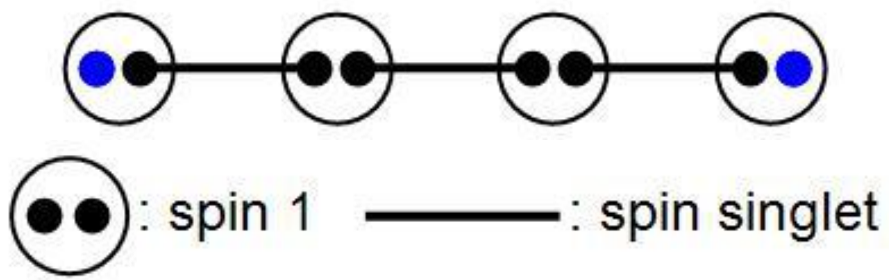
- Spin-1 AKLT chain Affleck, Kennedy, Lieb, and Tasaki, PRL (1987)



  – Gapped, nondegenerate ground state in p.b.c.

Haldane phase

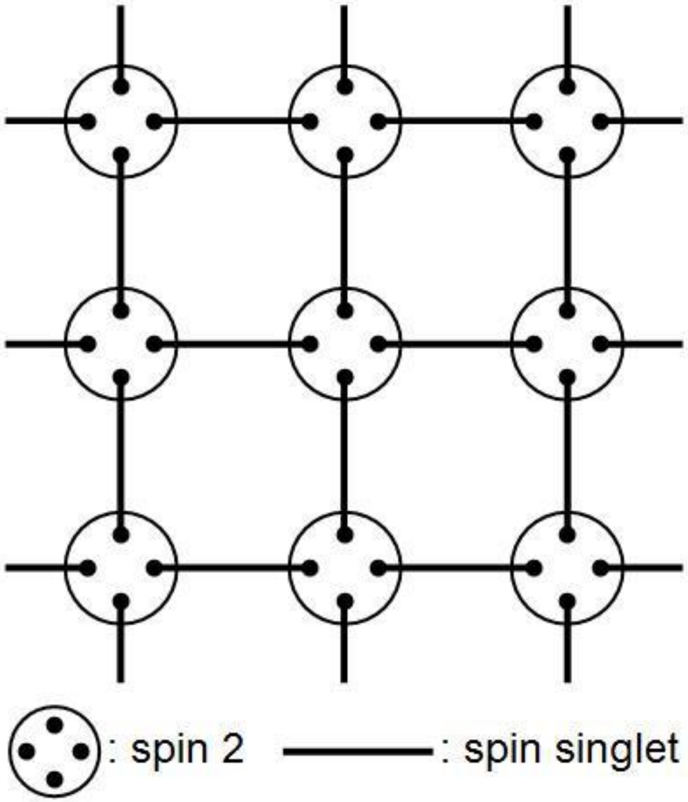
- Spin-1 AKLT chain Affleck, Kennedy, Lieb, and Tasaki, PRL (1987)



  - Gapped, nondegenerate ground state in p.b.c.
  - Dangling bonds in o.b.c.: gapless surface states
  - Surface states transform as spin 1/2
  - Protected by spin rotation symmetry Gu and Wen, PRB (2009)

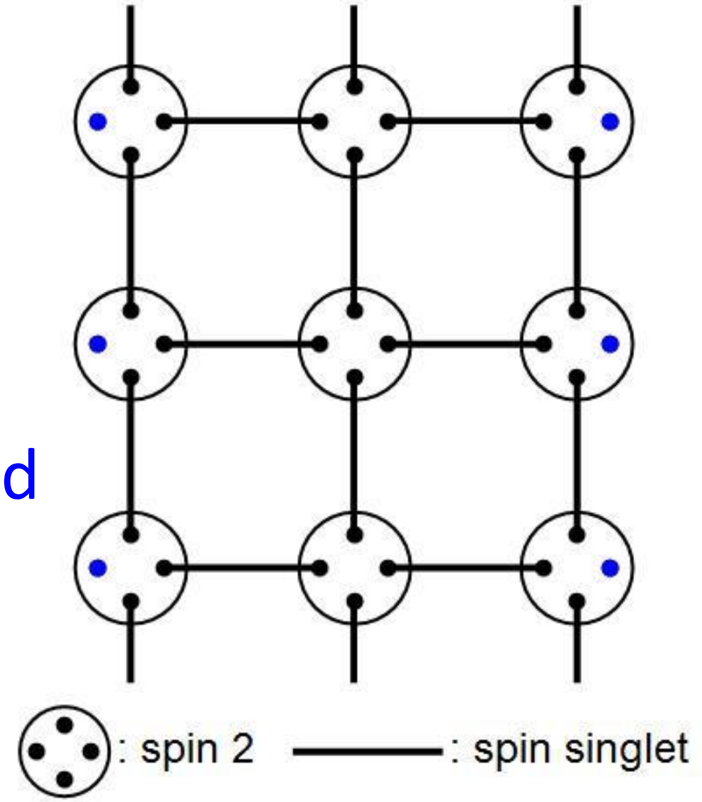
2D AKLT state

- 2D square lattice Affleck, Kennedy, Lieb, and Tasaki, PRL (1987)
  – Gapped ground state
  – Nondegenerate in p.b.c.



2D AKLT state

- 2D square lattice Affleck, Kennedy, Lieb, and Tasaki, PRL (1987)
  - Gapped ground state
  - Nondegenerate in p.b.c.
  - Spin-1/2 AF chain in o.b.c.
  - Gapless! Lieb et al, Ann. Phys. (1961)
  - Protected by spin rotation and translation symmetries Chen et al, PRB (2011)

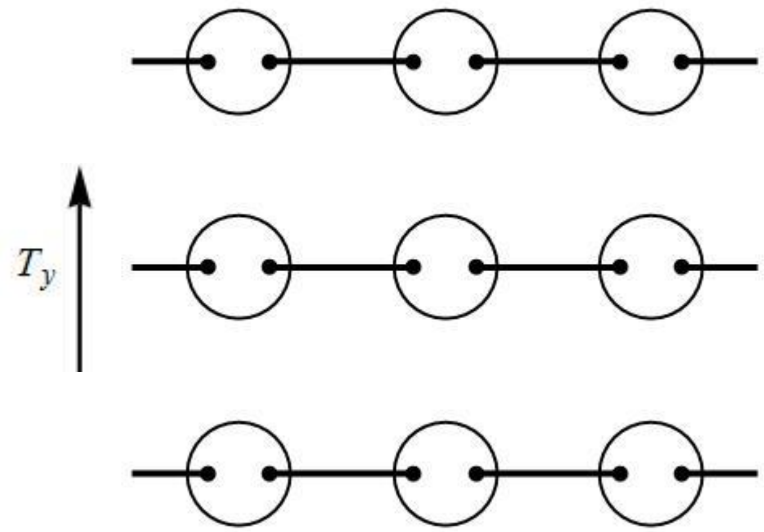


Relation of 1D and 2D AKLT

- 2D AKLT from stacked 1D AKLT, weak SPT

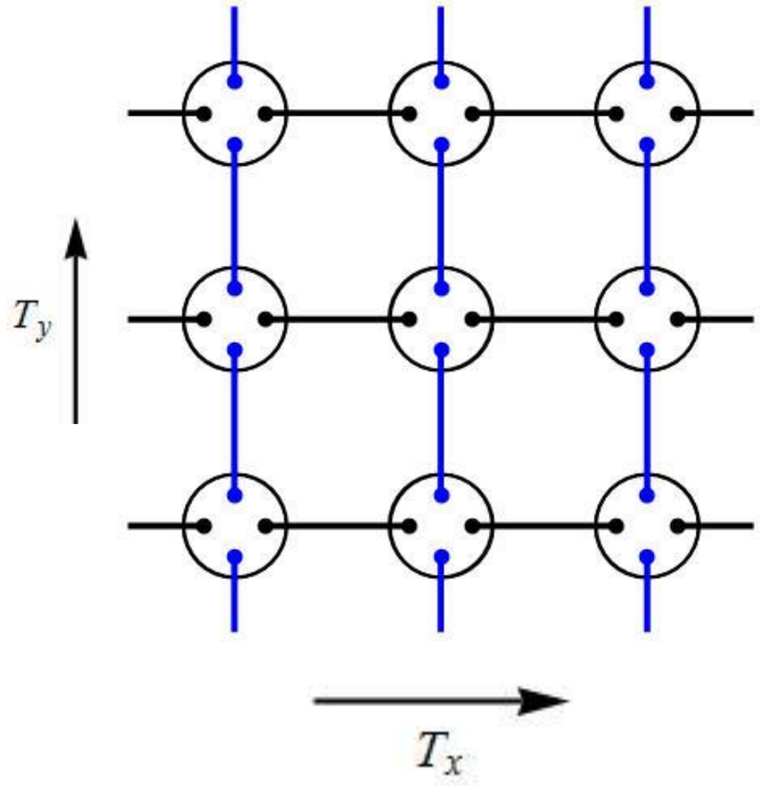
Relation of 1D and 2D AKLT

- 2D AKLT from stacked 1D AKLT, weak SPT
  - Protected by spin rotation and translation



Relation of 1D and 2D AKLT

- 2D AKLT from stacked 1D AKLT, weak SPT
  - Protected by spin rotation and translation

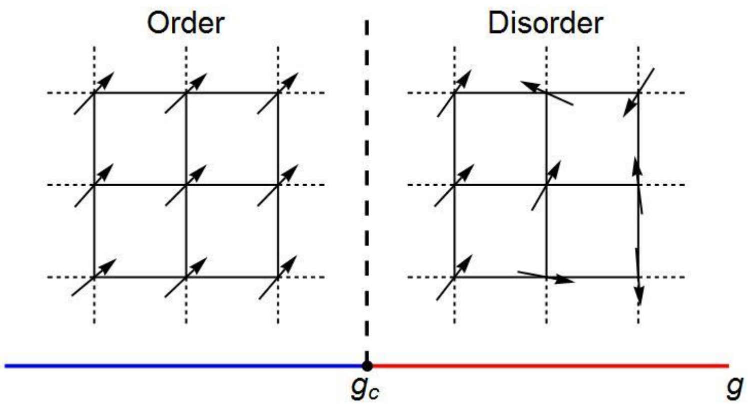


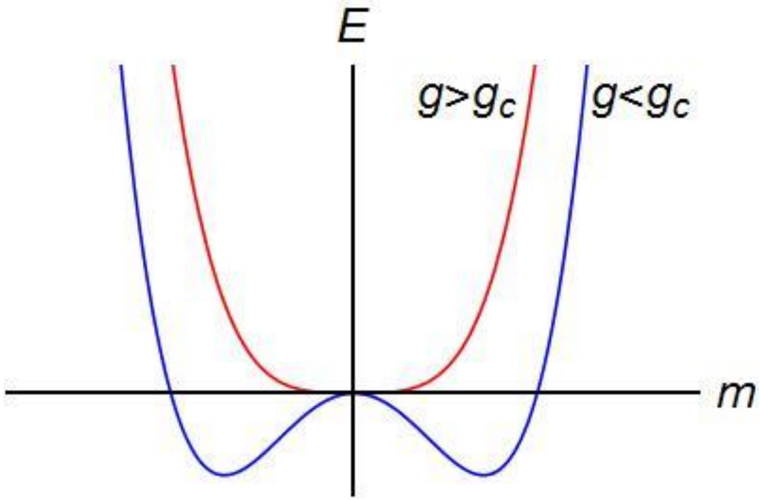
The physical consequence of breaking the symmetry that protects the SPT?

Phase transition and universality

- Spontaneous symmetry breaking
- Local order parameter
- Critical exponents are determined by symmetry and spatial dimension.

$$\xi \sim |g - g_c|^{-\nu}$$
$$m \sim (g_c - g)^{\beta}$$
$$C(r) \sim r^{-(d-2+\eta)}$$



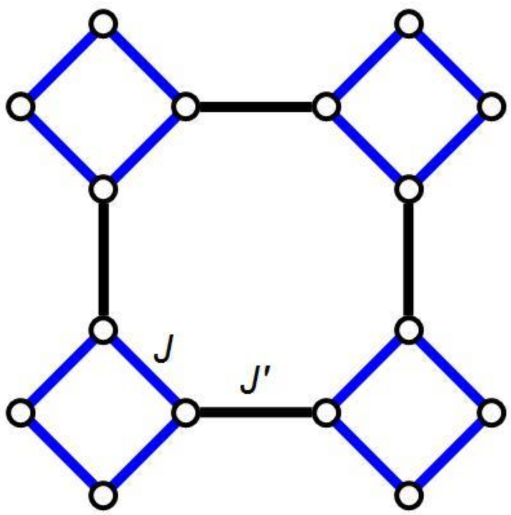


A simple model

- Heisenberg model on decorated square lattice

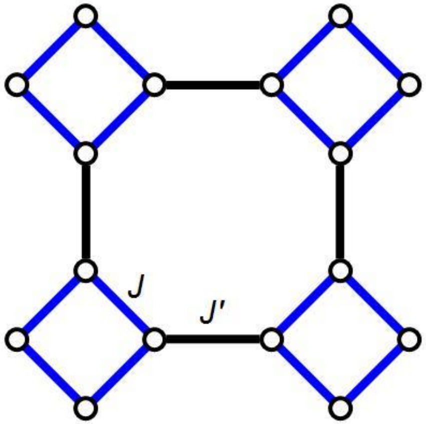
$$H = J \sum_{\langle ij \rangle} \mathbf{S}_i \cdot \mathbf{S}_j + J' \sum_{\langle ij \rangle'} \mathbf{S}_i \cdot \mathbf{S}_j,$$

- We consider AF inter-UC coupling, $J' = 1$.

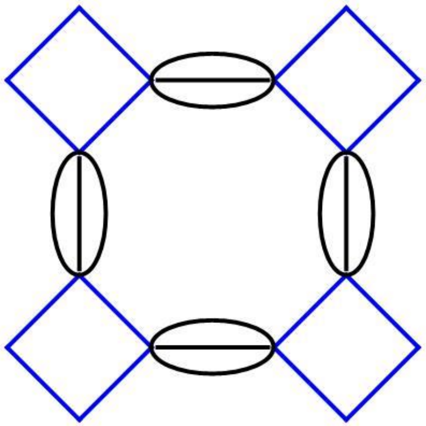


L. Zhang and F. Wang, PRL **118**, 087201 (2017).

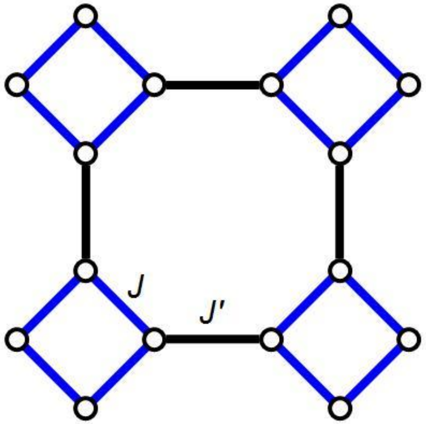
Quantum phase diagram



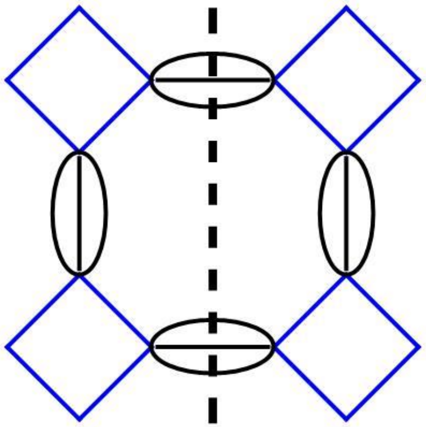
- $J = 0$: AKLT phase
  - Disjoint dimers



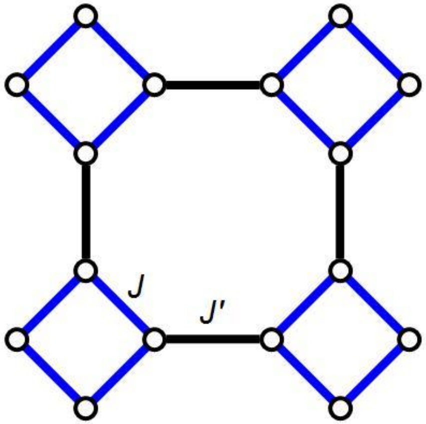
Quantum phase diagram



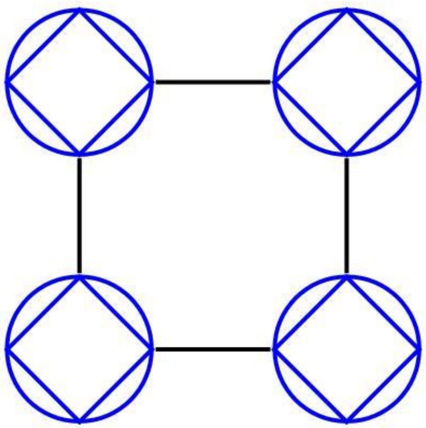
- $J = 0$: AKLT phase
  - Disjoint dimers
  - Free surface: dangling bonds
  - Spin-1/2 Heisenberg chain, gapless



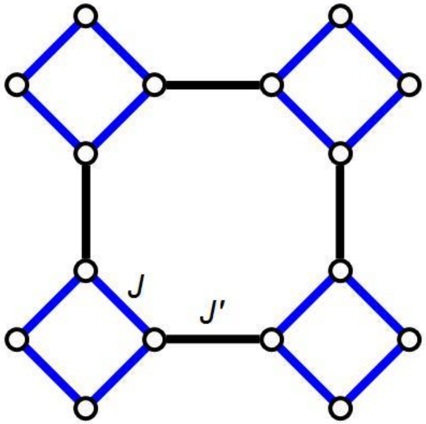
Quantum phase diagram



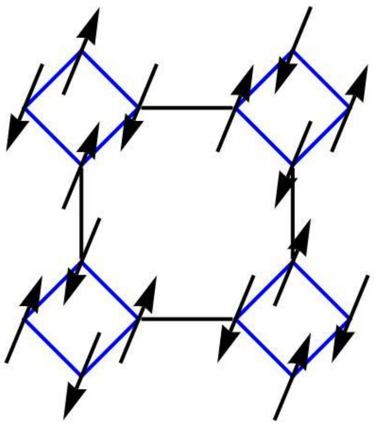
- $J = 0$: AKLT phase
  - Disjoint dimers
- $J = +\infty$: trivial phase
  - Disjoint plaquettes



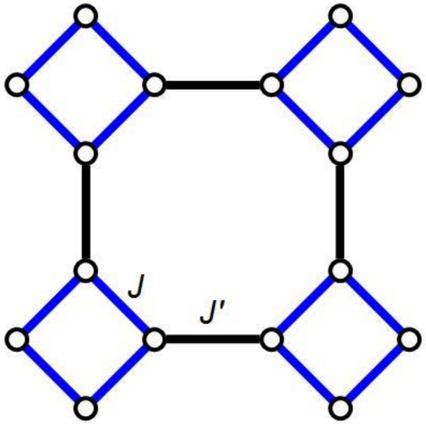
Quantum phase diagram



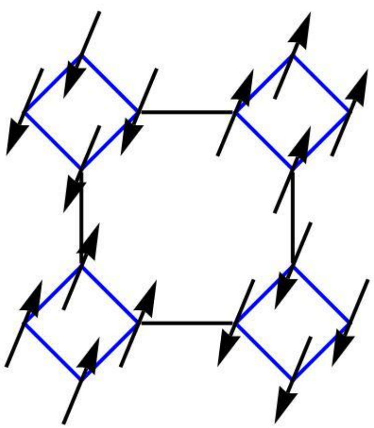
- $J = 0$: AKLT phase
  - Disjoint dimers
- $J = +\infty$: trivial phase
  - Disjoint plaquettes
- $J = 1$: $S = 1/2$ Neel order
  - CaV_4O_9 lattice Troyer et al, PRL (1996)



Quantum phase diagram

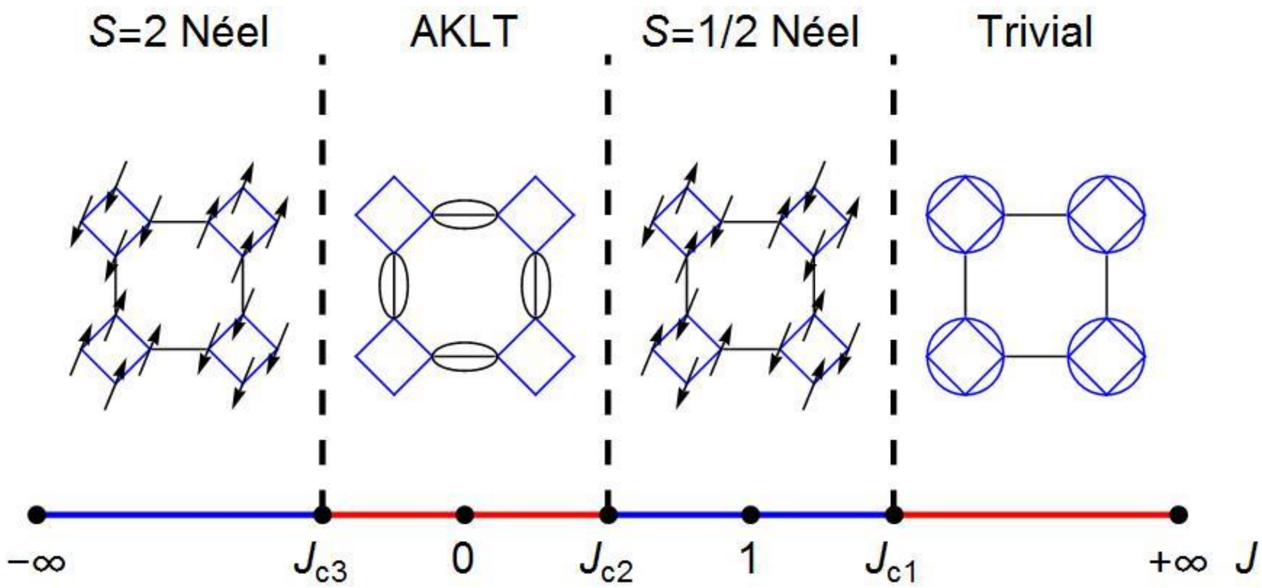


- $J = 0$: AKLT phase
  - Disjoint dimers
- $J = +\infty$: trivial phase
  - Disjoint plaquettes
- $J = 1$: $S = 1/2$ Neel order
- $J = -\infty$: $S = 2$ Neel order
  - AF coupled $S = 2$ clusters



Quantum phase diagram

- Three quantum critical points
  - Disorder to Neel order
  - Spin rotational symmetry breaking



Bulk QCP universality

- J_{c1}: trivial to Neel order
- J_{c2} and J_{c3}: AKLT to Neel order
- All consistent with 3D O(3) universality class dictated by Landau theory.

	J_c	z	ν	η	β
J_{c1}	1.064382(13)	1.0008(16)	0.7060(13)	0.0357(13)	0.3663(8)
J_{c2}	0.603520(10)	1.001(5)	0.7052(9)	0.031(4)	0.3642(13)
J_{c3}	−0.934251(11)	0.9999(13)	0.7052(15)	0.0365(10)	0.3659(9)
3D O(3)	–	–	0.7073(35)	0.0355(25)	0.3662(25)

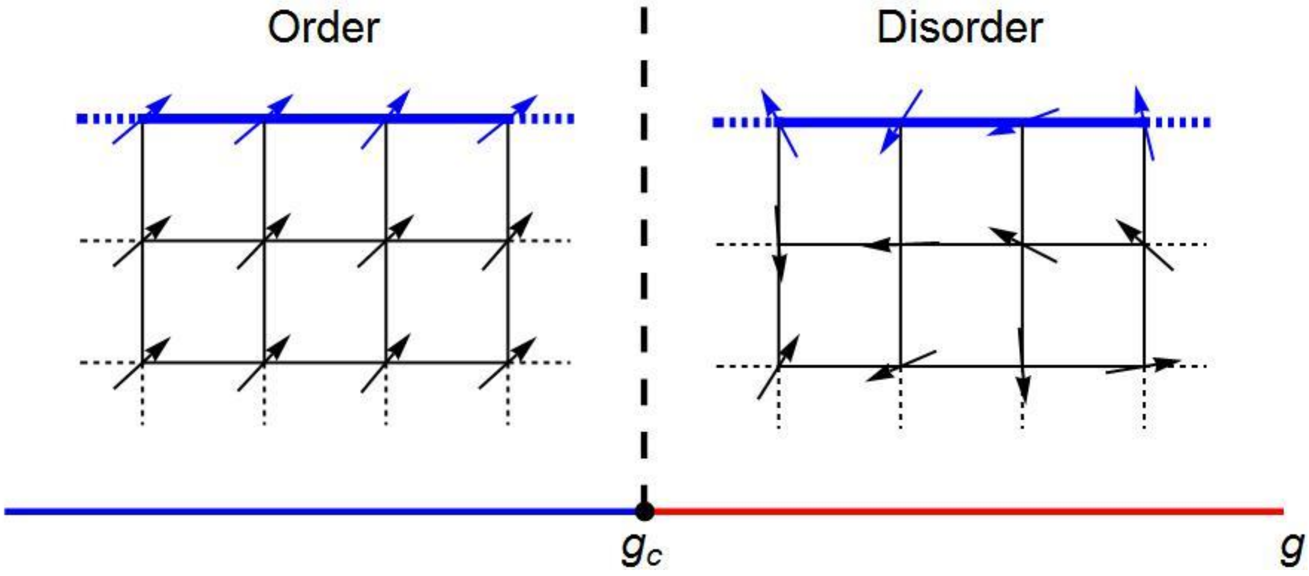
3D O(3): Guida and Zinn-Justin, J. Phys. A 31, 8103 (1998)

L. Zhang and F. Wang, PRL **118**, 087201 (2017).

What is the physical consequence of SPT?
What if gapless surface is formed?

Surface critical behavior

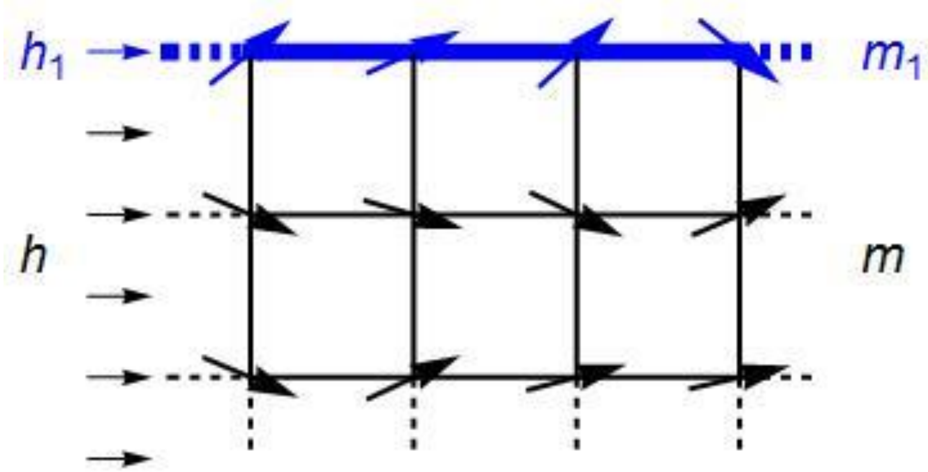
- Across bulk phase transition, long range order is induced on surfaces.
- Surface singularities at bulk critical point
- Intimately related to the bulk QCP.



Surface critical exponents

- Surface susceptibility

$$\chi_{1,1} = \partial m_1/\partial h_1 \sim L^{-(d+z-1-2y_{h1})}$$



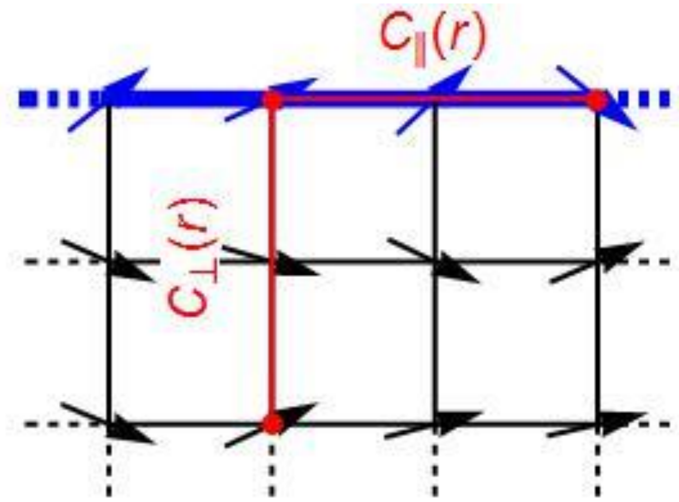
Surface critical exponents

- Surface susceptibility

$$\chi_{1,1} = \partial m_1/\partial h_1 \sim L^{-(d+z-1-2y_{h1})}$$

- Spin correlation functions

$$C_{\parallel}(r) \sim r^{-(d+z-2+\eta_{\parallel})}, \qquad C_{\perp}(r) \sim r^{-(d+z-2+\eta_{\perp})}$$



Surface critical exponents

- Surface susceptibility

$$\chi_{1,1} = \partial m_1 / \partial h_1 \sim L^{-(d+z-1-2y_{h1})}$$

- Spin correlation functions

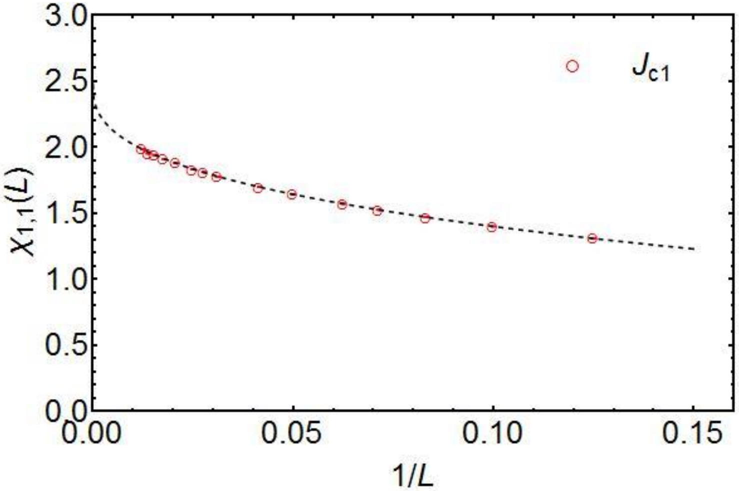
$$C_{\|}(r) \sim r^{-(d+z-2+\eta_{\|})}, \qquad C_{\perp}(r) \sim r^{-(d+z-2+\eta_{\perp})}$$

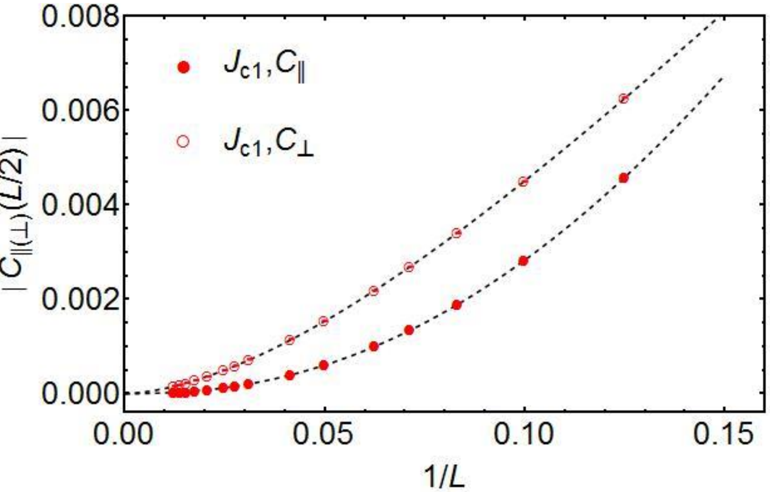
- Scaling relations Barber, PRB (1973); Lubensky and Rubin, PRB (1975)

$$1 - \eta_{\|} = -(d + z - 1 - 2y_{h_1}),$$
$$2\eta_{\perp} = \eta_{\|} + \eta.$$

J_{c1}: trivial phase to Neel order

- Surface susceptibility
  $\chi_{1,1}(L) \sim c + aL^{-(2-2y_{h1})}$
- Surface correlations
  $C_{\parallel,\perp}(L/2) \sim L^{-(1+\eta_{\parallel,\perp})}$
- Same as 3D classical O(3) model: universality





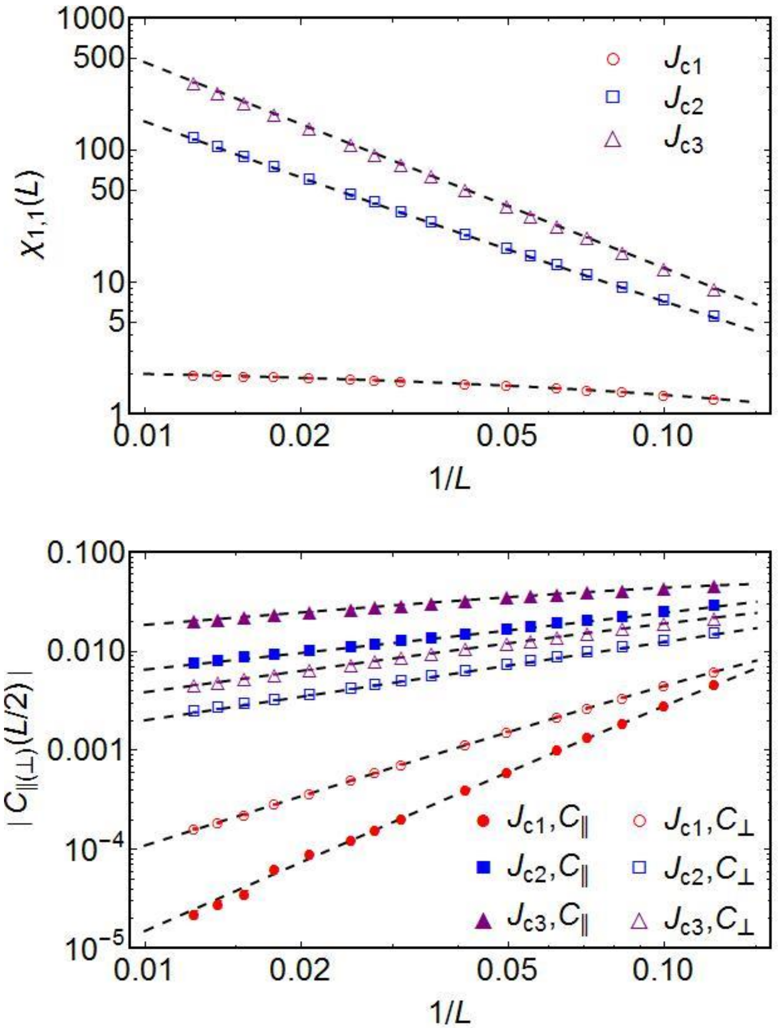
	y_{h_1}	$\eta_{\parallel}$	$\eta_{\perp}$
J_{c1}	0.810(20)	1.327(25)	0.680(8)
3D classical Heisenberg	0.813(2)		

L. Zhang and F. Wang, PRL **118**, 087201 (2017).

$J_{c2,3}$: AKLT to Neel order

- Qualitatively different from J_{c1}: "special"
- New universality classes of surface criticality due to gapless surface state

	y_{h_1}	$\eta_\parallel$	$\eta_\perp$
J_{c1}	0.810(20)	1.327(25)	0.680(8)
J_{c2}	1.7276(14)	−0.449(5)	−0.2090(15)
J_{c3}	1.7802(16)	−0.561(4)	−0.2707(24)
3D classical Heisenberg	0.813(2)		

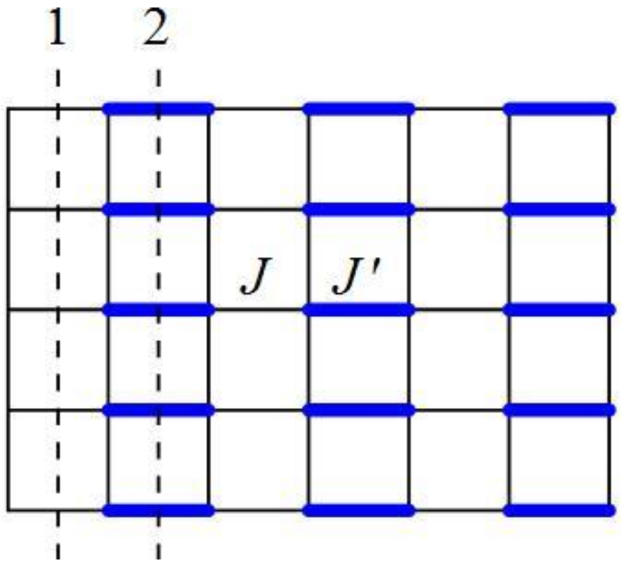


L. Zhang and F. Wang, PRL **118**, 087201 (2017).

Is this surface universality class generic for SPT phase transitions?

Columnar model

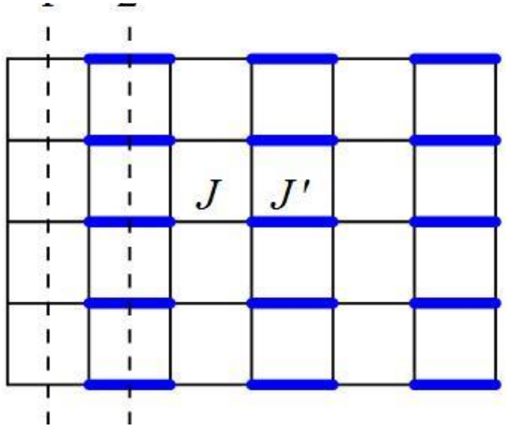
- Bulk dimer-Neel QCP: 3D O(3) class
- Surface configurations
  - Cut 1: trivial surface
  - Cut 2: dangling bonds, gapless surface

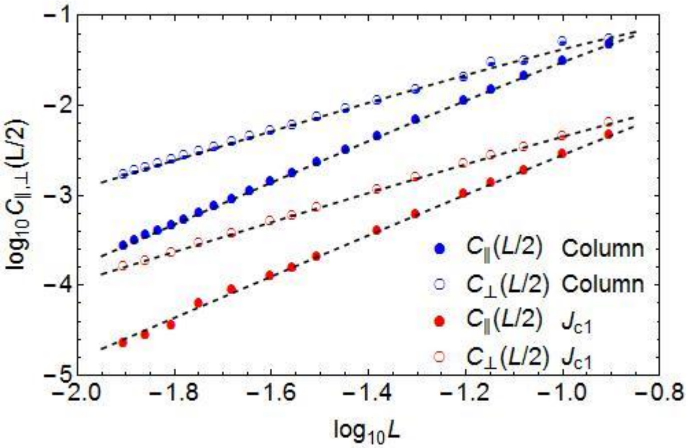


Trivial surface: ordinary transition

- Bulk QCP + trivial surface = ordinary 3D O(3)

Class	Model	y_{h1}	$\eta_{\parallel}$	$\eta_{\perp}$
Ord.	Column, cut-1	0.840(17)	1.387(4)	0.67(6)
	Stagger, cut-1	0.830(11)	1.340(21)	0.682(2)
	Deco.sq., J_{c1} [8]	0.810(20)	1.327(25)	0.680(8)
	3D classical [5]	0.813(2)		
	Field theory [4]	0.846	1.307	0.664



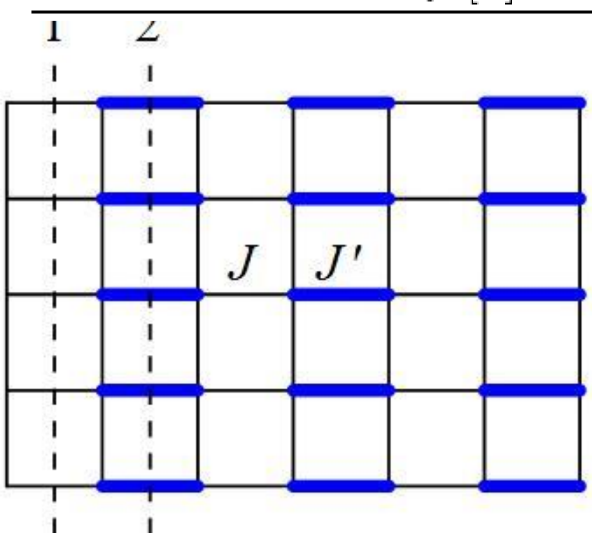


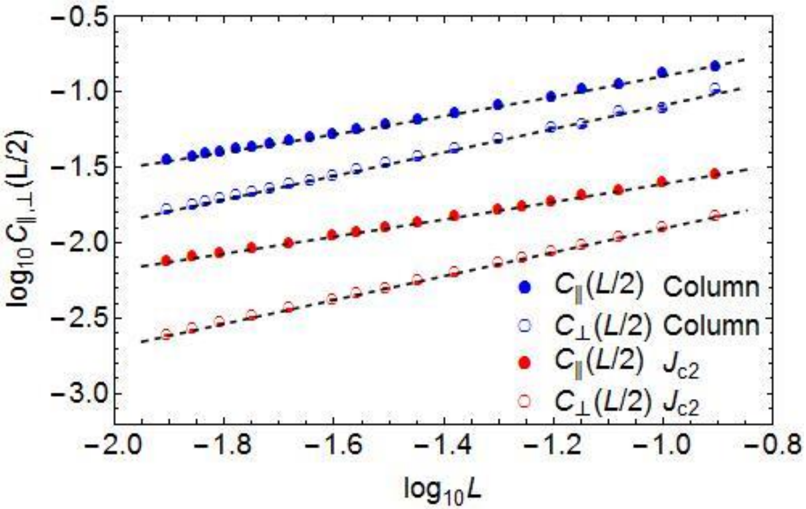
C. Ding, L. Zhang, and W. Guo, arXiv:1801.10035

SPT surface: special transition

- Bulk QCP + gapless surface = special 3D O(3)

Class	Model	y_{h1}	$\eta_{\parallel}$	$\eta_{\perp}$
Sp.	Column, cut-2	1.7339(12)	−0.445(15)	−0.218(8)
	Deco.sq., J_{c2} [8]	1.7276(14)	−0.449(5)	−0.2090(15)
	Field theory [4]	1.723	−0.445	−0.212

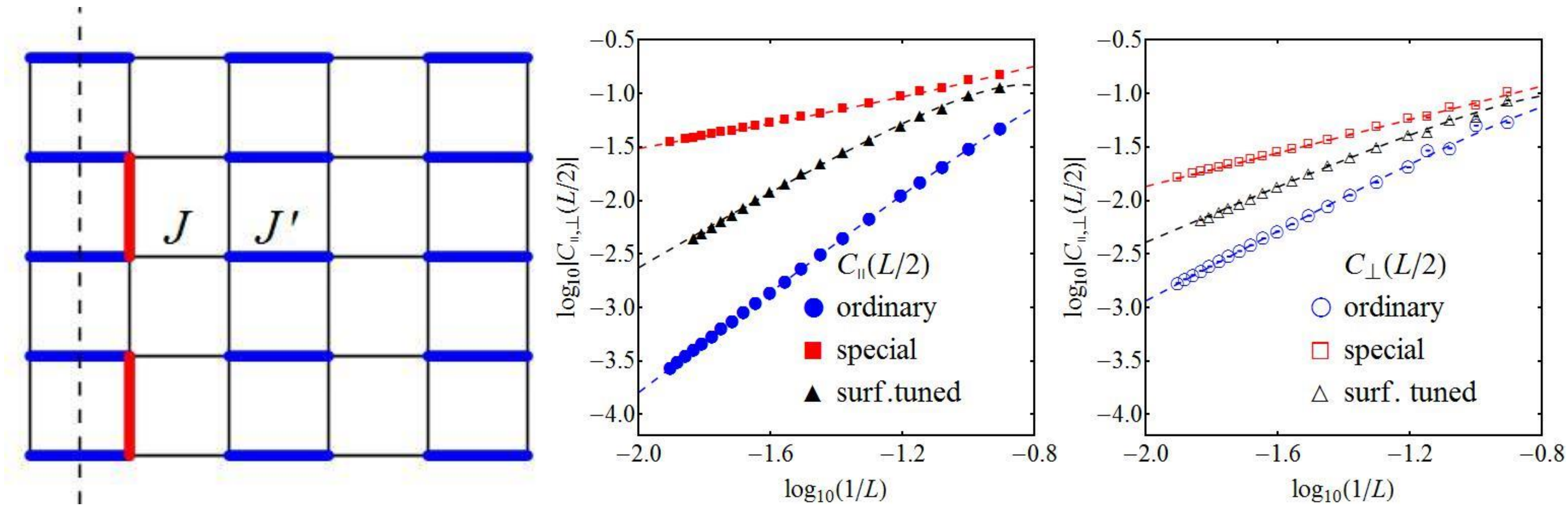




C. Ding, L. Zhang, and W. Guo, arXiv:1801.10035

Breaking translation: crossover

- AKLT: spin rotation and translation
- Breaking translation, surface state is gone.
- Surface critical behavior crosses over from special to ordinary class.



C. Ding and L. Zhang, to appear

Summary

- SPT does not change the bulk symmetry breaking universality class.
- SPT transition is characterized a new universality class of surface critical behavior.
  - Interplay of gapless surface and critical bulk states

Thank you for your attention!