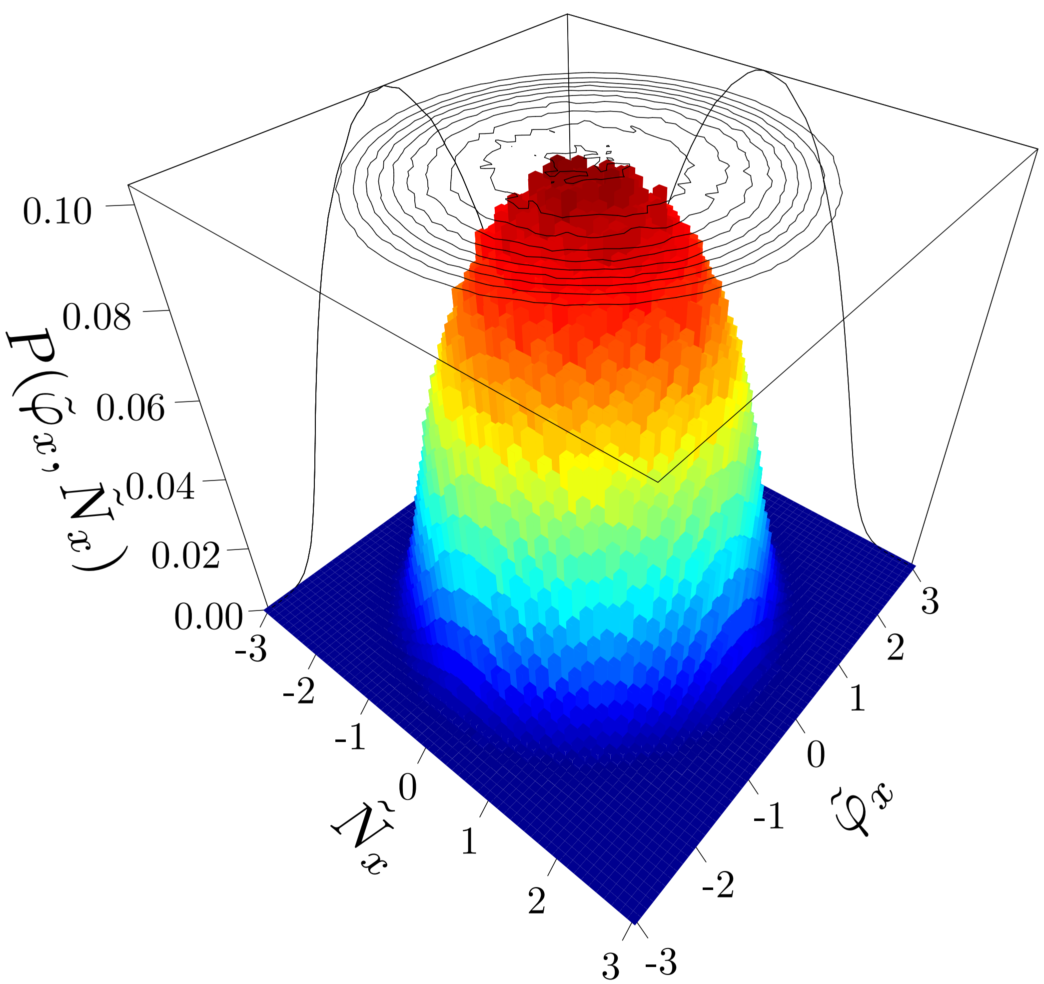
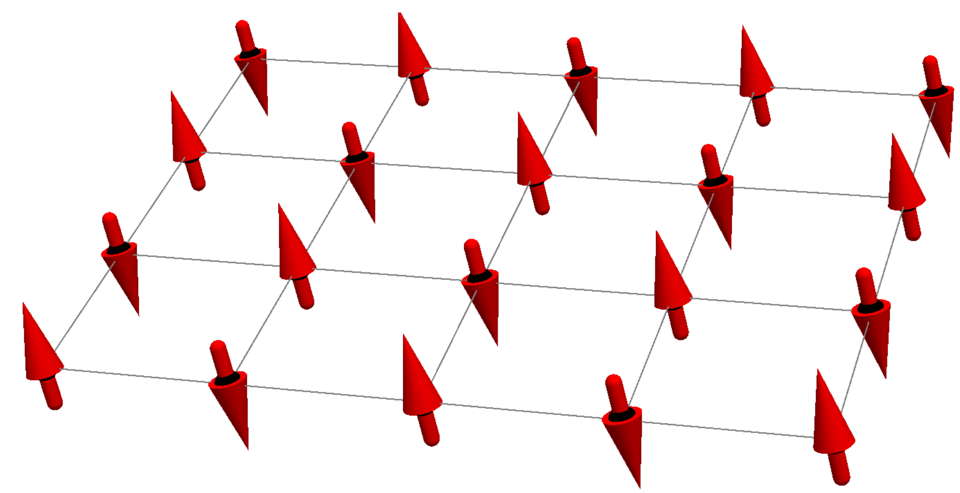
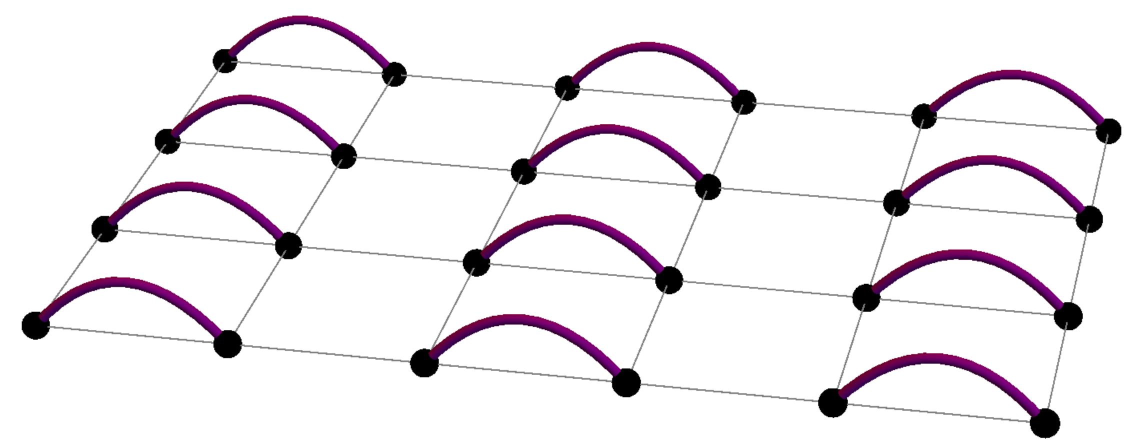
Emergent SO(5) symmetry and dualities at deconfined critical points







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Beijing, July 2017

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Emergent SO(5) symmetry at the Neel-VBS transition, PRL ’15
DQC, scaling violations, and classical loop models, PRX ’15

G. J. Sreejith
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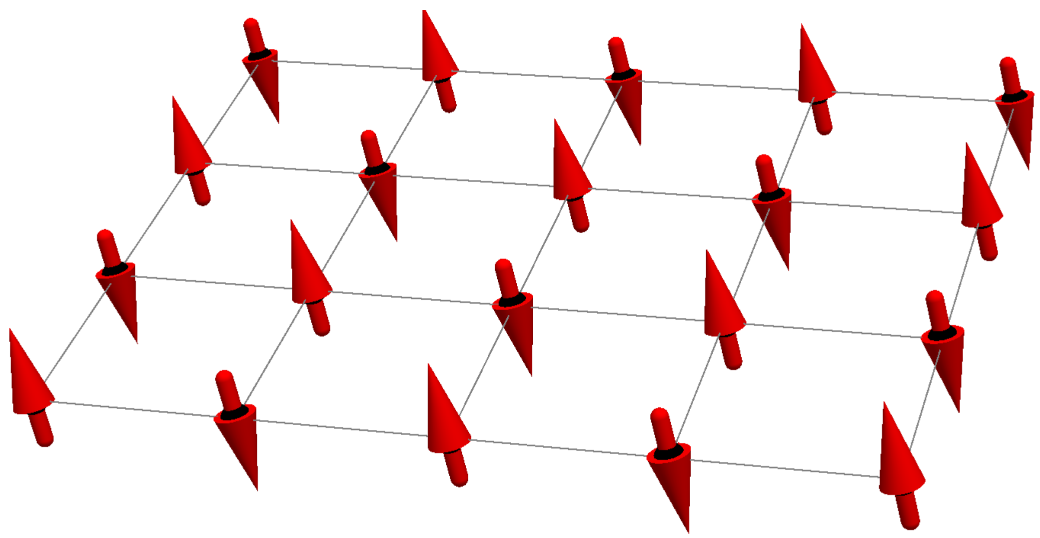
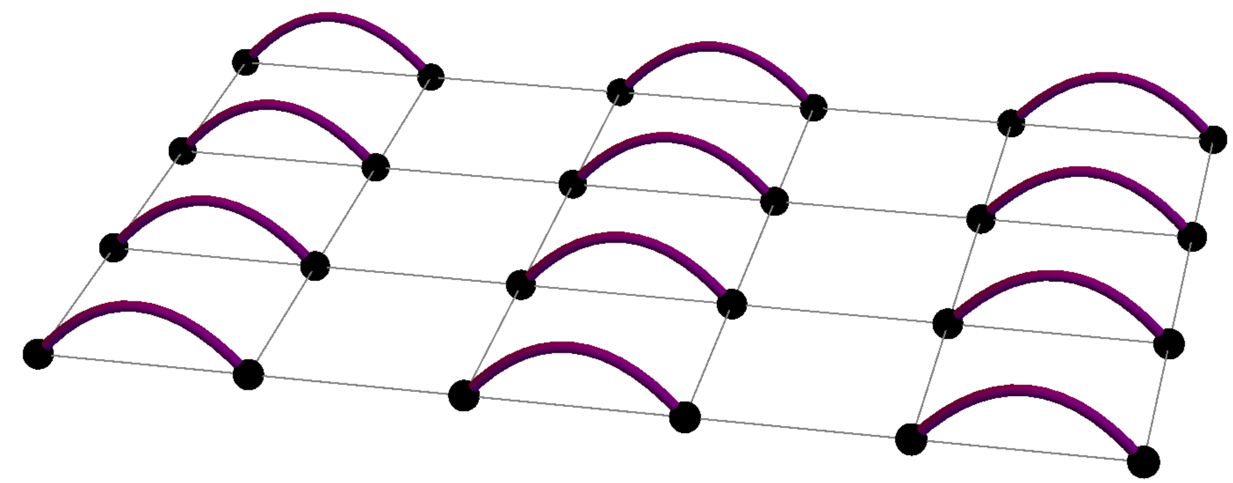
(MIT)

Deconfined quantum critical points: symmetries & dualities, ’17
(PRX, to appear)

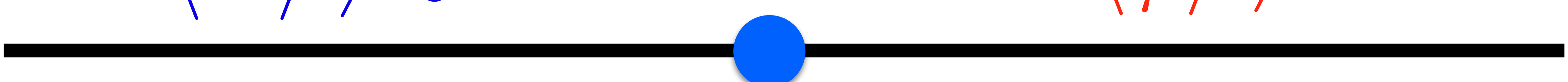
Spin-1/2s on square lattice: $H = J\sum \vec{S}_i.\vec{S}_j + \ldots$





$\langle \vec{N} \rangle \neq 0$

$\langle \vec{\varphi} \rangle \neq 0$



Neel order

DCP

VBS order

$\vec{N} = (N_x, N_y, N_z)$

$\vec{\varphi} = (\varphi_x, \varphi_y)$

Drive transition with e.g. 4-spin interaction (JQ model, Sandvik '07)

Senthil, Vishwanath, Balents, Sachdev, Fisher '04

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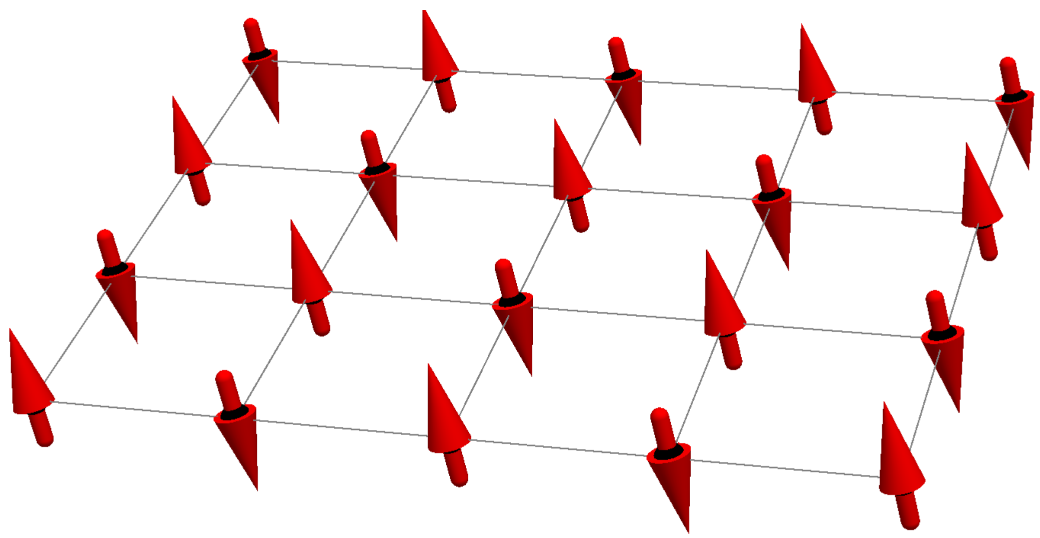

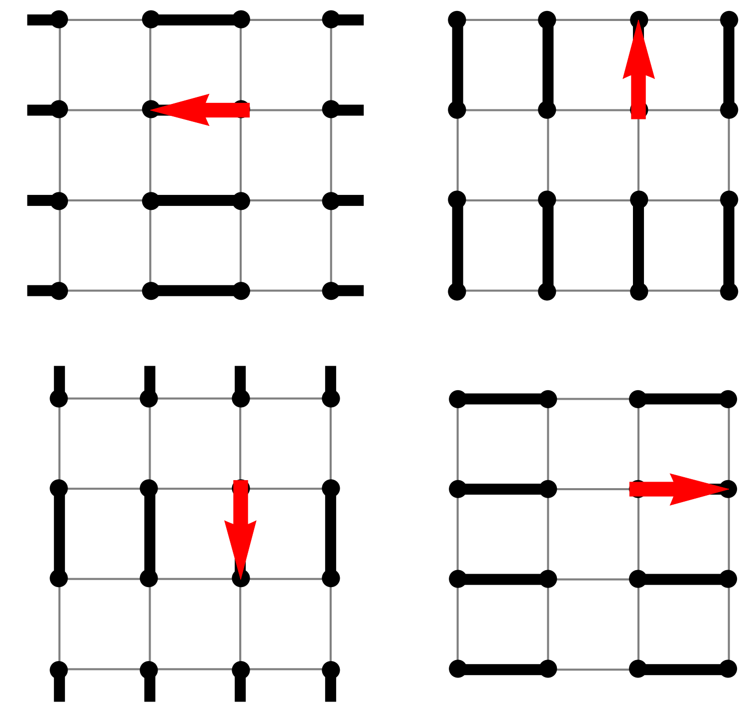

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Neel order DCP VBS order

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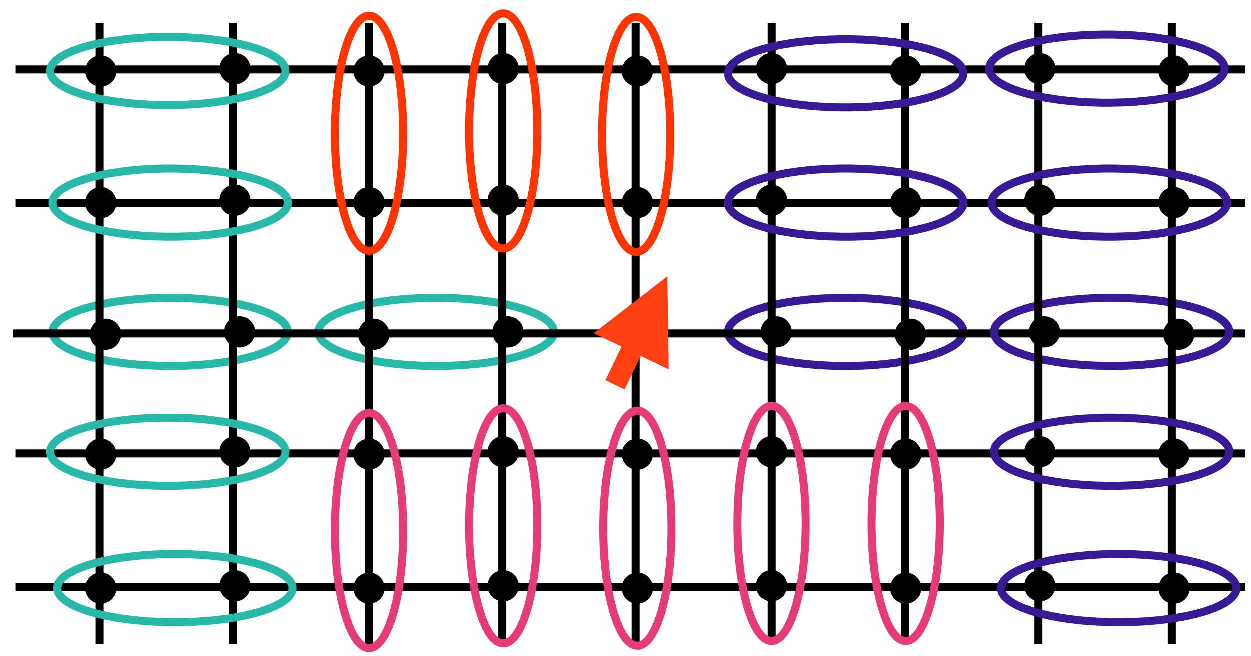
Neel-VBS transition

Landau-Ginsburg theory fails to capture quantum #s of topological defects.

Vortex in VBS order parameter $\vec{\varphi}$:



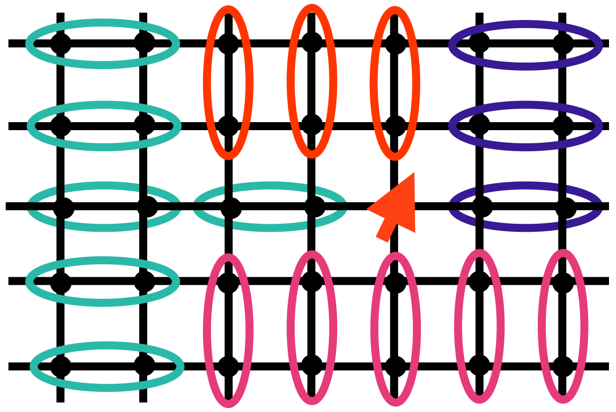
Captured with fractionalised representation of Neel order parameter:

$$\vec{N} = \mathbf{z}^\dagger \vec{\sigma} \mathbf{z} \qquad \mathbf{z} = (z_1, z_2), \qquad \text{U(1) gauge symmetry: } \mathbf{z} \sim e^{i\chi}\mathbf{z}$$

$\mathbf{z} \sim$ VBS vortex

Senthil, Vishwanath, Balents, Sachdev, Fisher '04 Levin & Senthil '04

NCCP^1 model



z_α ~ VBS vortex

$$\mathcal{L} = |(\partial - ia)\mathbf{z}|^2 + m^2|\mathbf{z}|^2 + \lambda(|\mathbf{z}|^2)^2$$

Operators

Neel order parameter $\vec{N} = \mathbf{z}^\dagger \vec{\sigma} \mathbf{z}$

VBS order parameter:
inserts monopoles in gauge field $\varphi_x + i\varphi_y = \mathcal{M}_a$

Internal symmetries

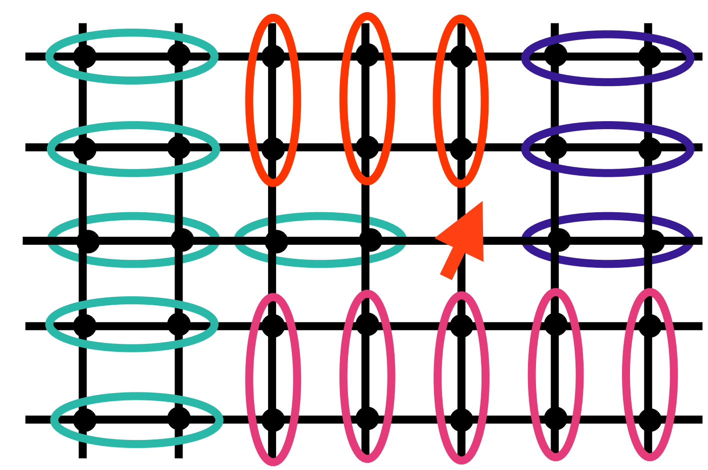
Spin symmetry:
SO(3) rotations of $\vec{N}$

Flux conservation (emergent):
U(1) rotations of $\vec{\varphi}$

Could there be an emergent symmetry relating $\vec{N}$ and $\vec{\varphi}$?

Neel-VBS superspin

Field theory directly in terms of $\vec{N}$, $\vec{\varphi}$?



$$\vec{n} = (N_x, N_y, N_z, \varphi_x, \varphi_y)$$

WZW term to attach spin-1/2 to VBS vortex:

More formally, to ensure correct anomalies for SO(3)xO(2)

Tanaka & Hu '05
Senthil & Fisher '06

$$S = \int \left(\frac{1}{g} (\nabla \vec{n})^2 + \text{strong anisotropies} \right) + S_{\mathrm{WZW}}$$

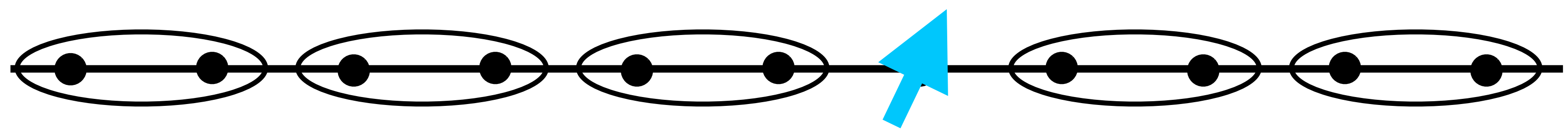
reduce symmetry to
SO(3)xO(2) or SO(3)xlattice

Motivation to speculate about SO(5)

- Numerically: SO(5) emerges to excellent precision at J_c

[1D reminder: spin-1/2 chain]

$$H = J \sum_i \vec{S}_i . \vec{S}_{i+1}$$



$\varphi = 1$ $\varphi = -1$

Flows to a conformal field theory with the emergent symmetry

$$\frac{SU(2) \times SU(2)}{\mathbb{Z}_2} = \mathrm{SO}(4)$$

(1+1D conformal invariance
$\Longrightarrow$ conserved currents doubled)

This SO(4) rotates the 'superspin' $\vec{n} = (\varphi, N_x, N_y, N_z)$

$$S = \frac{1}{g} \int \mathrm{d}x\mathrm{d}t \, (\nabla \vec{n})^2 + \frac{2\pi i \, \epsilon_{abcd}}{\mathrm{Area}(S^3)} \int \mathrm{d}u\mathrm{d}x\mathrm{d}t \, n_a \partial_x n_b \partial_t n_c \partial_u n_d$$

WZW term attaches spin-1/2 to VBS domain wall

Duality web

SO(5) can be understood in terms of a web of dualities

NCCP1 — dual NCCP1

SO(5)

QED—Gross-Neveu — dual QED—Gross-Neveu

None of these theories has explicit SO(5).

DCP also related to two theories with explicit SO(5):

- QCD with Nf=2, Nc=2
- Surface of 3+1D topological paramagnet (SPT) with SO(5)

Wang, AN, Metlitski, Xu, Senthil ’17

True critical point?

At first glance, numerics (up to L=640) show continuous transition. However, strong violations of finite-size scaling

Kuklov et al 08, Sandvik ’11, Banerjee et al ’10, Harada et al ’13…

AN, Chalker, Serna, Ortuno, Somoza ’15 Shao, Guo, Sandvik ’16

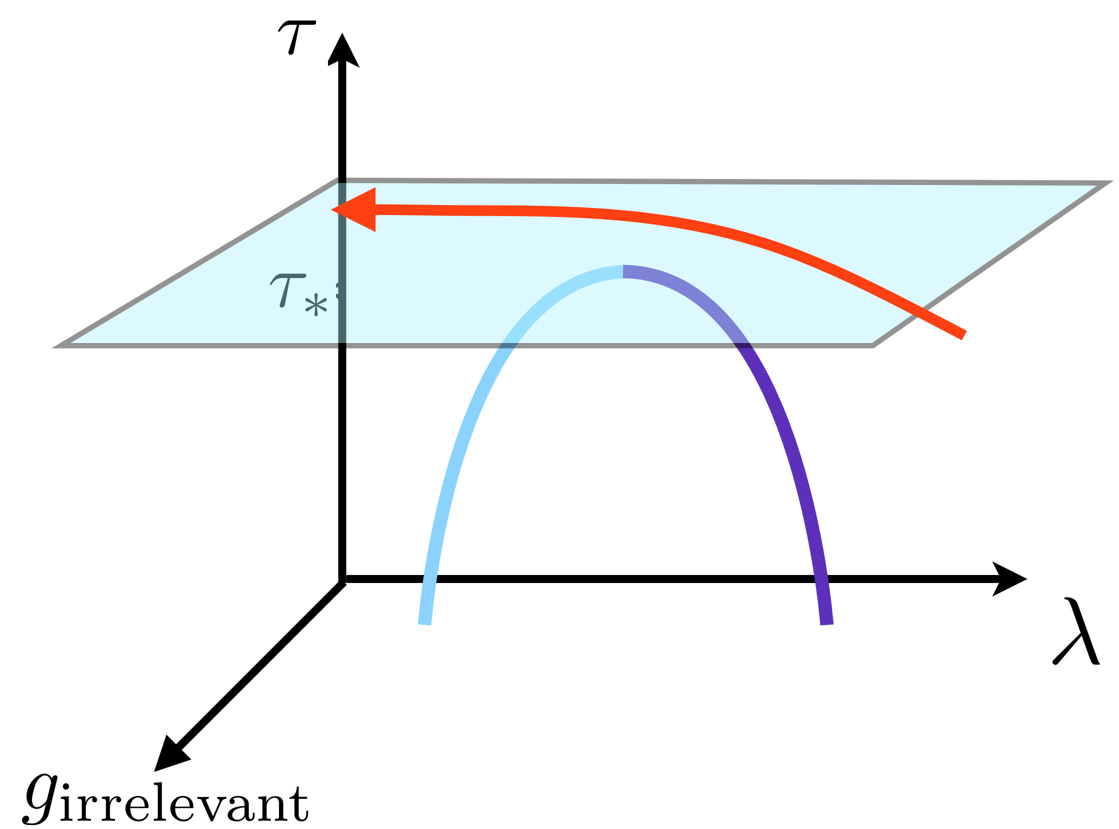
also, tension with conformal bootstrap bounds for SO(5)

Simmons-Duffin; Nakayama, Ohtsuki ’16

One scenario is that transition is ultimately first order, with $\xi \gg 640$. If so, SO(5) & dualities are approximate, not exact.

RG analysis: even in this scenario there is ‘quasiuniversality’:

Large ξ and SO(5) symmetry are *robust*, not due to fine-tuning. SO(5) holds up to accuracy “$1/\xi^{\text{const}}$”



AN, Chalker, Serna, Ortuno, Somoza ’15 Wang, AN, Metlitski, Xu, Senthil ’17

Plan

Introduction

Evidence for SO(5) at the Neel-VBS transition

Dualities

NCCP1 — dual NCCP1

SO(5)-invariant theory

QED—Gross-Neveu — dual QED—Gross-Neveu

Possibility of ‘pseudocritical’ behaviour

SO(5) in a very different microscopic model

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Evidence for SO(5) at the Neel-VBS transition

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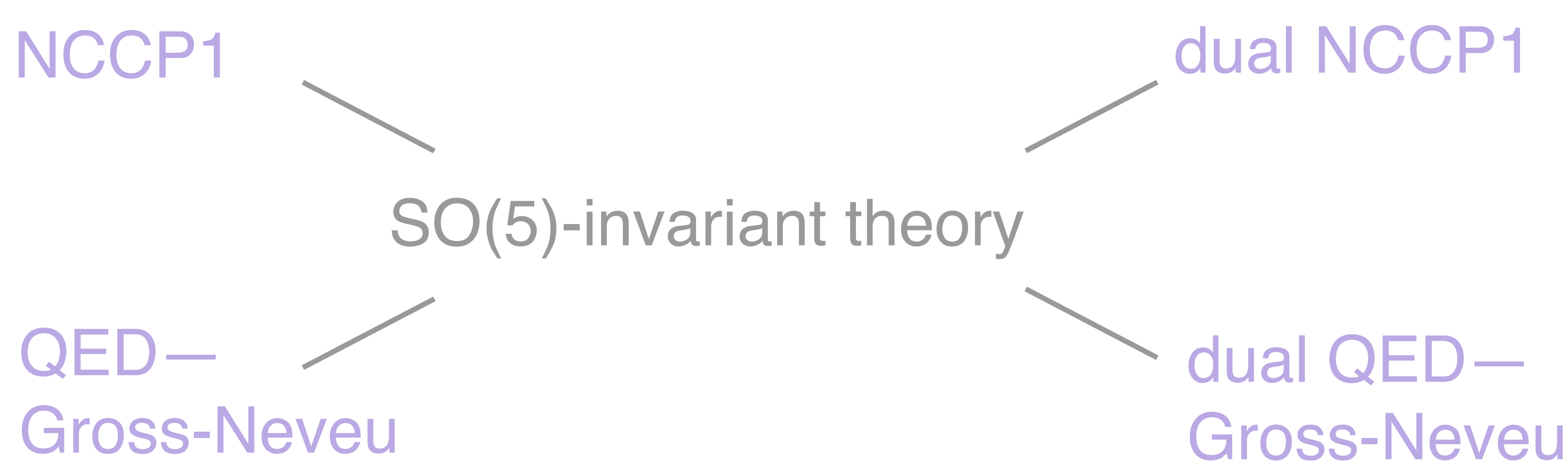


Possibility of ‘pseudocritical’ behaviour

SO(5) in a very different microscopic model

Plan

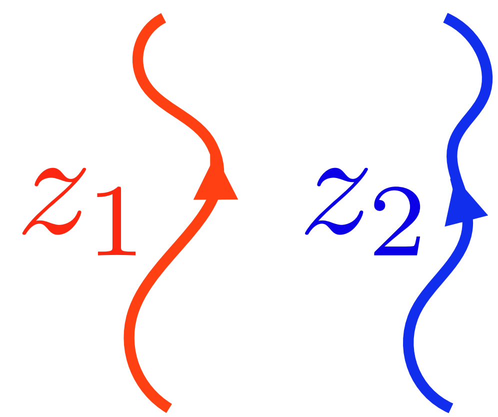
Evidence for SO(5) at the Neel-VBS transition

- Strategy for simulations
- Numerical results
- ‘Naive’ RG interpretation

Strategy for simulations

Mapping between Heisenberg AFM
and stat mech of loops in one higher dimension.

loops = worldlines of spinons



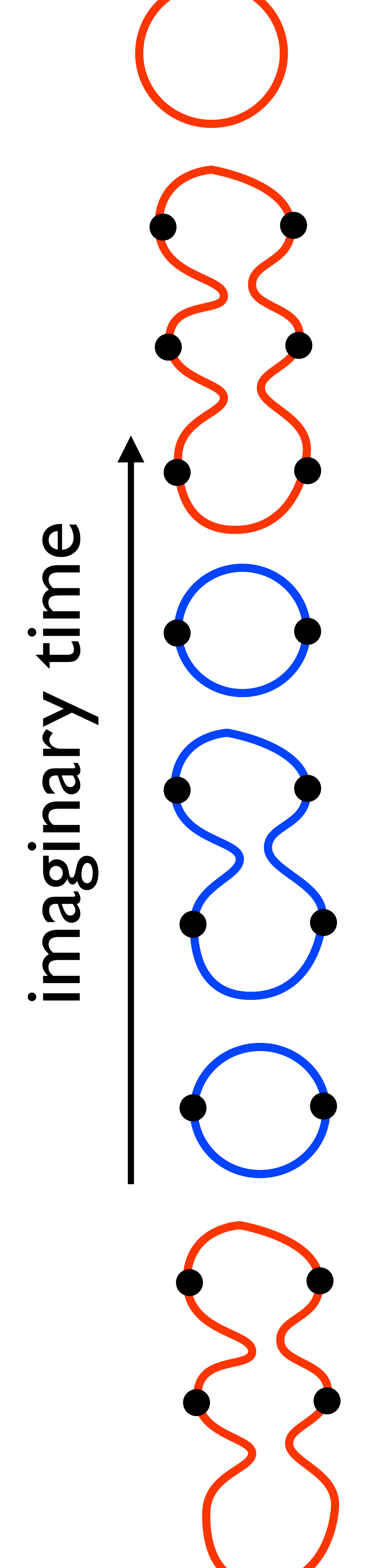
$\vec{N} = \mathbf{z}^\dagger \vec{\sigma} \mathbf{z}$

For Heisenberg AFM,

$$Z(\beta) = \mathrm{Tr}\left(e^{-\delta t H}\right)^{\beta/\delta t}$$

$=$ partition function for loops in
discrete space, continuous time

Simplify: construct loop model with same universal
properties, but **isotropic in spacetime**



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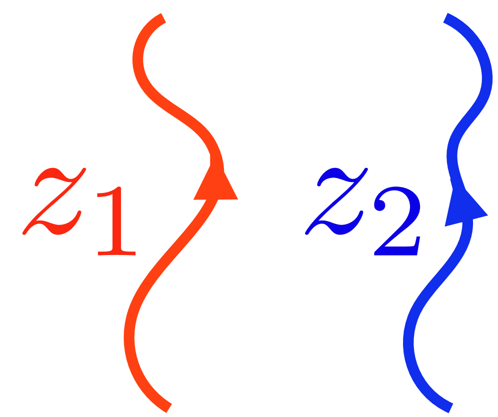

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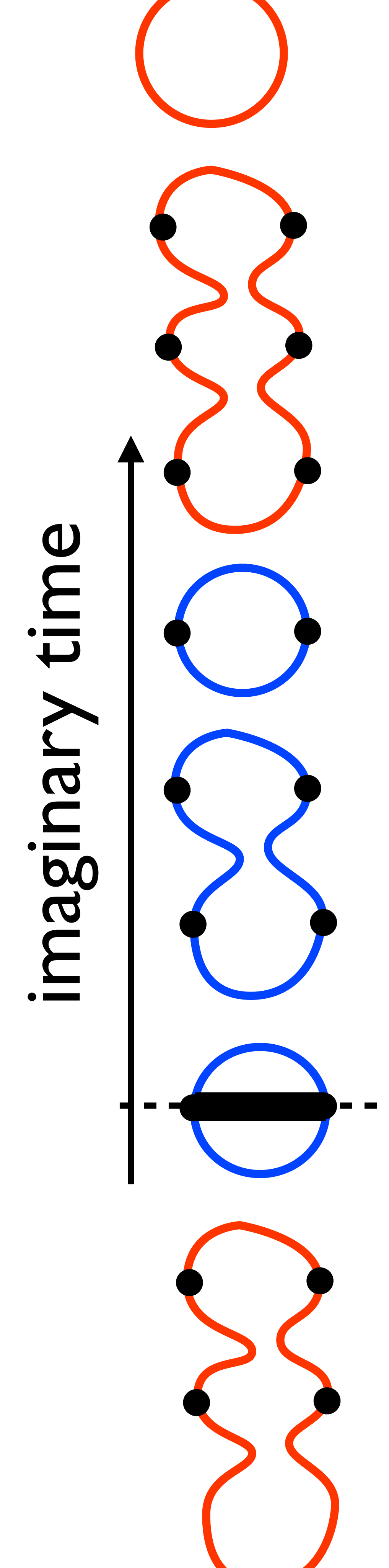
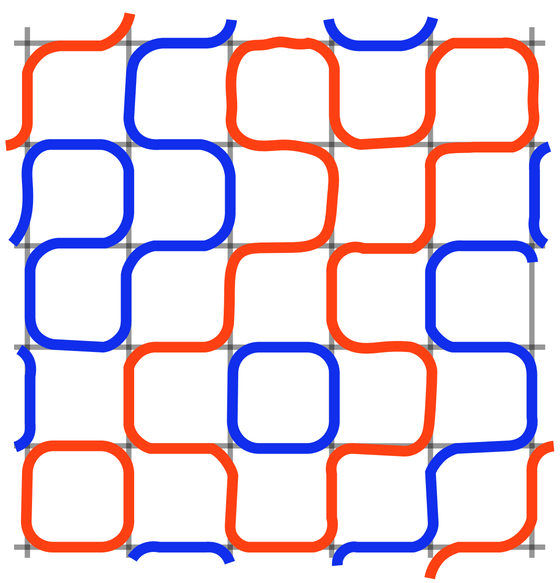


1+1D example: spin-1/2 chain

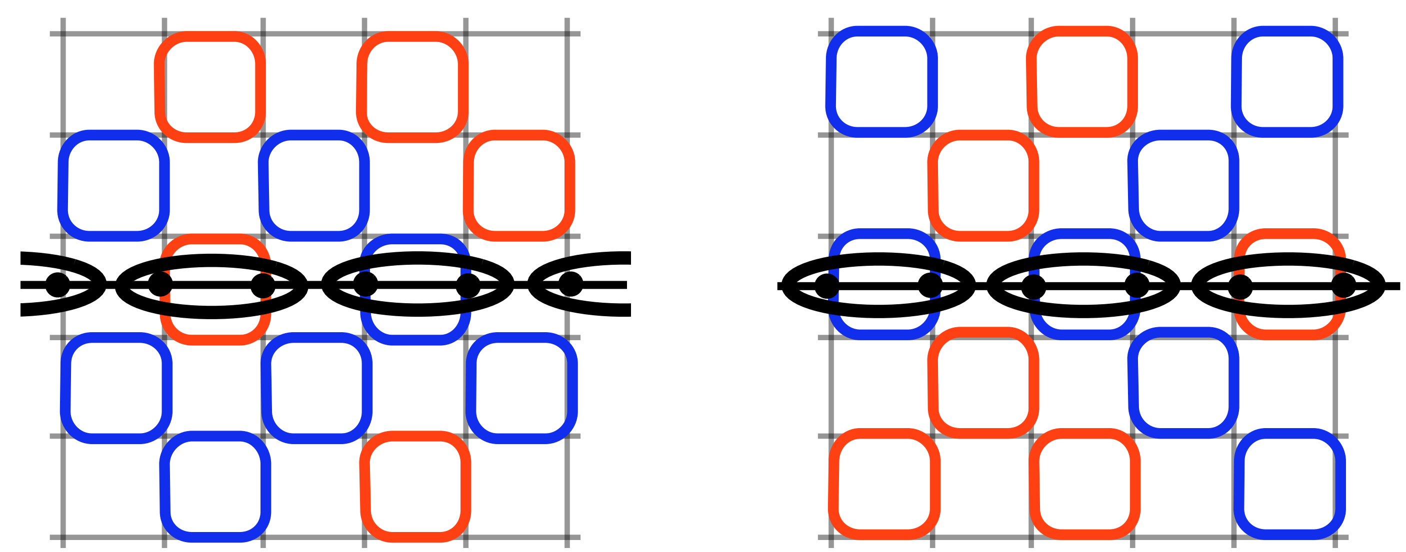
Standard 2D loop model
→ universal physics of spin-1/2 chain

Gapless spin-1/2 chain



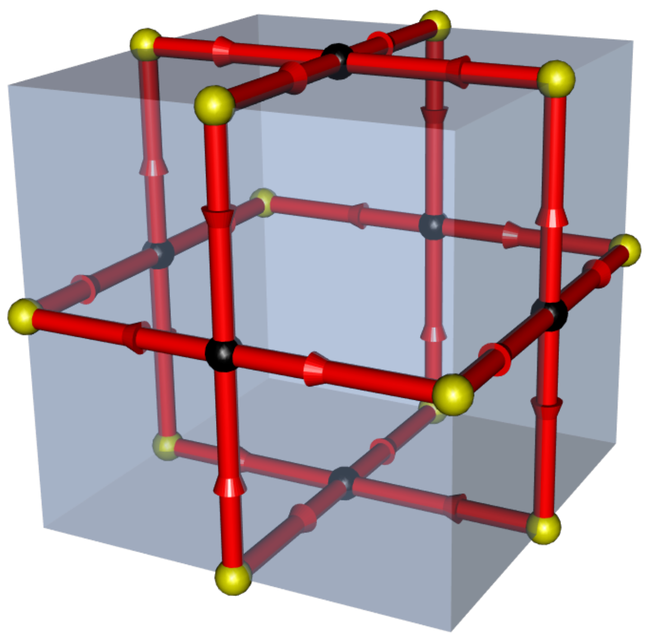
Long loops mediate power law $\langle \vec{N}(0).\vec{N}(r) \rangle$ correlations

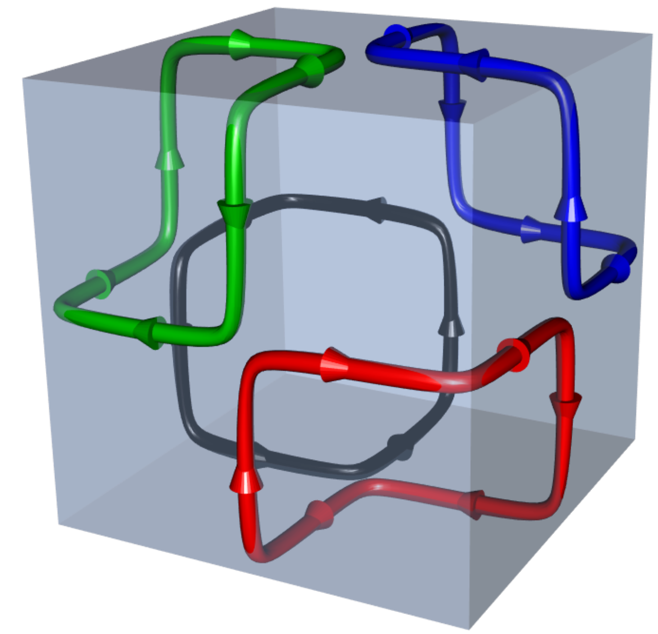
Gapped VBS states



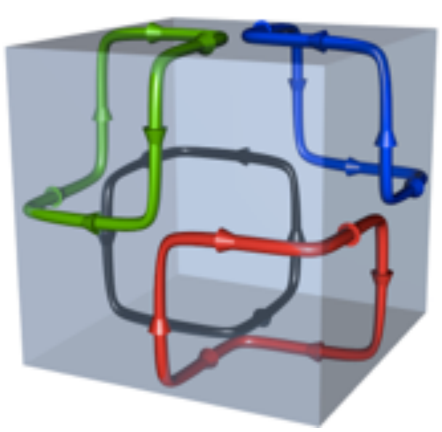
Two packings of short loops → two VBS configurations

Model for 2+1D Neel-VBS transition

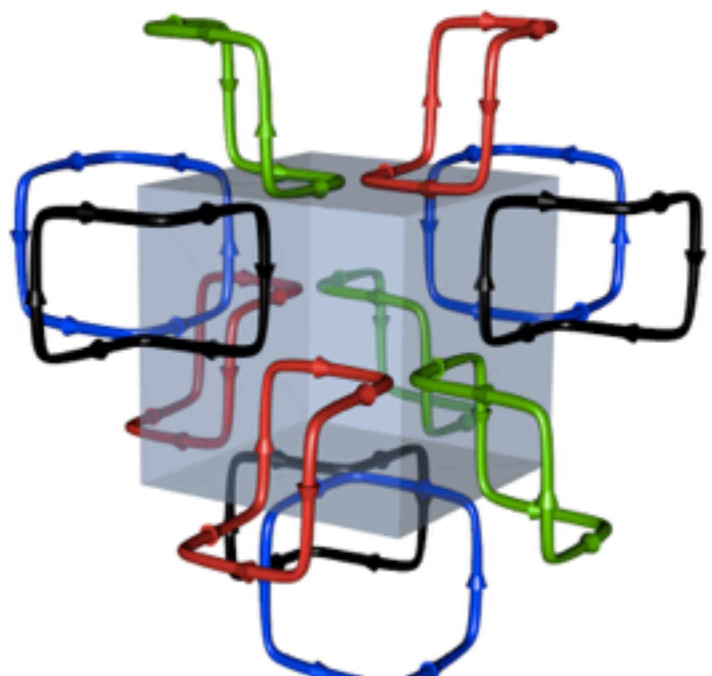




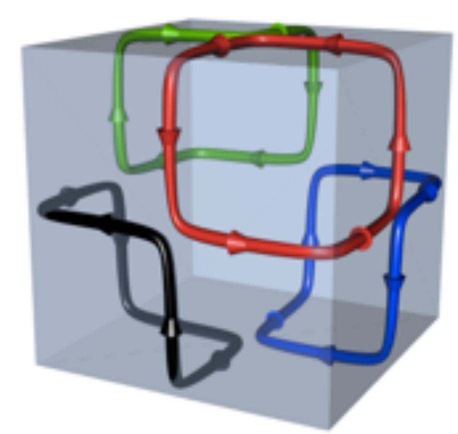
Neel → long loops. VBS → short loops. 4 'VBS' states:



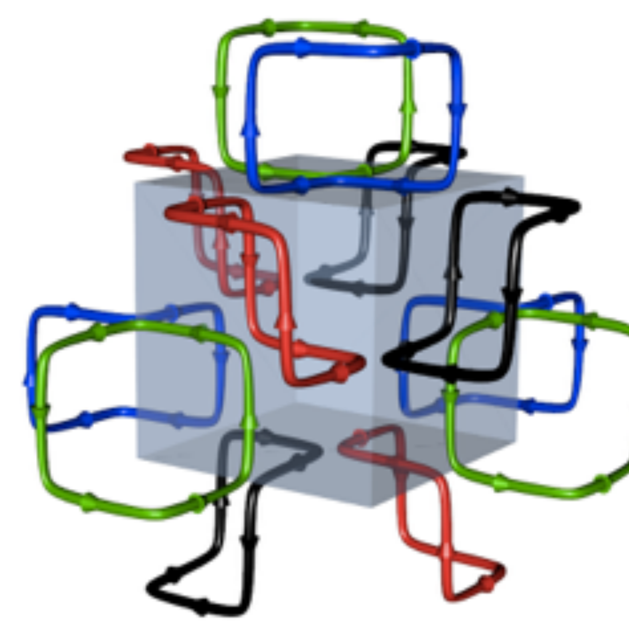
$\vec{\varphi} = (1, 1)$



$\vec{\varphi} = (-1, 1)$



$\vec{\varphi} = (1, -1)$



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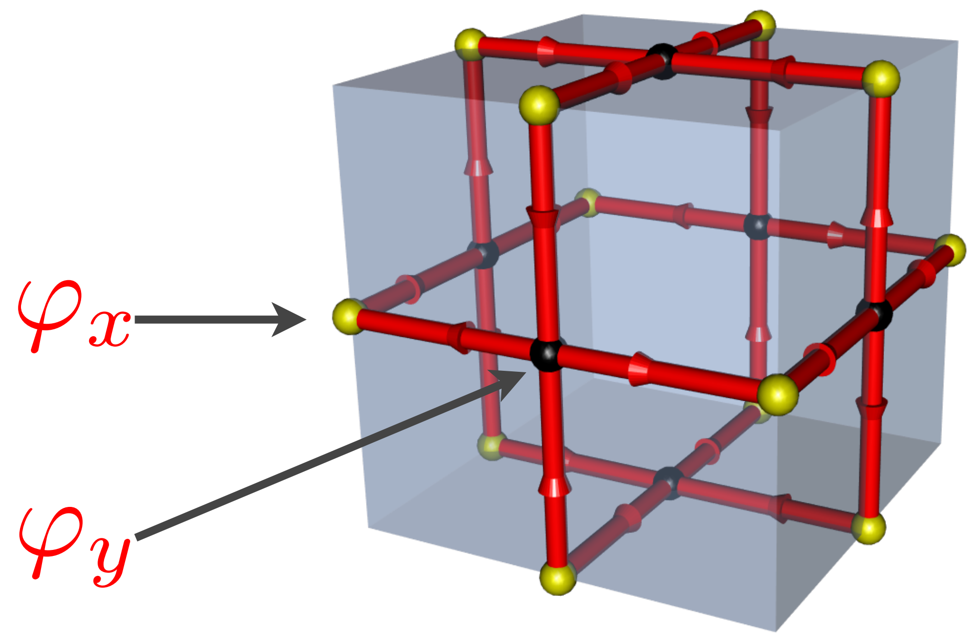

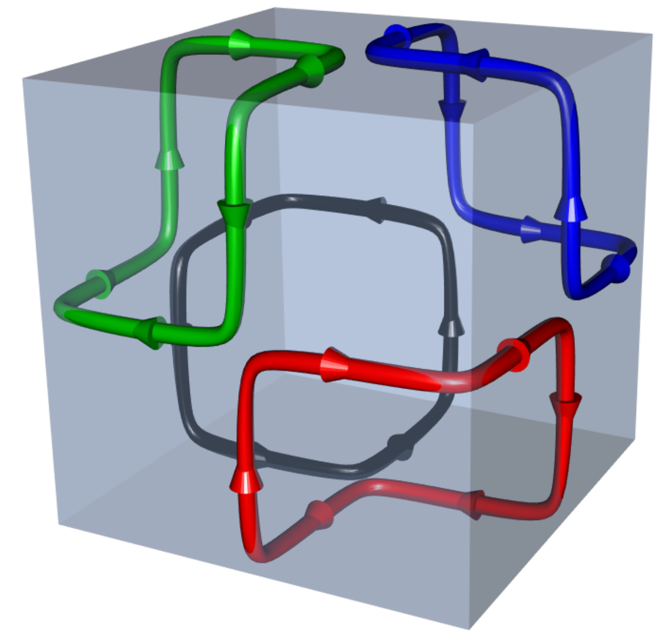

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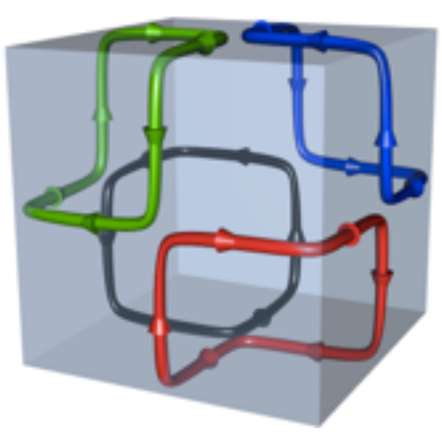

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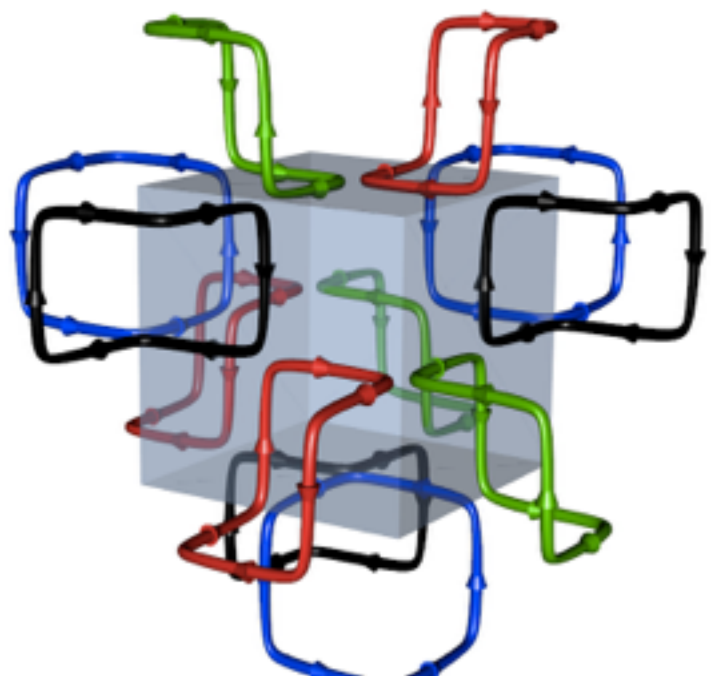

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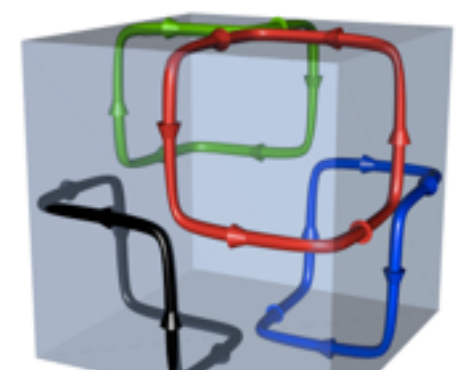

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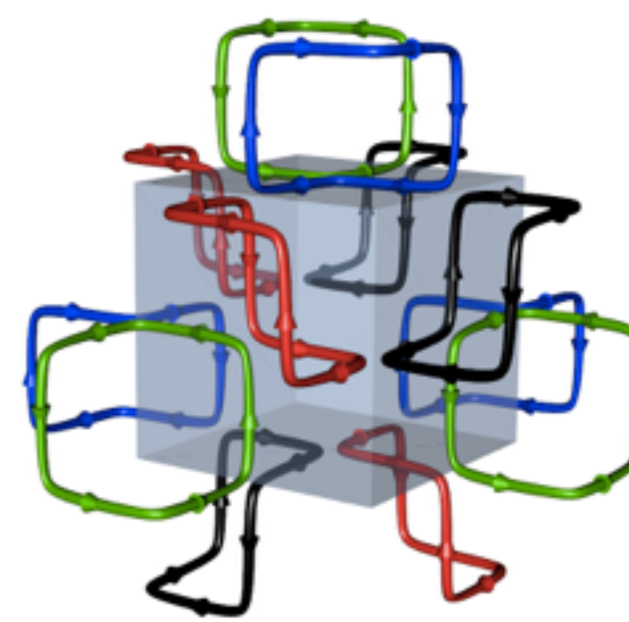

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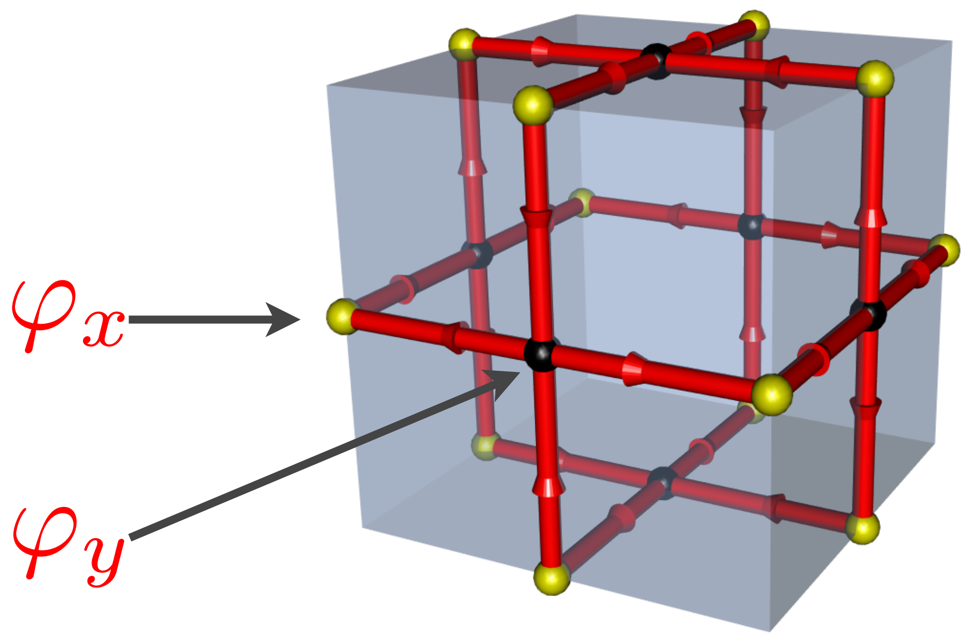
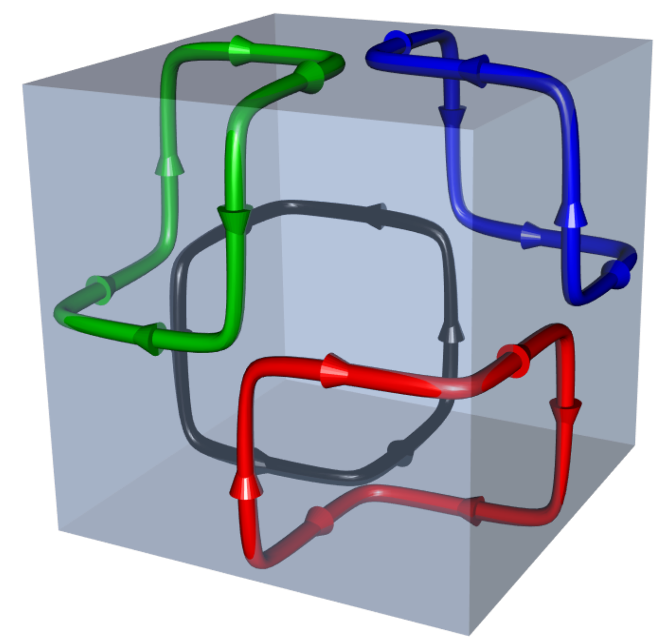




Neel → long loops. VBS → short loops.

Drive Neel—VBS transition using interaction between φs

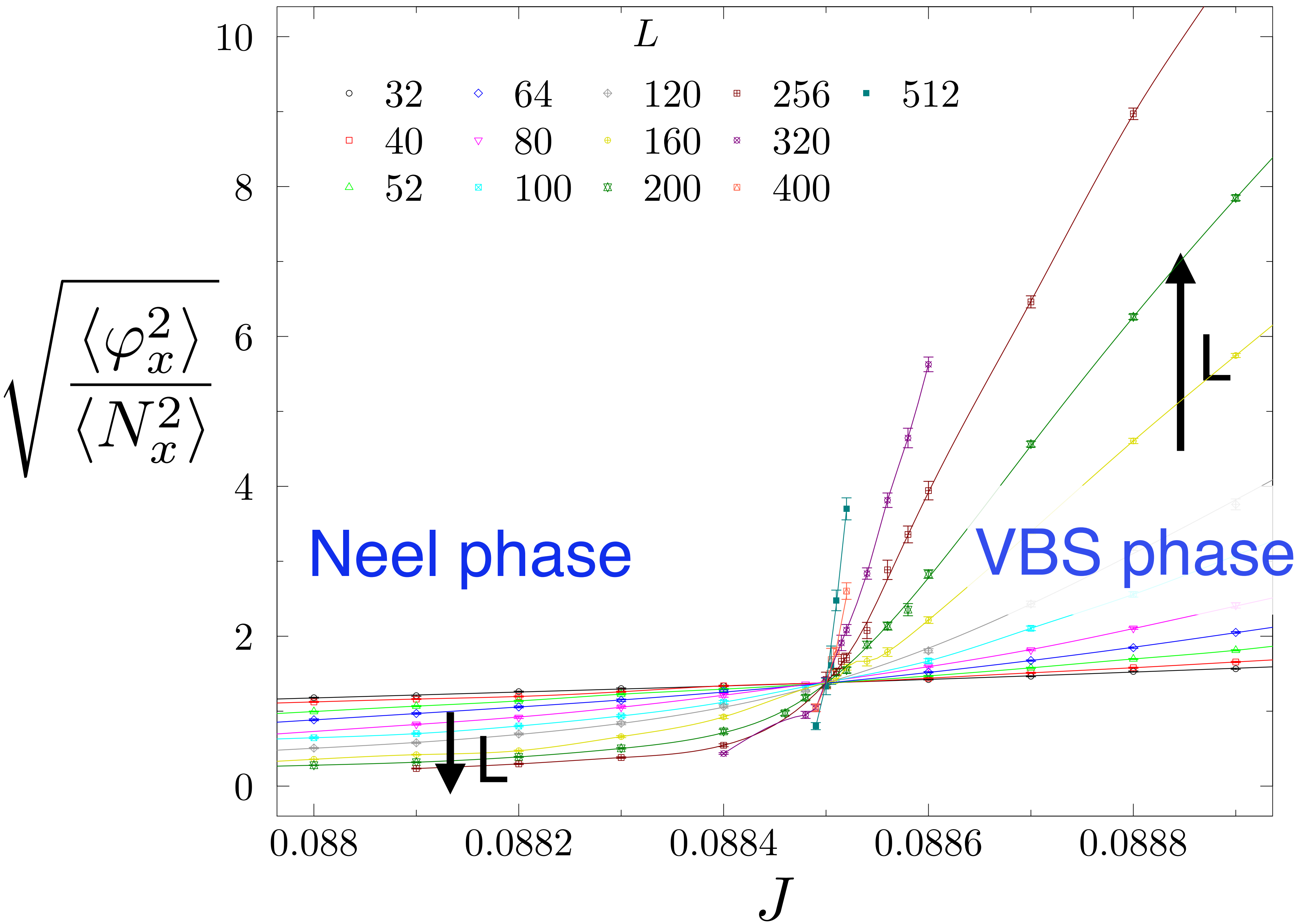
Long-distance properties as in JQ model, but more efficient to simulate. Exact spacetime isotropy also removes some scaling corrections.

SO(5)

$$\vec{n} = (\overbrace{\varphi_x, \underbrace{\varphi_y}_{}}^{\mathrm{U}(1)} \underbrace{, N_x}_{\mathrm{U}(1)?}, \overbrace{N_y, N_z}^{\mathrm{SO}(3)})$$

Look for emergent $\mathrm{U}(1)$ in (φ_x, N_x) plane

Scaling of Neel and VBS fluctuations (up to L=512)



Fluctuations scale identically at J_c

Contrast with $\sqrt{\langle \varphi_x^2 \rangle} / \sqrt{\langle N_x^2 \rangle} \sim L^{x_N - x_\varphi}$

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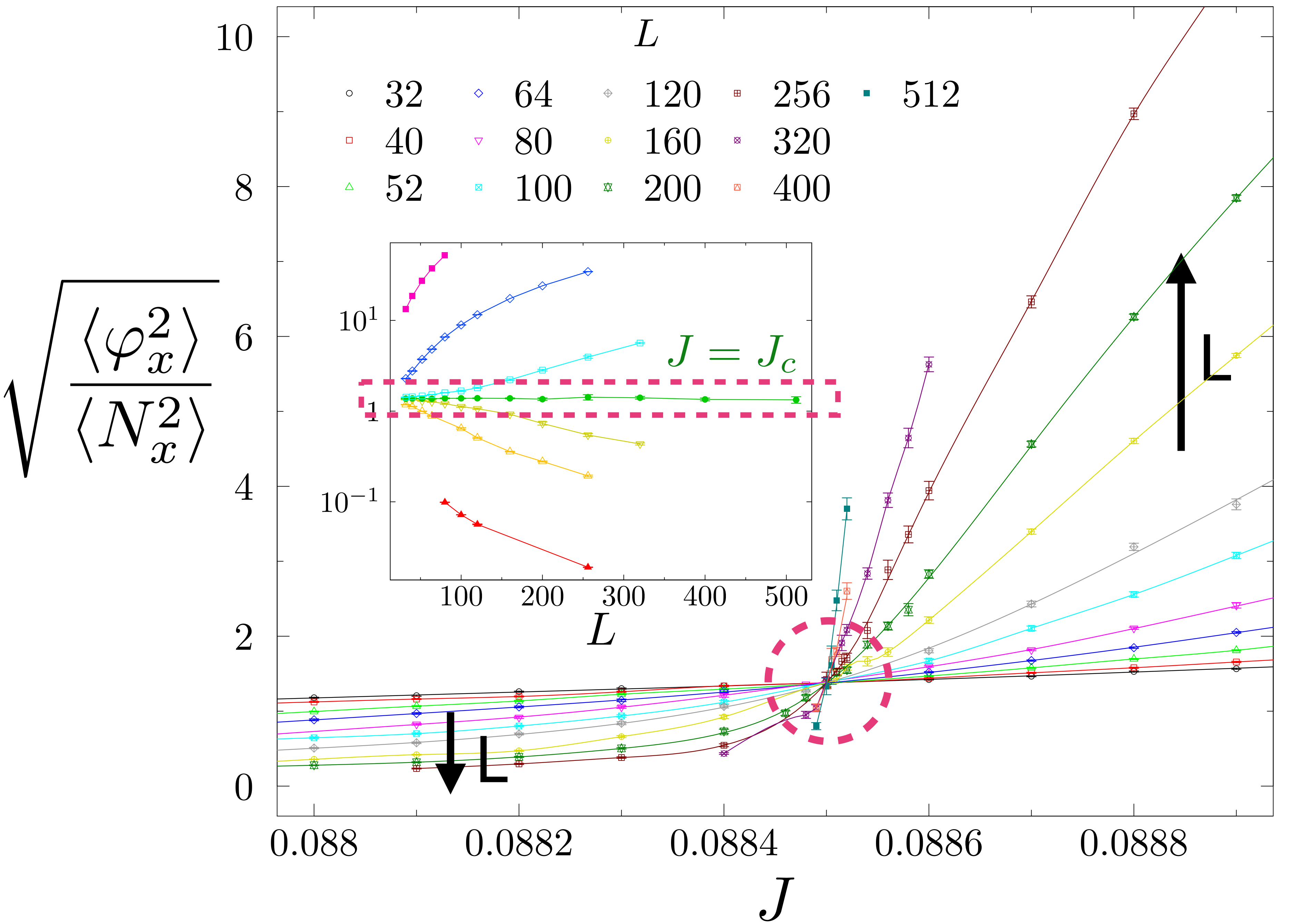

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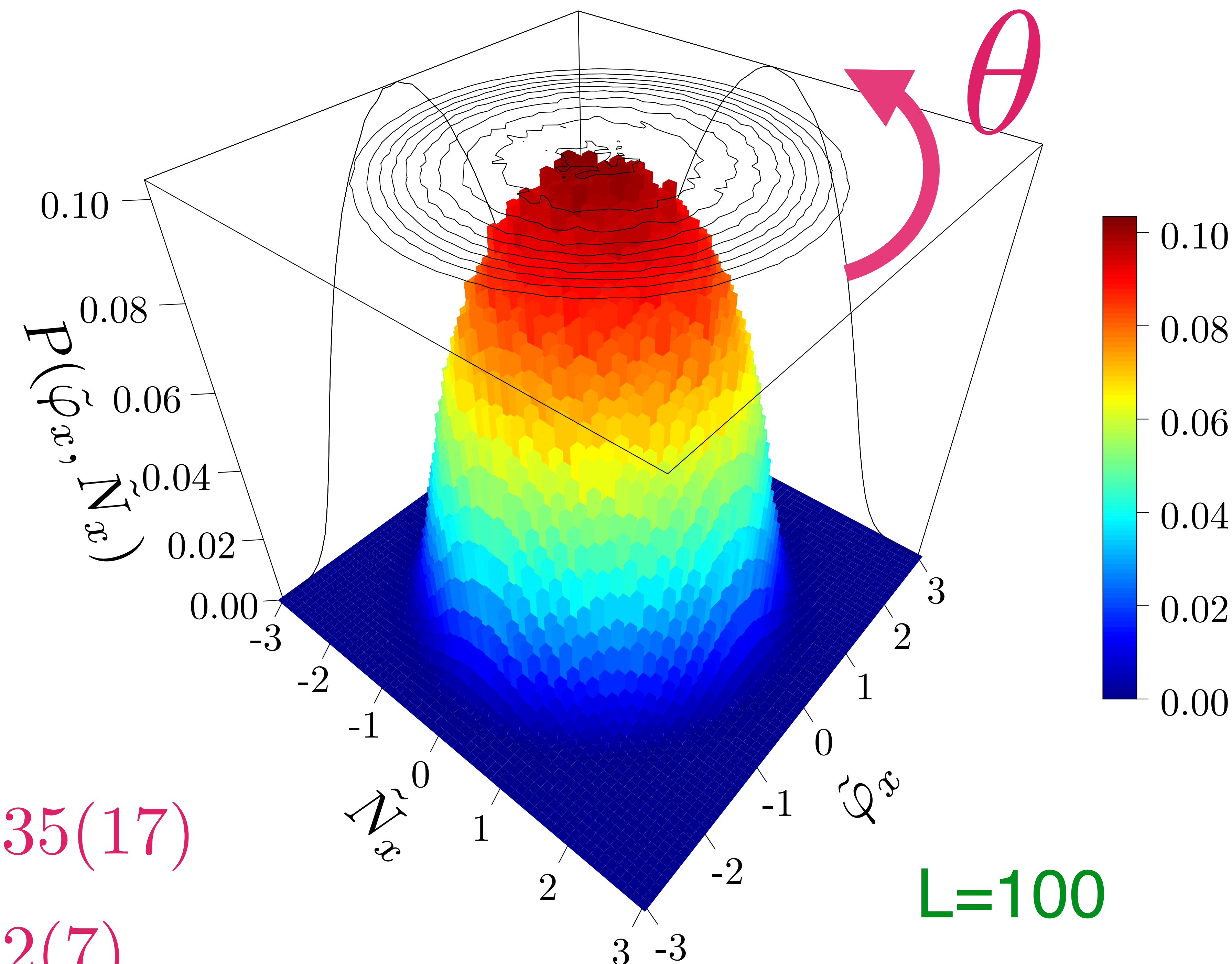
Joint Neel/VBS probability distribution



$\langle \cos 2\theta \rangle = 0.00035(17)$

$\langle \cos 4\theta \rangle = 0.0002(7)$

$\langle \cos 6\theta \rangle = 0.0002(5)$

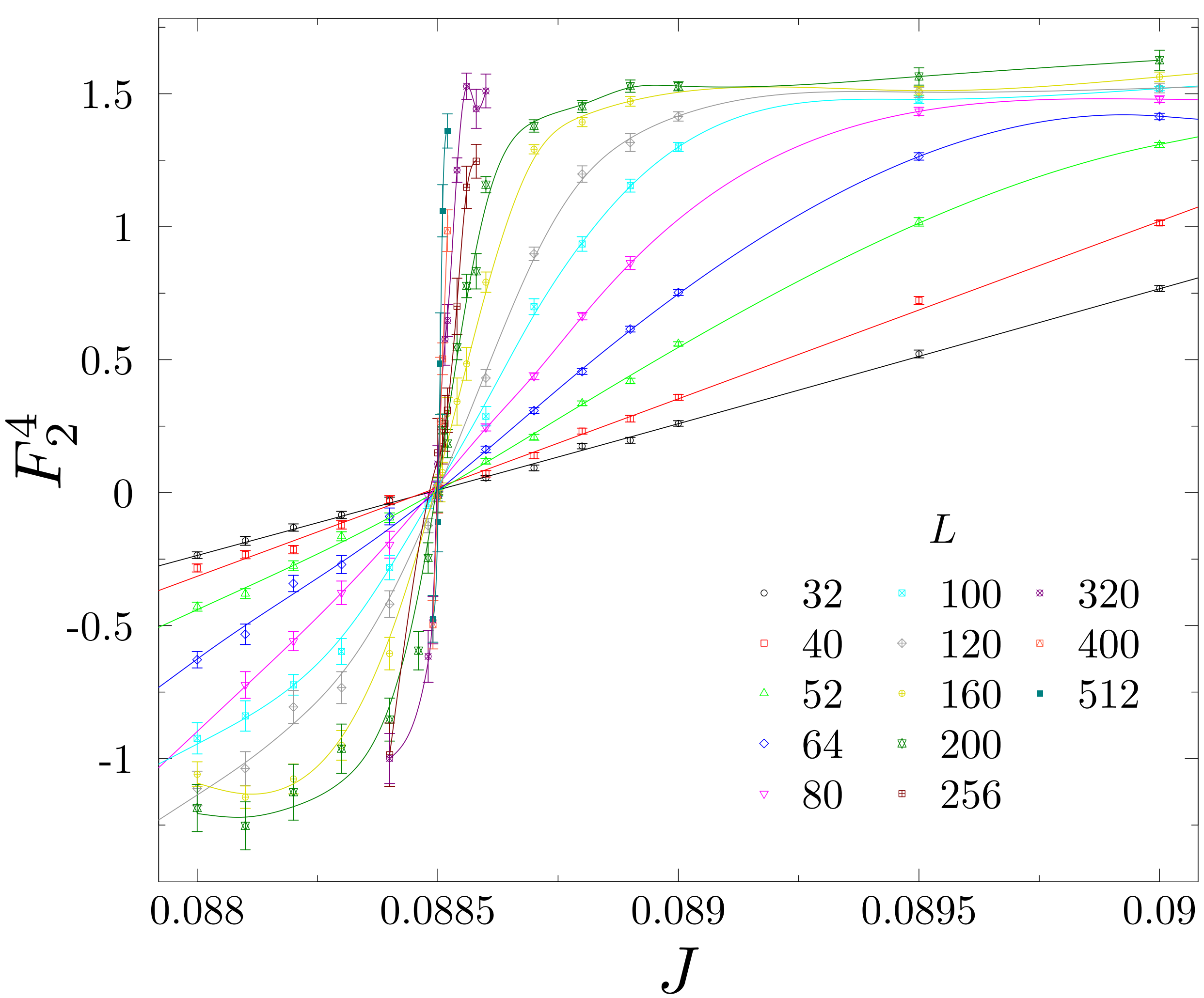
L=100

(Variances normalized to one)

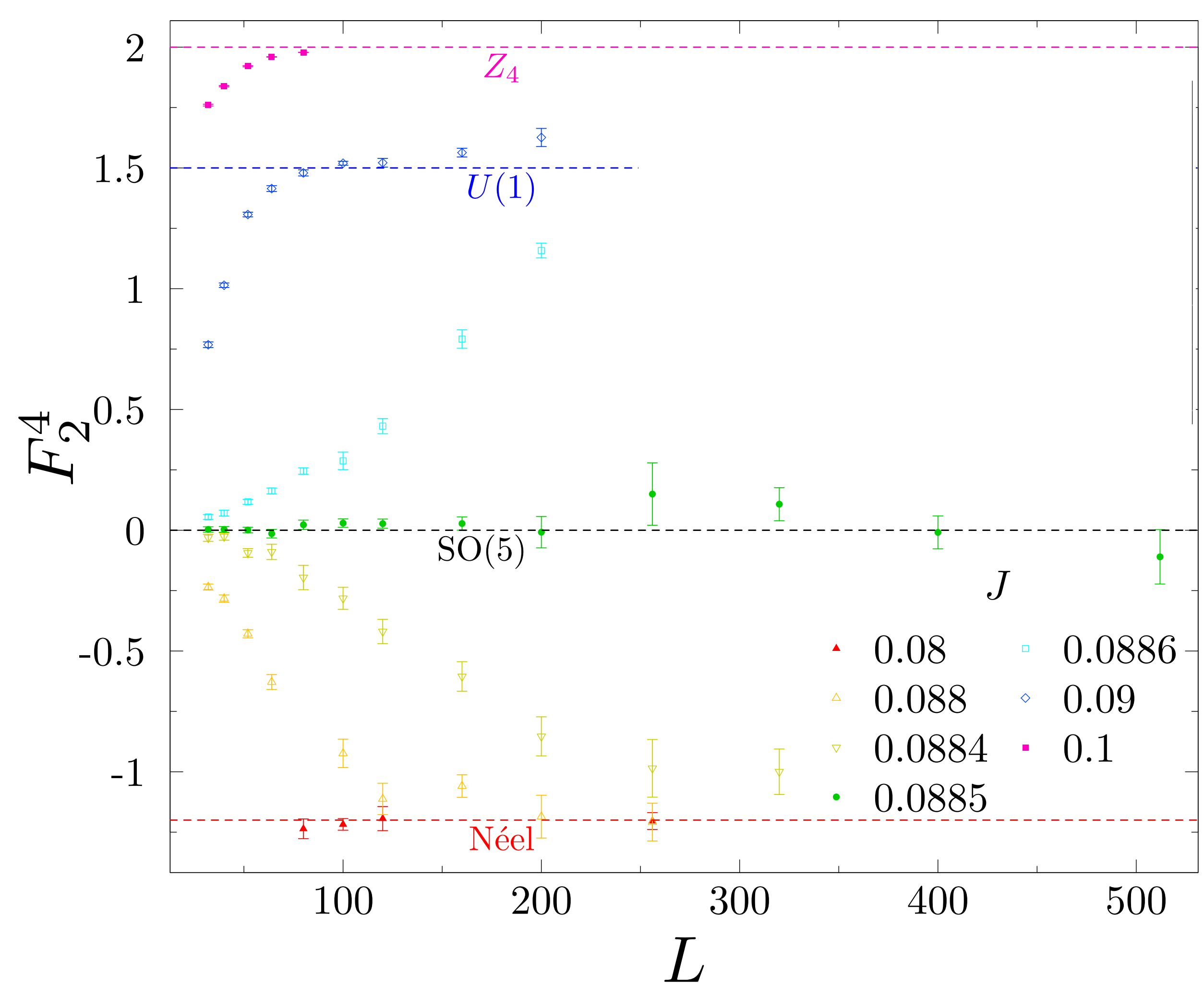
Scaling of Neel and VBS fluctuations

$$F_2^4 = \langle \tilde{N}_x^4 - \tilde{\varphi}_x^4 \rangle = \begin{cases} -1.2 & \text{Neel phase} \\ 0 & \text{SO(5)} \\ 1.5 & \text{VBS, U(1) regime} \\ 2 & \text{VBS, “Z}_4\text{” regime} \end{cases}$$

As a function of coupling



As a function of L



Operators

Classify operators into SO(5) multiplets

Vector $$\vec{n} = (\varphi_x, \varphi_y, N_x, N_y, N_z)$$

Symmetric traceless 2-cpt and 4-cpt tensors

$$X^{(2)}_{ab} = n_a n_b - \frac{1}{5}\delta_{ab} n^2 \qquad X^{(4)}_{abcd} = n_a n_b n_c n_d - (\ldots)$$

We have also tested for symmetry relations between two-point correlation functions of **n** and of $\mathbf{X}^{(2)}$

Operators

Classify operators into SO(5) multiplets

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Symmetric traceless 2-cpt and 4-cpt tensors

$$X^{(2)}_{ab} = n_a n_b - \frac{1}{5}\delta_{ab} n^2$$

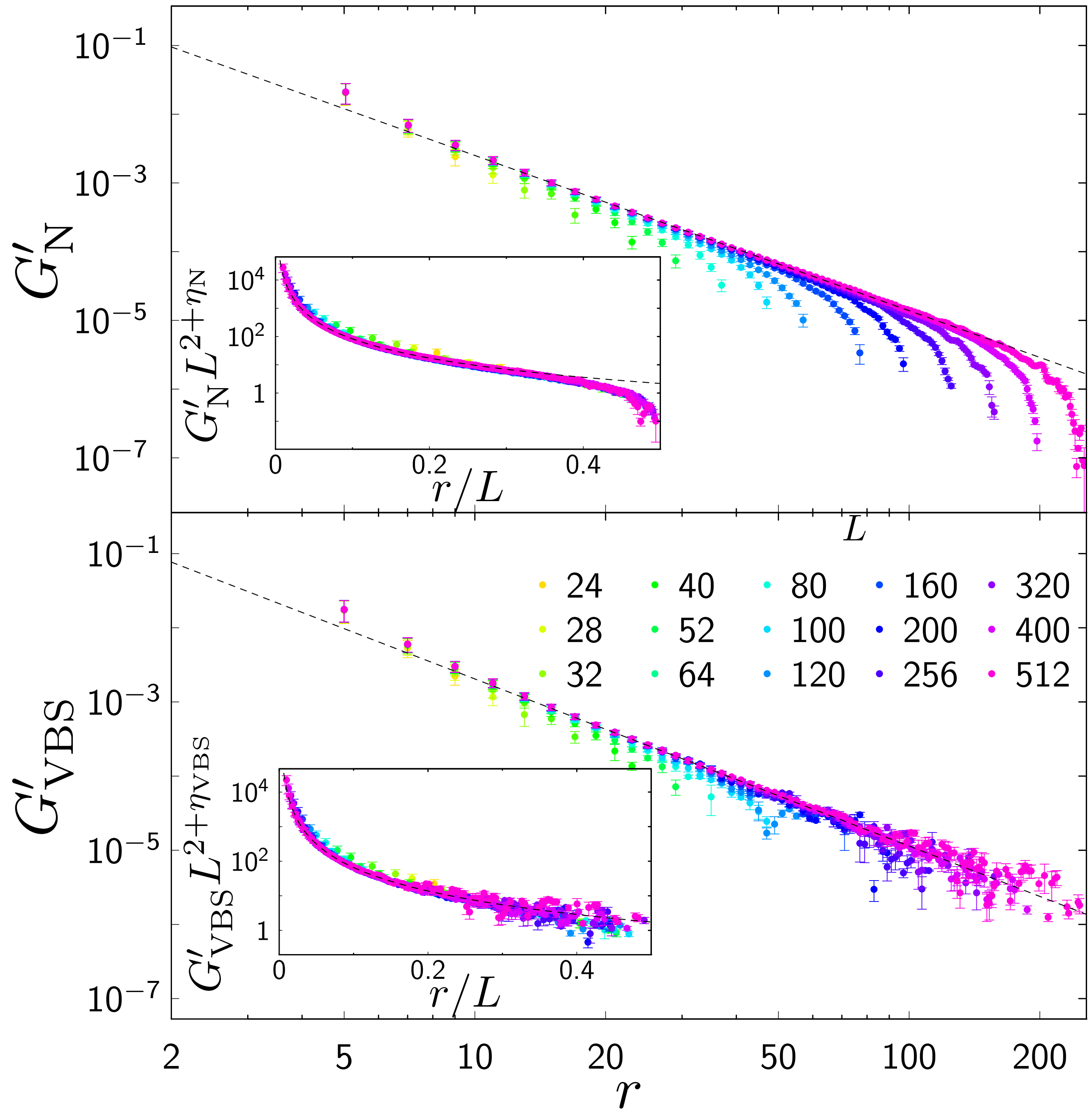
$$\boxed{\varphi_x N_z} \qquad \boxed{\varphi_x \varphi_y} \qquad \boxed{\frac{3}{5}\vec{\varphi}^2 - \frac{2}{5}\vec{N}^2}$$

$$X^{(4)}_{abcd} = n_a n_b n_c n_d - (\ldots)$$

We have also tested for symmetry relations between two-point correlation functions of **n** and of $\mathbf{X}^{(2)}$

Operators

Symmetry between components of vector



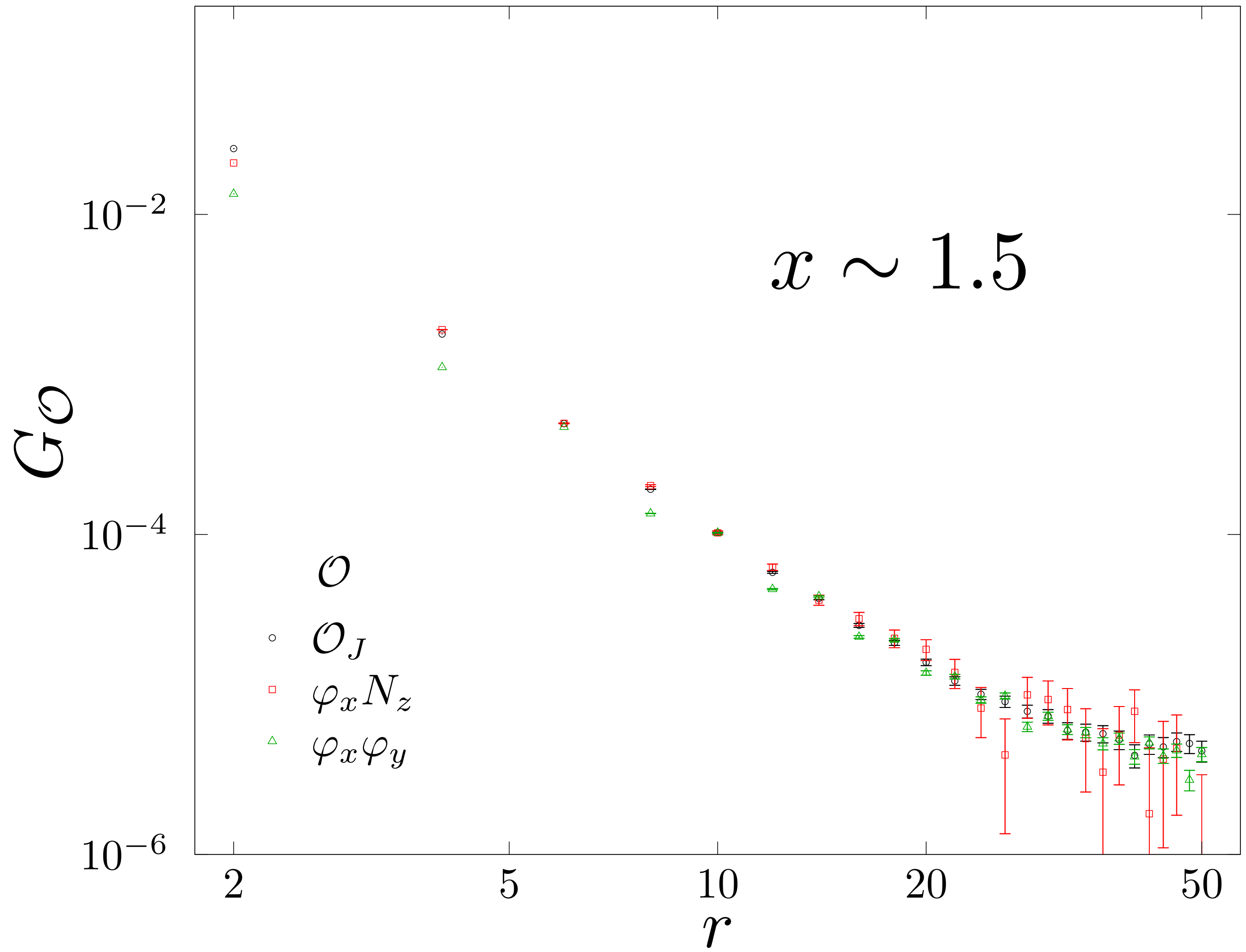
$$\eta_{\text{Néel}} = 0.259(6)$$

$$\eta_{\rm VBS} = 0.25(3)$$

Operators

Symmetry between cpts of symmetric tensor: $X^{(2)}_{ab}$

$$\langle \mathcal{O}(0)\mathcal{O}(r)\rangle \ \text{ for } \ \mathcal{O} = \begin{cases} \varphi_x N_z \\ \varphi_x \varphi_y \\ \frac{3}{5}\vec{\varphi}^2 - \frac{2}{5}\vec{N}^2 \end{cases}$$



Emergence of SO(5)

$$\vec{n} = (\varphi_x, \varphi_y, N_x, N_y, N_z)$$

For exact SO(5) to emerge, need sufficiently stable fixed point.

$$X^{(2)}_{ab} = n_a n_b - \frac{1}{5}\delta_{ab} n^2$$

RG relevant

$$X^{(4)}_{abcd} = n_a n_b n_c n_d - (\ldots)$$

SO(5) singlets

Must be RG irrelevant

Contrast Wilson-Fisher CFTs where singlet mass strongly relevant

$$\mathcal{L} = \mathcal{L}_{\mathrm{SO}(5)} - \delta J \sum_{a=1,2} X^{(2)}_{aa} + \lambda \sum_{a,b=1,2} X^{(4)}_{aabb} + \kappa \sum_{a=1,2} X^{(4)}_{aaaa} + \ldots$$

Tuning parameter $\sim \vec{\varphi}^2 - \frac{2}{3}\vec{N}^2$

Higher Neel-VBS anisotropy $\sim (\vec{\varphi}^2)^2 + \ldots$

"Z_4" VBS anisotropy $\sim \varphi_x^4 + \varphi_y^4 + \ldots$

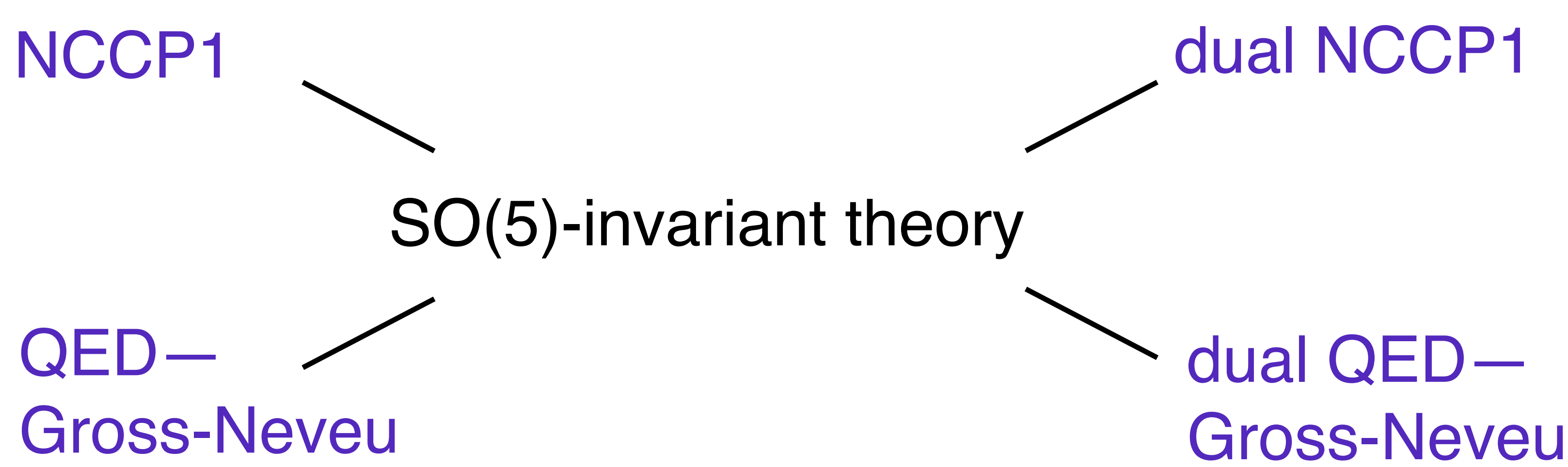
irrelevant

Plan

Introduction

Evidence for SO(5) at the Neel-VBS transition

Dualities



Possibility of ‘pseudocritical’ behaviour

SO(5) in a very different microscopic model

Two duality webs

NCCP1 — SO(5)-invariant theory — dual NCCP1

QED—Gross-Neveu — SO(5)-invariant theory — dual QED—Gross-Neveu

More speculatively:

but see recent simulations:
Qin, He, You, Lu, Sen, Sandvik, Xu, & Meng '17
Zhang, He, Eggert, Moessner, & Pollmann '17

easy-plane NCCP1 — O(4)-invariant theory? — dual easy-plane NCCP1

QED — O(4)-invariant theory? — dual QED

Particle-vortex duality for complex scalar field

$$\mathcal{L} = |\partial\Phi|^2 + m^2|\Phi|^2 + \lambda|\Phi|^4$$

$$\mathcal{L}_{\rm dual} = |(\partial - ia)w|^2 - m^2|w|^2 + \lambda'|w|^4 + \ldots$$

Gauge invariant U(1) order parameter:

$$\Phi \quad \longleftrightarrow \quad \mathcal{M}_a$$ inserts monopole in a

L and L_{dual} describe same system with different bare couplings.
May have same IR behaviour and in fact they do.

Recently, extension of particle-vortex duality idea to
Dirac fermion

Wang & Senthil '15; Metlitski & Vishwanath '16;
Seiberg et al '16; Karch & Tong '16; Son '15;

[Aside: strong and weak duality]

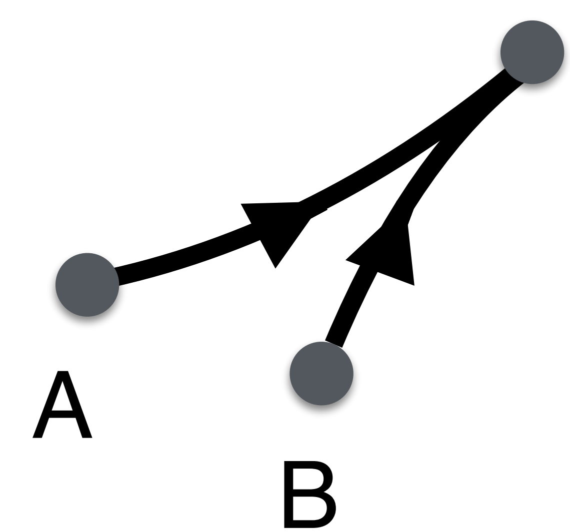
for two field theories A and B which share the 'microscopic' symmetry $G = G_A \cap G_B$:

Weak statement (derived):

Theories A and B have the same operator content and anomalies.

Strong statement (conjectured):

Theories A and B become the same under RG



For DCPs, it is the strong dualities that imply large emergent symmetries.

Dualities

NCCP1 dual NCCP1

SO(5)-invariant theory

QED—
Gross-Neveu

dual QED—
Gross-Neveu

Self-duality of NCCP[1]

$$\mathcal{L} = |(\partial - ia)\mathbf{z}|^2 + m^2|\mathbf{z}|^2 + \lambda(|\mathbf{z}|^2)^2$$

Apply particle-vortex duality to both species of spinon $\begin{array}{l} z_1 \longrightarrow w_2 \\ z_2 \longrightarrow w_1^* \end{array}$

$$\mathcal{L}_{\text{dual}} = |(\partial - i\tilde{a})\mathbf{w}|^2 - m^2|\mathbf{w}|^2 + \lambda(|\mathbf{w}|^2)^2 + \kappa(\mathbf{w}^\dagger\sigma_z\mathbf{w})^2 + \ldots$$

We have SU(2) for **z**. But a priori, do **not** have SU(2) for **w.**

	$\varphi_x + i\varphi_y$	$N_x + iN_y$	N_z
in $\mathcal{L}$	$\mathcal{M}_a$	$2z_1^* z_2$	$\lvert z_1\rvert^2 - \lvert z_2\rvert^2$
in $\mathcal{L}_{\text{dual}}$	$\underline{2w_2^* w_1}$	$\mathcal{M}_{\tilde{a}}^*$	$\underline{\lvert w_1\rvert^2 - \lvert w_2\rvert^2}$

Self-duality of NCCP[1]

Conjecture: at the critical point of $\mathcal{L}$,
duality becomes a self—duality

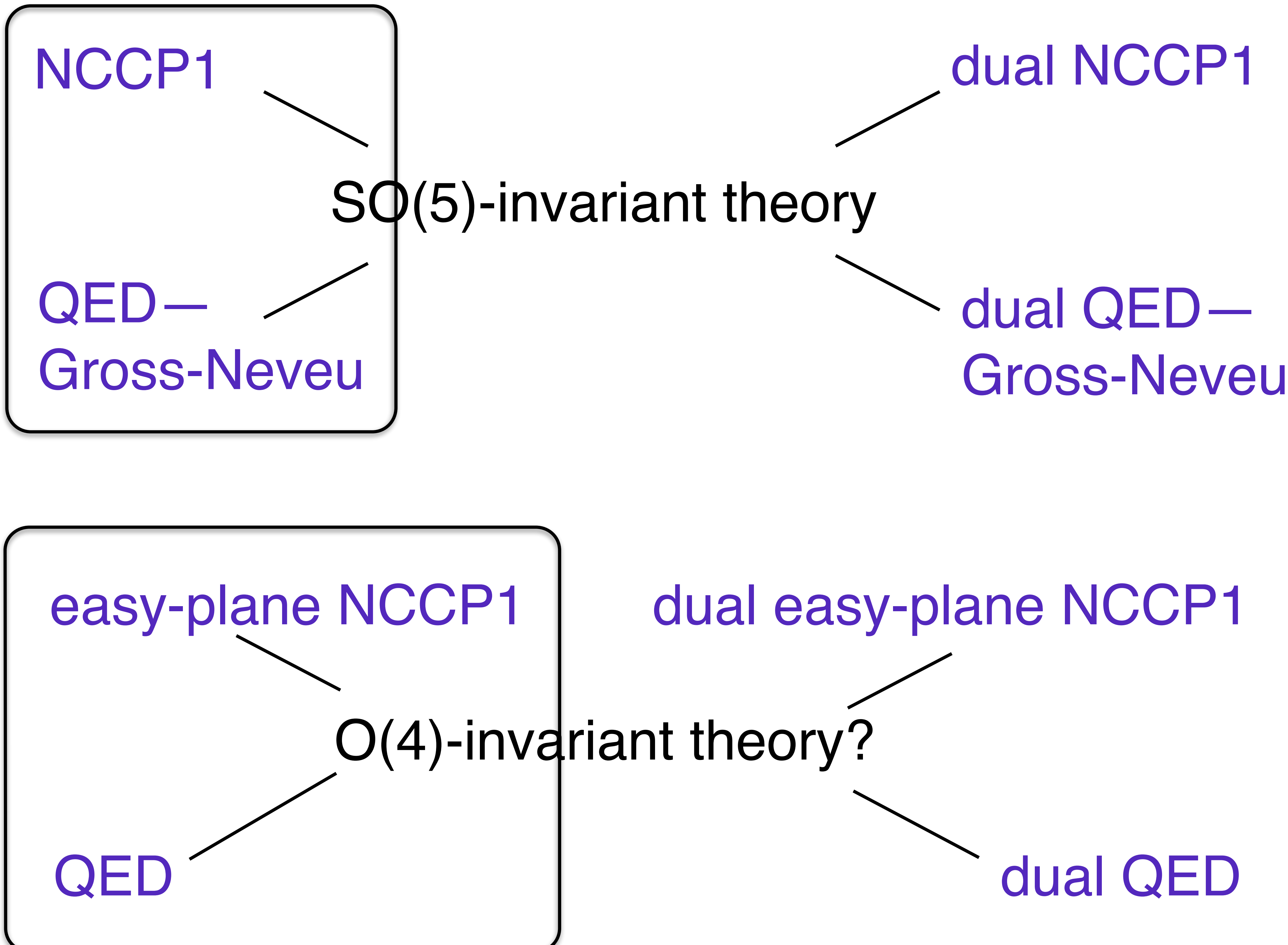
$$\mathcal{L} = |(\partial - ia)\mathbf{z}|^2 + \lambda(|\mathbf{z}|^2)^2 \longleftrightarrow \mathcal{L}_{\mathrm{dual}} = |(\partial - i\tilde{a})\mathbf{w}|^2 + \lambda(|\mathbf{w}|^2)^2$$

Dual field **w** has **emergent** SU(2) symmetry. This rotates Neel into VBS

	$\varphi_x + i\varphi_y$	$N_x + iN_y$	N_z
in $\mathcal{L}$	$\mathcal{M}_a$	$2z_1^* z_2$	$\lvert z_1\rvert^2 - \lvert z_2\rvert^2$
in $\mathcal{L}_{\mathrm{dual}}$	$2w_2^* w_1$	$\mathcal{M}_{\tilde{a}}^*$	$\lvert w_1\rvert^2 - \lvert w_2\rvert^2$

This implies SO(5) (and vice versa).

Boson-fermion dualities



QED

Two 2-component Dirac fermions, dynamical U(1) gauge field

$$\mathcal{L}_{qed} = \sum_{j=1}^{2} i\bar{\psi}_j \not{D}_a \psi_j + \cdots$$ SU(2) global flavour symmetry

This is a theory of bosons: all gauge invariant operators are bosonic. What are the ‘elementary’ ones?

Naively, S=1 bilinears: $\bar{\psi}_i \vec{\sigma}_{ij} \psi_j$

In fact monopole is an S=1/2 boson: $f_i^\dagger \mathcal{M}_a$

Borokhov, Kapustin, Wu ’02

Possible duality with easy-plane NCCP1 model

Karch & Tong ‘16 Wang, AN, Metlitski, Xu, Senthil ’17

QED—Gross-Neveu

Two 2-component Dirac fermions, dynamical U(1) gauge field

$$\mathcal{L}_{qed-gn} = \sum_{j=1}^{2} i\bar{\psi}_j \not{D}_a \psi_j + \phi\bar{\psi}\psi + V(\phi)$$

Ising scalar tuned to critical point

Elementary 'order parameters'

$$f_1^\dagger \mathcal{M}_a \qquad f_2^\dagger \mathcal{M}_a \qquad \phi$$

5 real components…

Dual representations of 5 order parameters

Conjecture: duality to NCCP^1, emergent SO(5)

QED—GN symmetries

NCCP1 symmetries

$$
\begin{array}{llcll}
SU(2)_{\text{flavour}} & f_1^\dagger\mathcal{M}_a & \longleftrightarrow & \varphi_x - i\varphi_y & U(1)_{\text{VBS}} \\
 & f_2^\dagger\mathcal{M}_a & \longleftrightarrow & N_x + iN_y & SO(3)_{\text{spin}} \\
T & \phi & \longleftrightarrow & N_z &
\end{array}
$$

Under previous assumptions about stability of SO(5) fixed point, *either* symmetry can protect emergent SO(5) (at critical point)

Wang, AN, Metlitski, Xu, Senthil '17

Dual representations of 5 order parameters

QED—GN symmetries

NCCP1 symmetries

$$SU(2)_{\rm flavour} \begin{cases} f_1^\dagger \mathcal{M}_a \longleftrightarrow \varphi_x - i\varphi_y & U(1)_{\rm VBS} \\ f_2^\dagger \mathcal{M}_a \longleftrightarrow N_x + iN_y & \end{cases}$$

$$T \quad \phi \longleftrightarrow N_z$$

$$\left.\begin{matrix} N_x + iN_y \\ N_z \end{matrix}\right\} SO(3)_{\rm spin}$$

NCCP¹ $\mathcal{L}_{\rm SO(5)} - \delta J \sum_{a=1,2} X^{(2)}_{aa} + \lambda \sum_{a,b=1,2} X^{(4)}_{aabb} + \kappa \sum_{a=1,2} X^{(4)}_{aaaa} + \ldots$

Tuning parameter Neel-VBS anisotropy "Z_4" VBS anisotropy

QED-GN $\mathcal{L}_{\rm SO(5)} + m^2 X^{(2)}_{55} + \lambda' X^{(4)}_{5555} + \ldots$

Ising mass Higher anisotropy

Dual representations of 5 order parameters

QED—GN symmetries

NCCP1 symmetries

$$SU(2)_{\text{flavour}} \begin{cases} f_1^\dagger \mathcal{M}_a \longleftrightarrow \varphi_x - i\varphi_y & U(1)_{\text{VBS}} \\ f_2^\dagger \mathcal{M}_a \longleftrightarrow N_x + iN_y & \end{cases}$$

$$T \qquad \phi \longleftrightarrow N_z$$

$$\left.\begin{matrix} N_x + iN_y \\ N_z \end{matrix}\right\} SO(3)_{\text{spin}}$$

NCCP1 $$\mathcal{L}_{\text{SO}(5)} - \delta J \sum_{a=1,2} X^{(2)}_{aa} + \lambda \sum_{a,b=1,2} X^{(4)}_{aabb} + \kappa \sum_{a=1,2} X^{(4)}_{aaaa} + \ldots$$

Tuning parameter | Neel-VBS anisotropy | "Z_4" VBS anisotropy

QED-GN $$\mathcal{L}_{\text{SO}(5)} + m^2 X^{(2)}_{55} + \lambda' X^{(4)}_{5555} + \ldots$$

Ising mass | Higher anisotropy

assumed irrelevant

[Aside: duality building blocks]

elementary bosonic duality

$$|D_A\Phi|^2 - |\Phi|^4 \longleftrightarrow |D_a w|^2 - |w|^4 + \frac{1}{2\pi} a\mathrm{d}A$$

elementary boson-fermion duality

$$i\bar{\psi}\not{D}_A\psi \longleftrightarrow |D_a w|^2 - |w|^4 + \frac{1}{2\pi} a\mathrm{d}A + \frac{1}{4\pi} a\mathrm{d}a + \frac{1}{8\pi} A\mathrm{d}A$$

$$\longleftrightarrow |D_{\tilde{a}}\tilde{w}|^2 - |\tilde{w}|^4 - \frac{1}{2\pi}\tilde{a}\mathrm{d}A - \frac{1}{4\pi}\tilde{a}\mathrm{d}\tilde{a} - \frac{1}{8\pi} A\mathrm{d}A$$

"S" and "T" operations

— make background gauge field dynamical

— add level 1 Chern Simons term for background gauge field

integrate out gauge field $\displaystyle\int_a e^{\frac{i}{2\pi}\int a\mathrm{d}A} = \delta(A)$

more handwavingly: deform each side to reach new fixed point

Can we make SO(5) explicit?

— in field theory?

— in lattice model?

Can we make SO(5) explicit?

WZW model?

$$S = \int \left(\frac{1}{g} (\nabla \vec{n})^2 + \text{strong } \cancel{\text{anisotropies}} \right) + S_{\text{WZW}}$$

Not well-defined ctm theory (nonrenormalizable), not useful for calculations

Possible alternative: QCD with $N_f = N_c = 2$

$$\mathcal{L} = \sum_{v=1,2} i \bar{\psi}_v \gamma^\mu (\partial_\mu - i a_\mu) \psi_v + \dots$$

SU(2) global

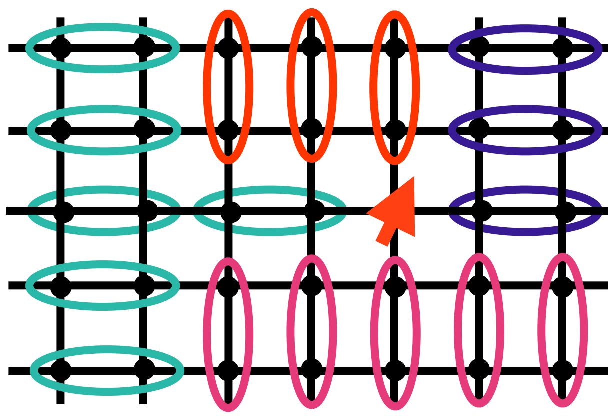
SU(2) gauge

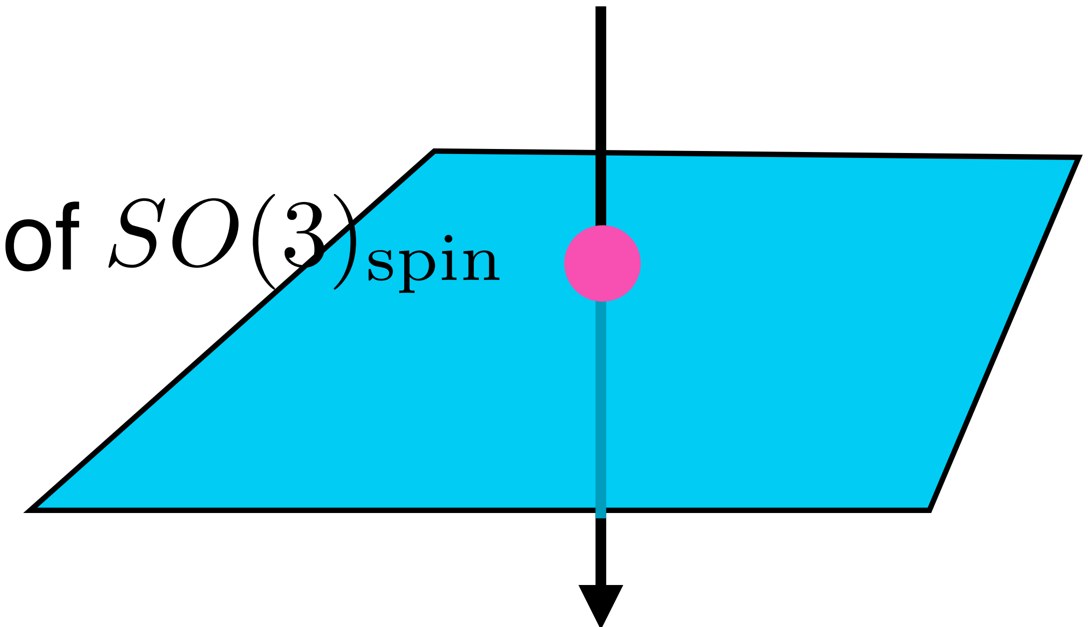
This theory has exact SO(5) and the same anomalies as DCP

One way to see: couple to $\vec{n}$ and integrate out fermions in large mass expansion (Abanov-Wiegmannization): gives WZW model

SO(5) anomaly

SO(5) cannot be incorporated in a microscopic 2+1D model, due to anomaly.





Must live at surface of SO(5) SPT ('topological paramagnet') with correct response to SO(5) gauge field $(n_1, n_2, n_3, n_4, n_5)$

QCD description implies a 3+1D parton construction for this SPT. Is there a nicer picture for its wavefunction?

Plan

Introduction

Evidence for SO(5) at the Neel-VBS transition

Dualities

NCCP1

dual NCCP1

SO(5)-invariant theory

QED—Gross-Neveu

dual QED—Gross-Neveu

Possibility of ‘pseudocritical’ behaviour

SO(5) in a very different microscopic model

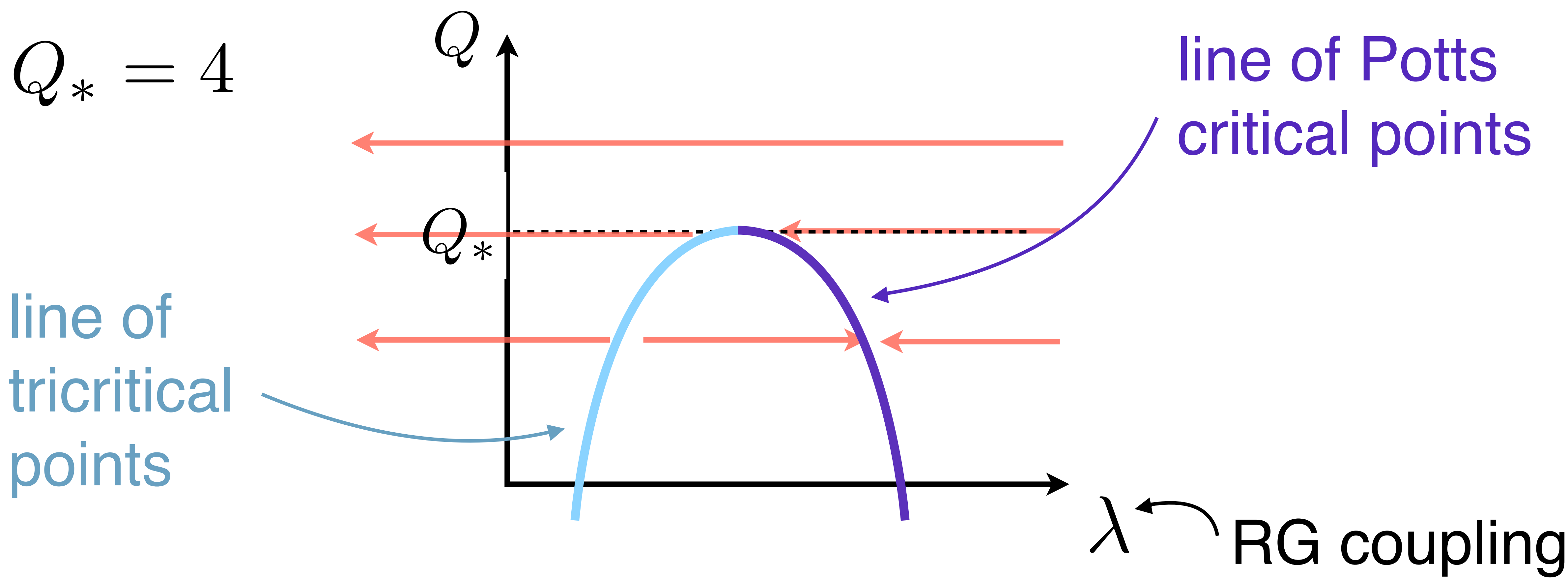
Pseudocriticality

Quasi—universality at a weak first order transition, due to nearby unphysical fixed point.

Illustrate using Q-state Potts model in 2d

Nienhuis et al ’79
Cardy, Nauenberg, Scalapino ’80

$$Q_* = 4$$



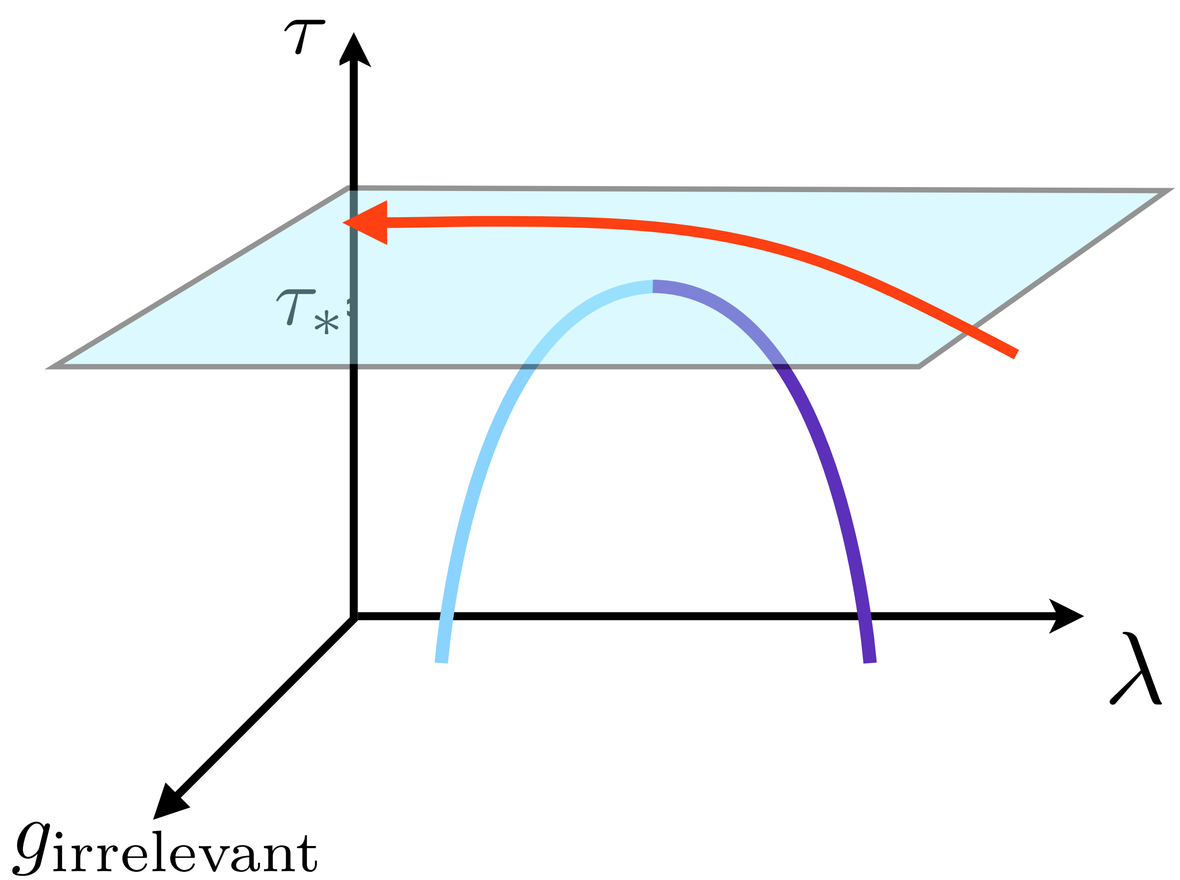
For $Q \gtrsim Q_*$, slow RG flow due to inaccessible nearby fixed point.

$$\xi \sim \exp\left(\frac{\pi^2}{\sqrt{Q - Q_*}}\right)$$

Pseudocriticality

Same topology of RG flows in many other theories, with different 'deformation parameter' τ

Small $\tau - \tau_*$: RG flows attracted to quasiuniversal flow **line**



$$\xi \sim e^{\frac{a}{\sqrt{\tau - \tau_*}}}$$

$$g_{\text{irrelevant}} \sim \xi^{-\text{const}}$$

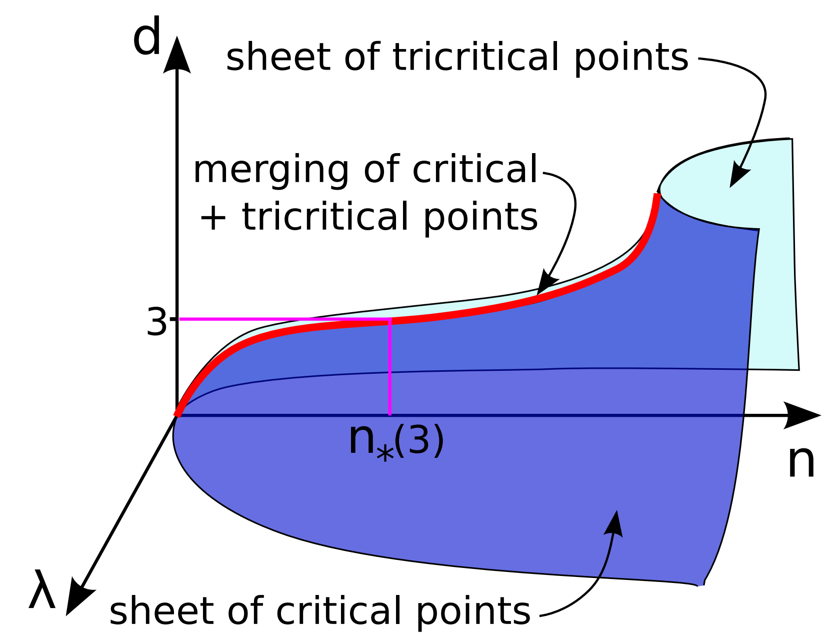
- Effective exponents, etc. drift as λ flows
- However, these drifts are quasiuniversal when ξ is large

AN, Chalker, Serna, Ortuno, Somoza '15   Wang, AN, Metlitski, Xu, Senthil '17

Application to NCCP^1

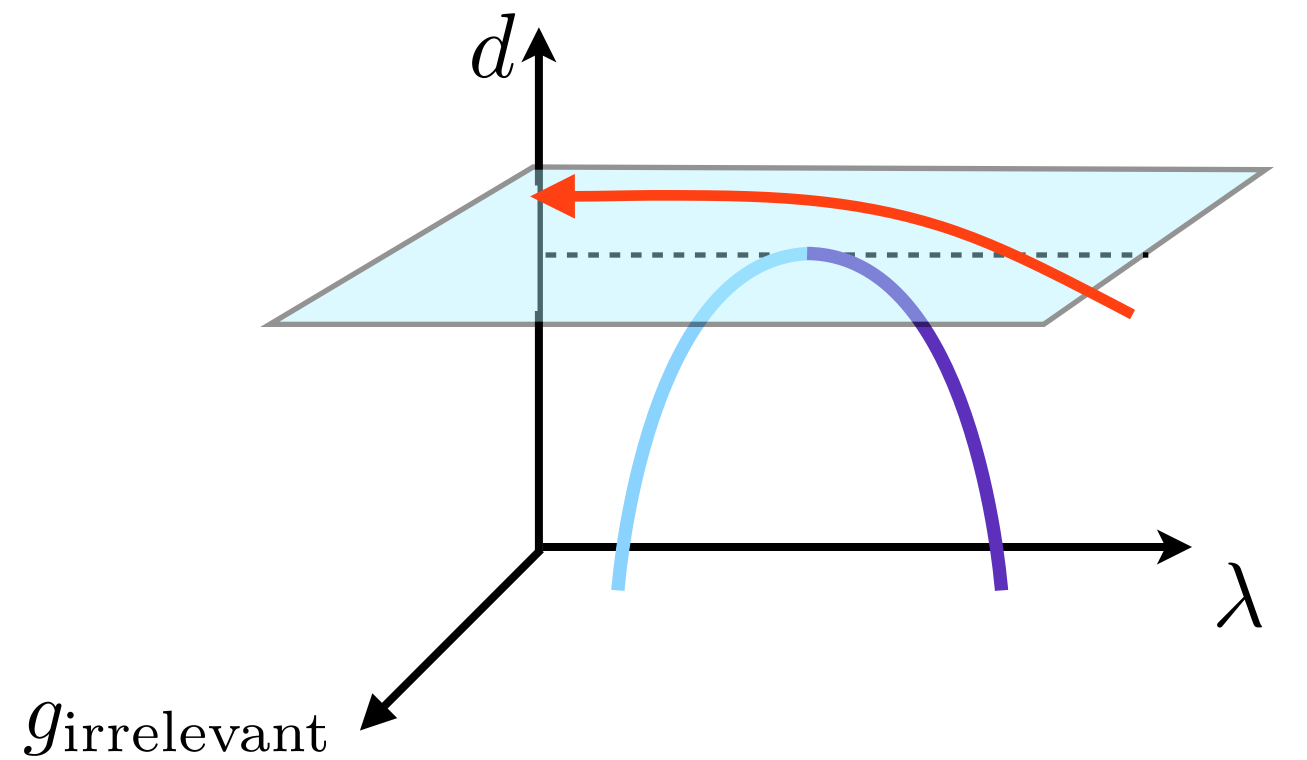
NCCP^1 in d=3 plausibly in this regime with $\xi \gg 640$

RG fixed points for NCCP^{n-1}
in d dimensions



Conjecture based on large n,
2+ϵ, 4-ϵ, replica limit.

Slice at constant n=2 (NCCP^1):



Can explain ubiquitous
'drifts' at DCP as quasiuniversal
feature of flow line

Can also be made
compatible with SO(5)

AN, Chalker, Serna, Ortuno, Somoza '15   Wang, AN, Metlitski, Xu, Senthil '17

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Possibility of ‘pseudocritical’ behaviour

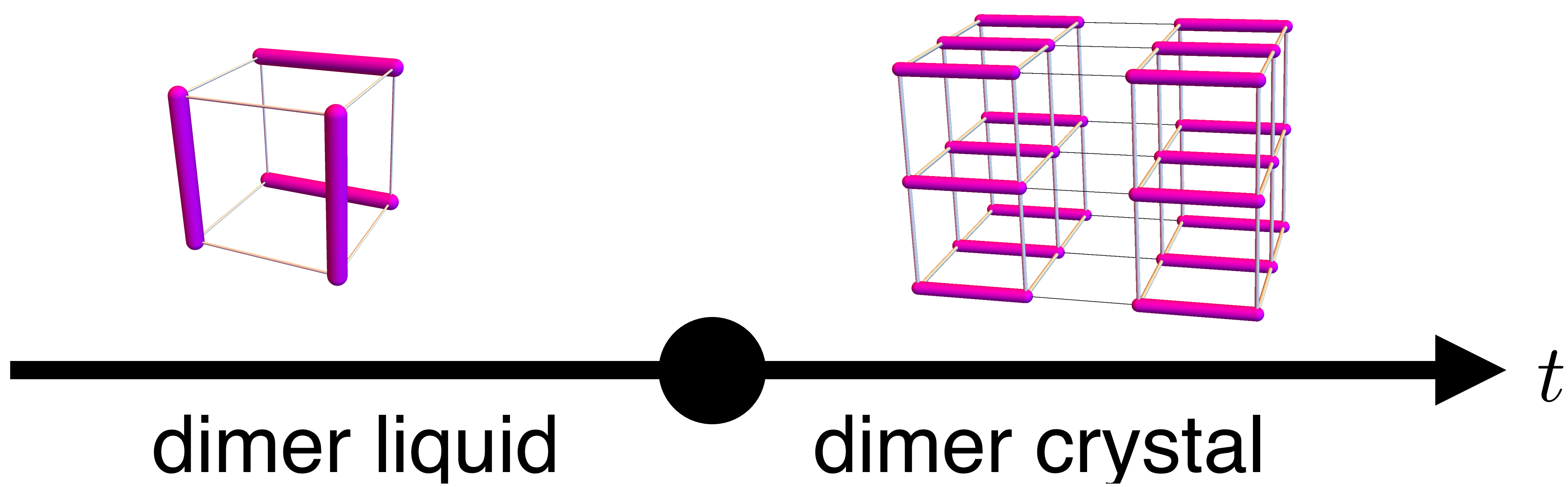
SO(5) in a very different microscopic model

SO(5) in a very different model

Classical dimers in 3D: columnar ordering transition

Argued to be described by NCCP[1] Powell, Chalker '08

Chen, Gukelberger, Trebst, Alet, Balents, '09



$$Z = \sum_{\substack{\text{fully-packed} \\ \text{dimer configs}}} e^{-E/T}$$

Vary interaction t on squares

Also, fixed interaction on cubes

Charrier & Alet

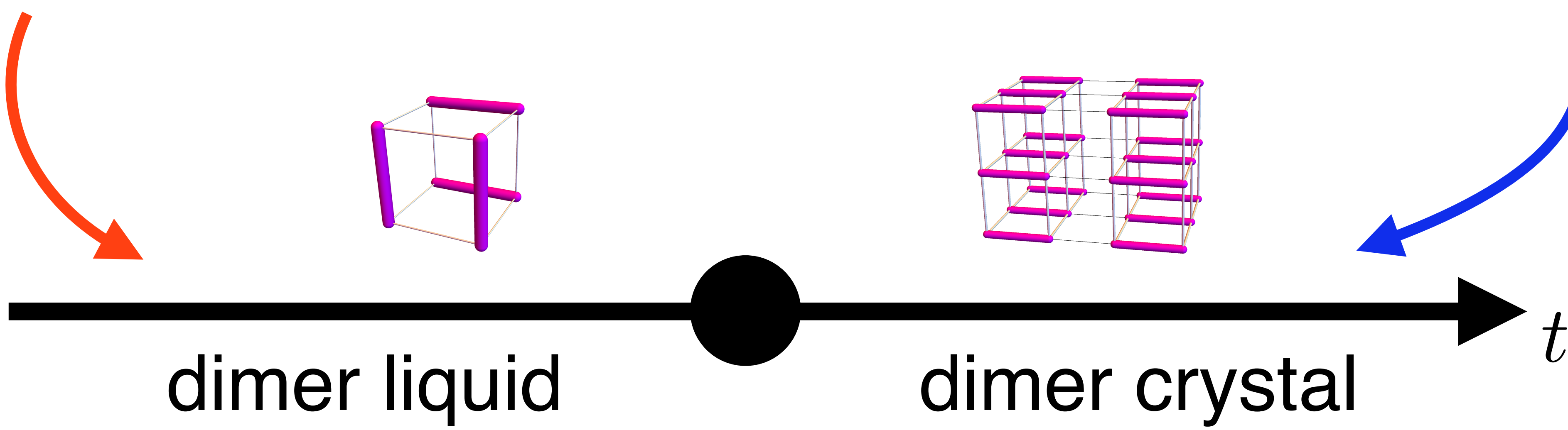
JQ model and loop model had SO(3) x lattice symmetry.

Here, only lattice x U(1).

SO(5) in a very different model

LRO for $\varphi_x + i\varphi_y$
operator which inserts
monomer (NCCP1 monopole)

Columnar order parameter
$\vec{N} = (N_x, N_y, N_z)$
6-fold anisotropy



Microscopic symmetry $\left(\varphi_x, \varphi_y, N_x, N_y, N_z\right)$ → **SO(5)?**

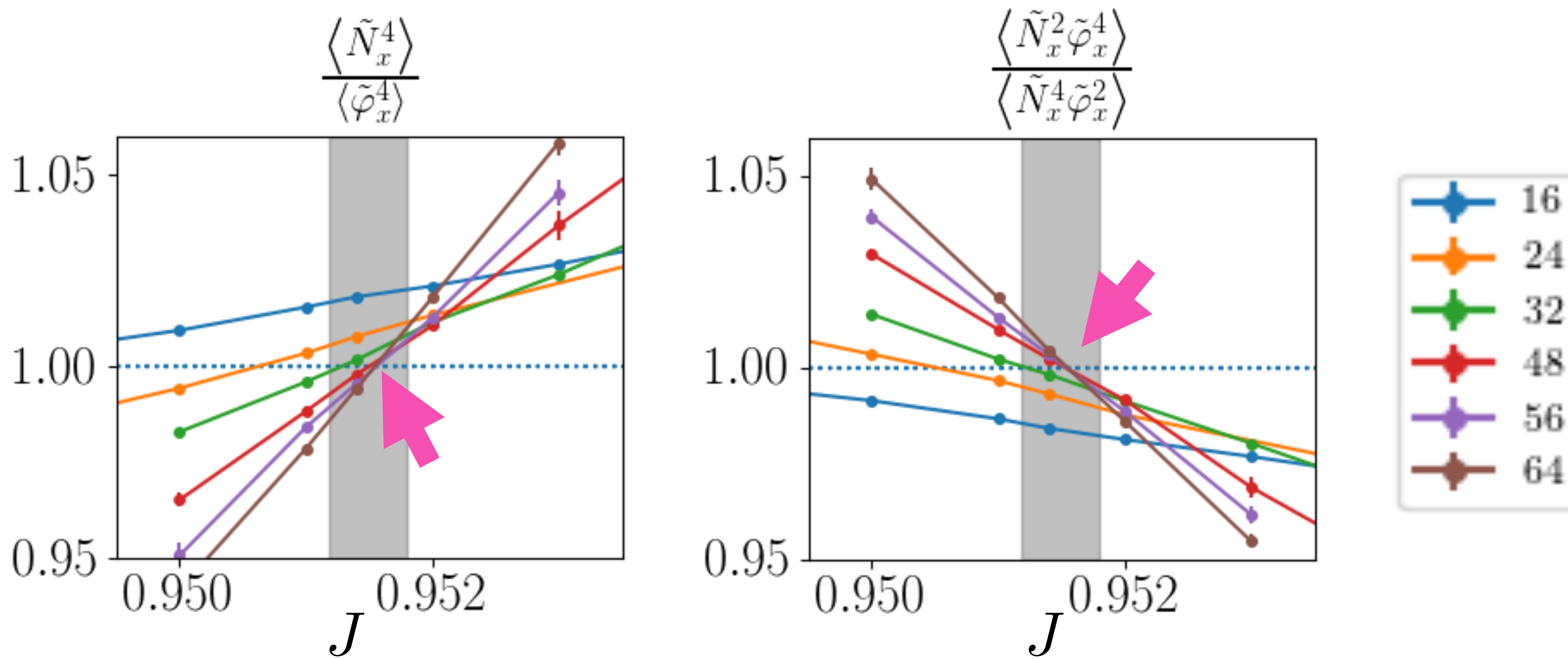
U(1) $\rtimes$ lattice

Emergence of SO(5) is allowed here:

$$\mathcal{L} = \mathcal{L}_{\mathrm{SO}(5)} + \delta J \sum_{a=1,2} X^{(2)}_{aa} + \lambda \sum_{a=1,2} X^{(4)}_{aabb} + \kappa \sum_{\underline{a=3,4,5}} X^{(4)}_{aaaa} + \dots$$

Classical dimers: numerics

Check emergent U(1) symmetry for (N_x , φ_x)



Moment ratios consistent with SO(5) to very good precision

Confirmation that SO(5) is generic

Sreejith, AN, Powell, in preparation

Outlook

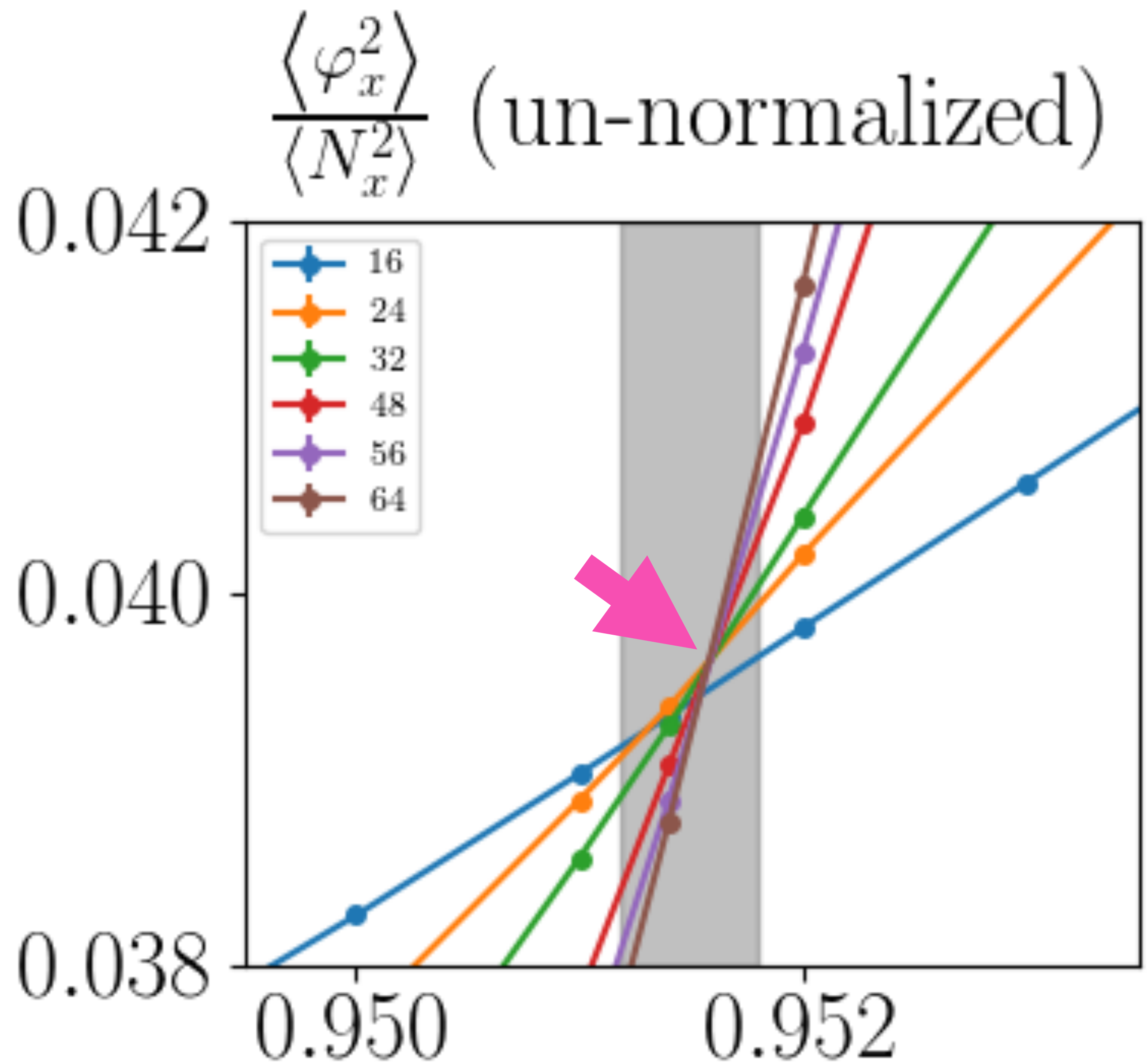
Emergent SO(5) in Neel—VBS transition and related models to very high precision

Conjectured duality webs for SO(3)-symmetric and easy-plane deconfined criticality. Further simulations for fermionic theories?

Picture for 'quasiuniversal' behaviour due to nearby fixed-point annihilation. Applications to other quantum phase transitions?

Connection to 3+1D SO(5) topological paramagnet
Nice model for this?

extra slides



Strategy for simulations

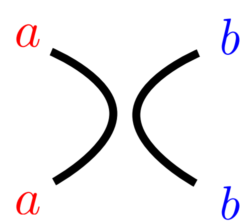
Heisenberg AFM → stat mech of loops in spacetime

Change basis on B sublattice so singlet is

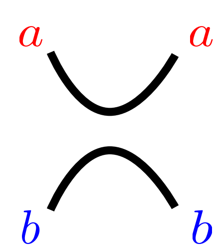
$$\frac{1}{\sqrt{2}}\left(|{\color{red}\bullet}\rangle_A|{\color{red}\bullet}\rangle_B + |{\color{blue}\bullet}\rangle_A|{\color{blue}\bullet}\rangle_B\right)$$

Two spins with S.S coupling:

$$e^{-\delta t\; H} \simeq$$

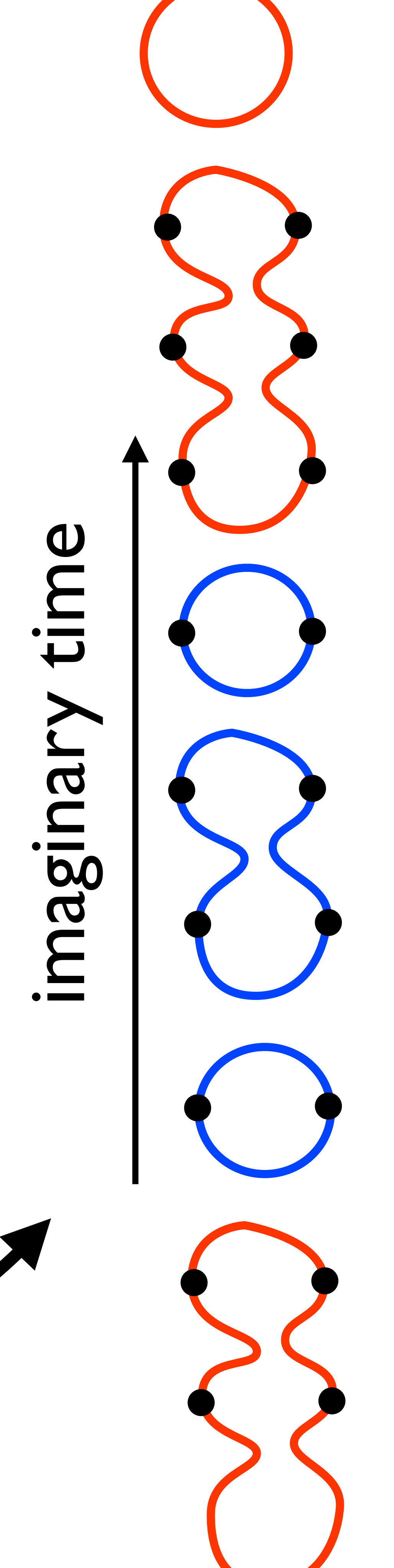


$$+\,\delta t\, J$$



Graphical representation for partition function:

$$Z(\beta) = \mathrm{Tr}\left(e^{-\delta t H}\right)^{\beta/\delta t} = \sum_{\substack{\text{coloured}\\ \text{loop configs}}} (\ldots)$$



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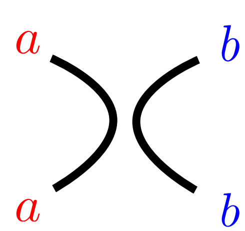

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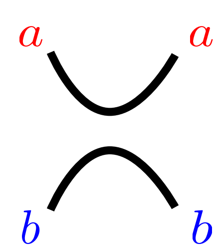

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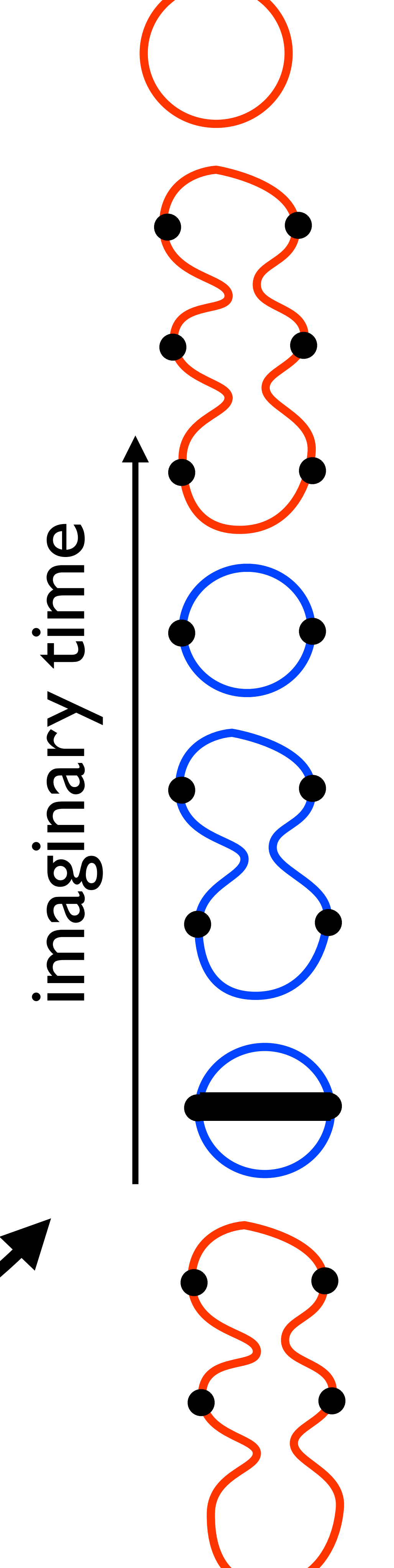
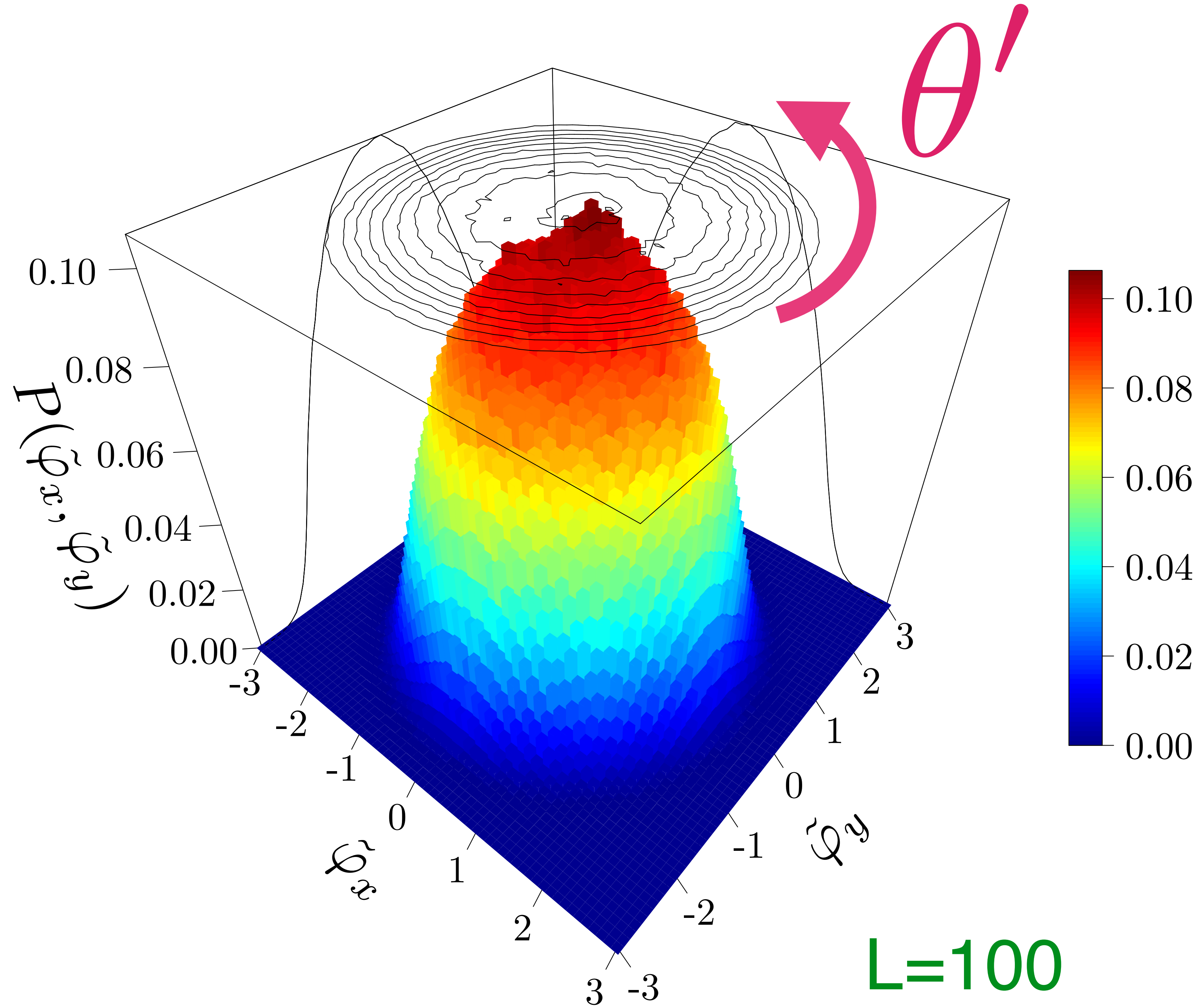


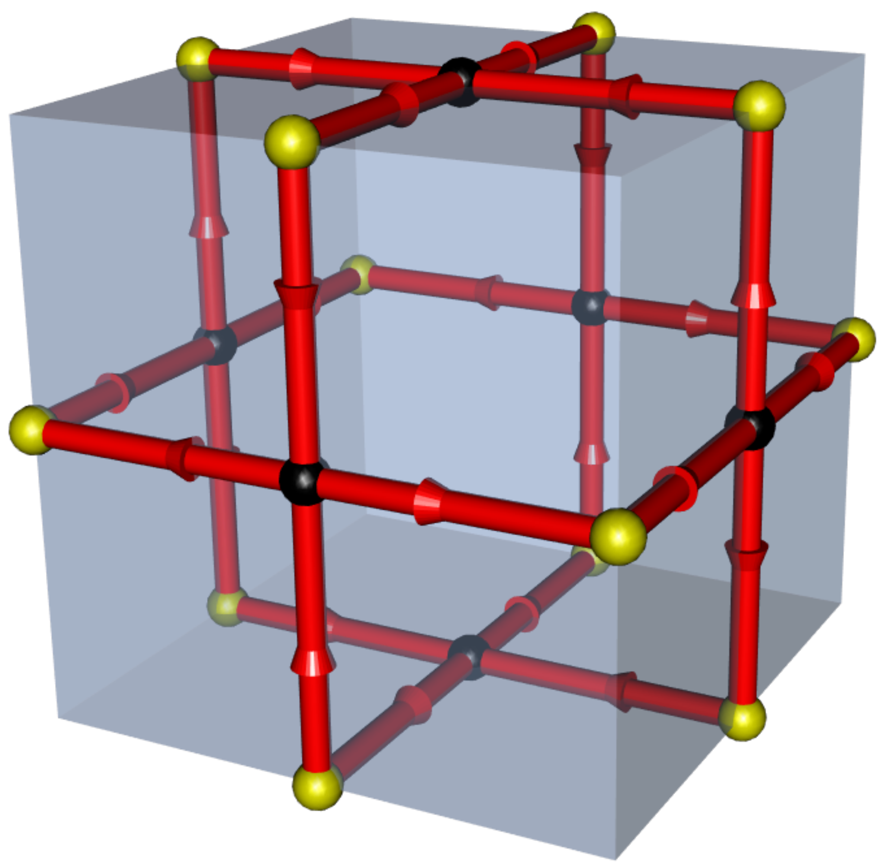
‘Old’ emergent U(1) symmetry for VBS



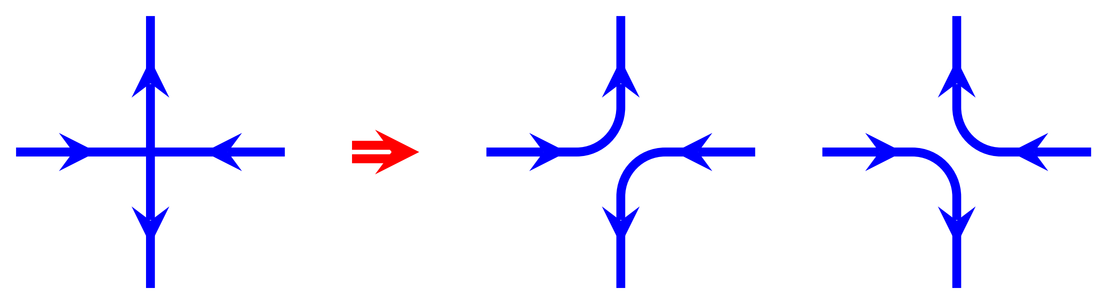
$\langle \cos 4\theta' \rangle = 0.0028(14)$

(Variances normalized to one)

Interactions



Loop configuration given by breaking up each node



$$\boxed{Z = \sum_{\substack{\text{coloured}\\ \text{loop configs}}} \exp\left(J \sum \varphi\varphi\right)}$$

Interaction is between n.n. nodes on same sublattice

Mapping to $NCCP^1$ model

Analogous to Heisenberg magnet.

(transfer matrix ➜ spin-$^1/_2$ on square lattice)

Exact mapping of loop model to lattice CP^1 model.

Microscopic gauge group is compact, but:

Explicit Berry phase calculation for hedgehogs:

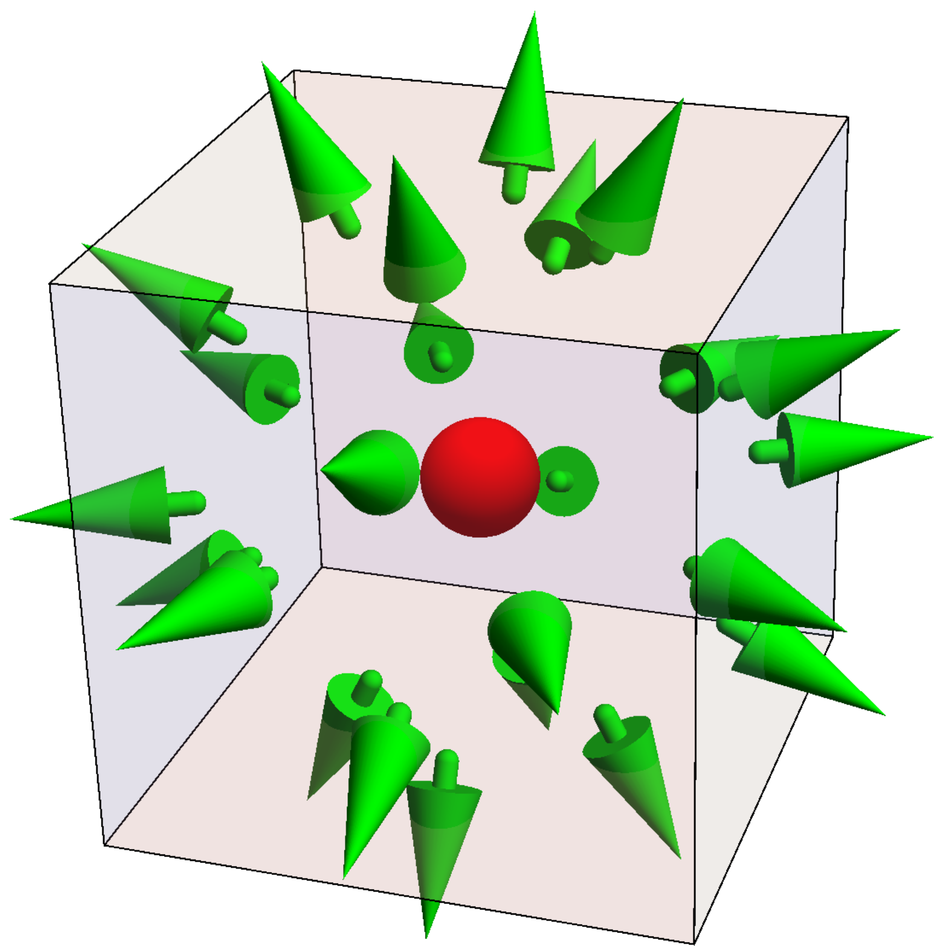
$$1, i, -1, -i$$

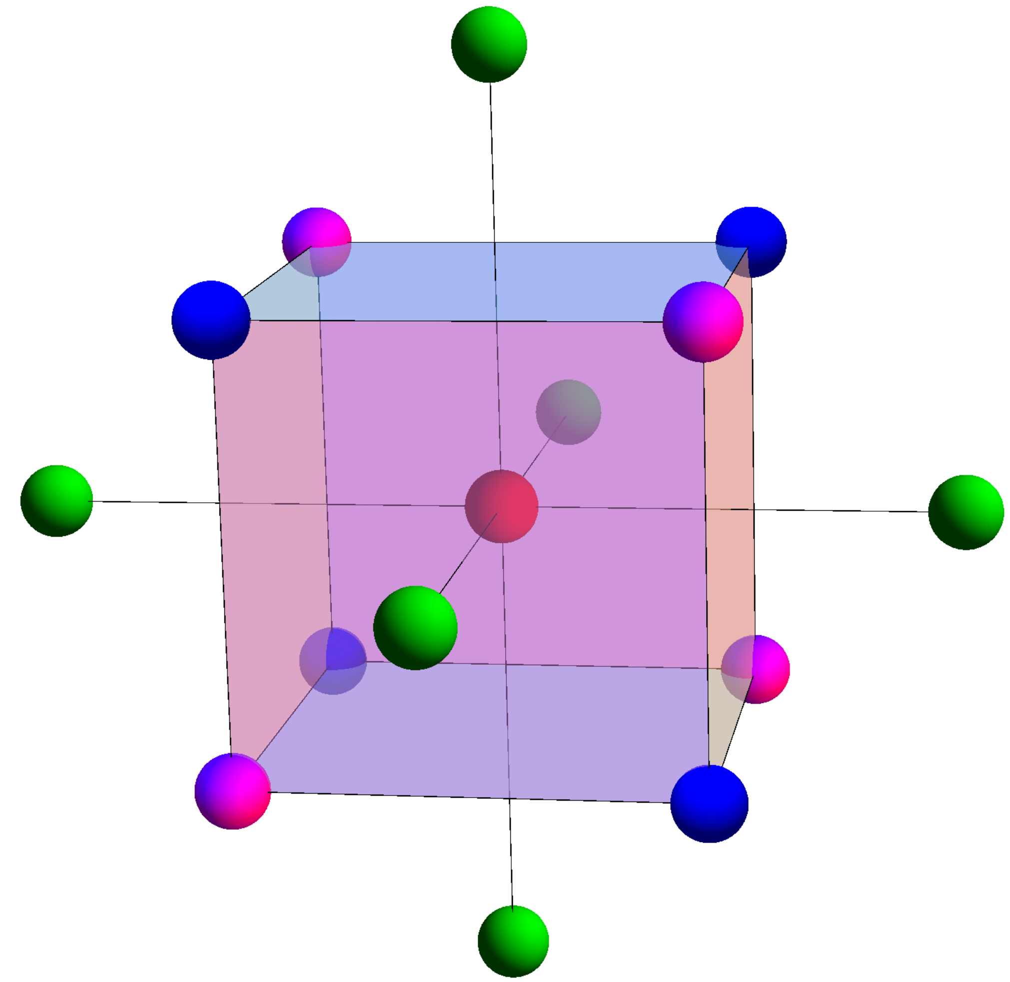
only quadrupled hedgehogs in continuum.

(Dual point of view: VBS vortex = spinon worldline)

PRL 107, 110601 (2011), PRB 88, 134411 (2013), and arXiv: 1506.06798

Hedgehogs



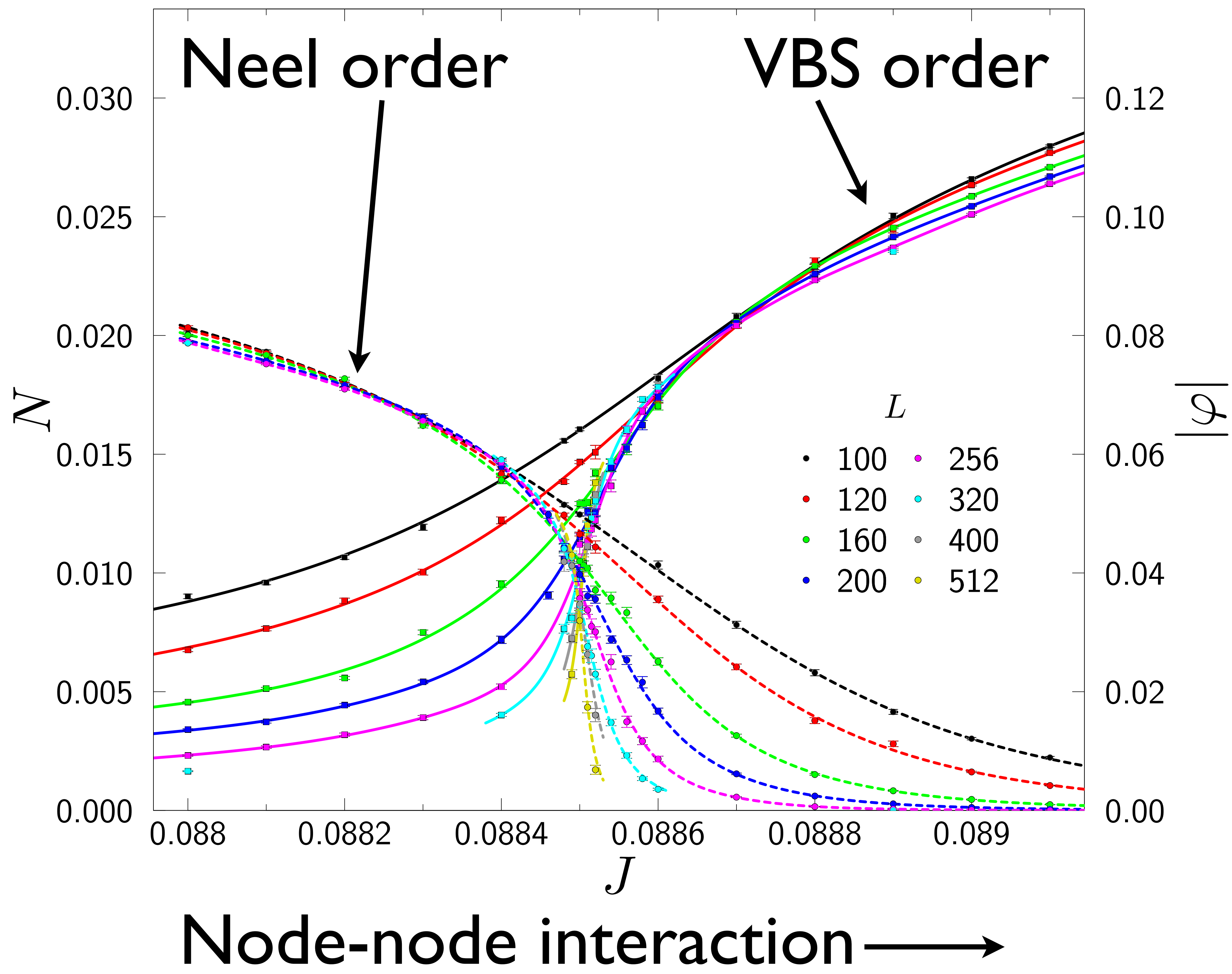


Microscopic hedgehog fugacity proportional to $1, i, -1, -i$

Coarse-grained hedgehog fugacity vanishes $\Rightarrow$ NCCP^1

(More precisely: only quadrupled hedgehogs in continuum)

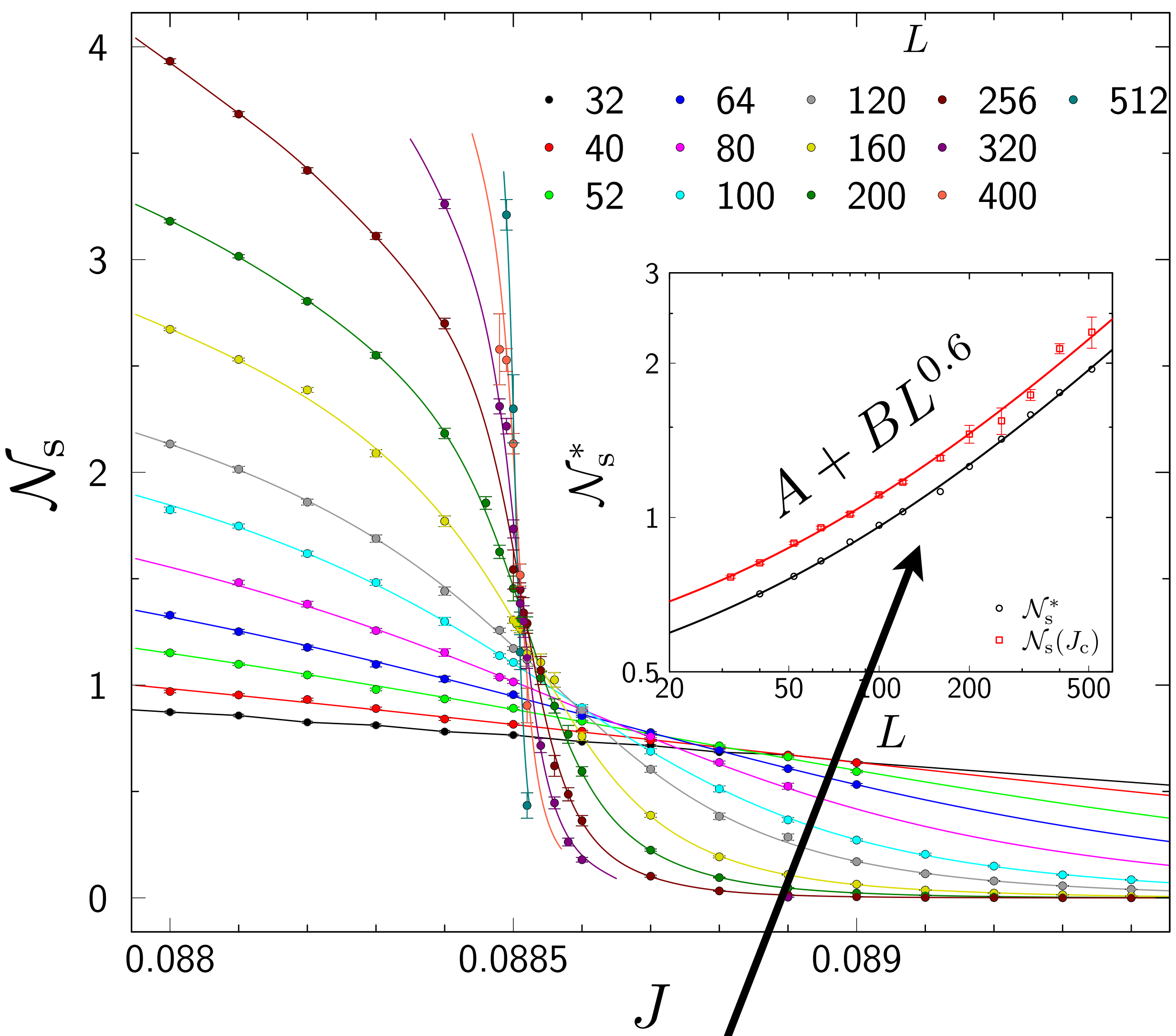
Direct, apparently continuous transition



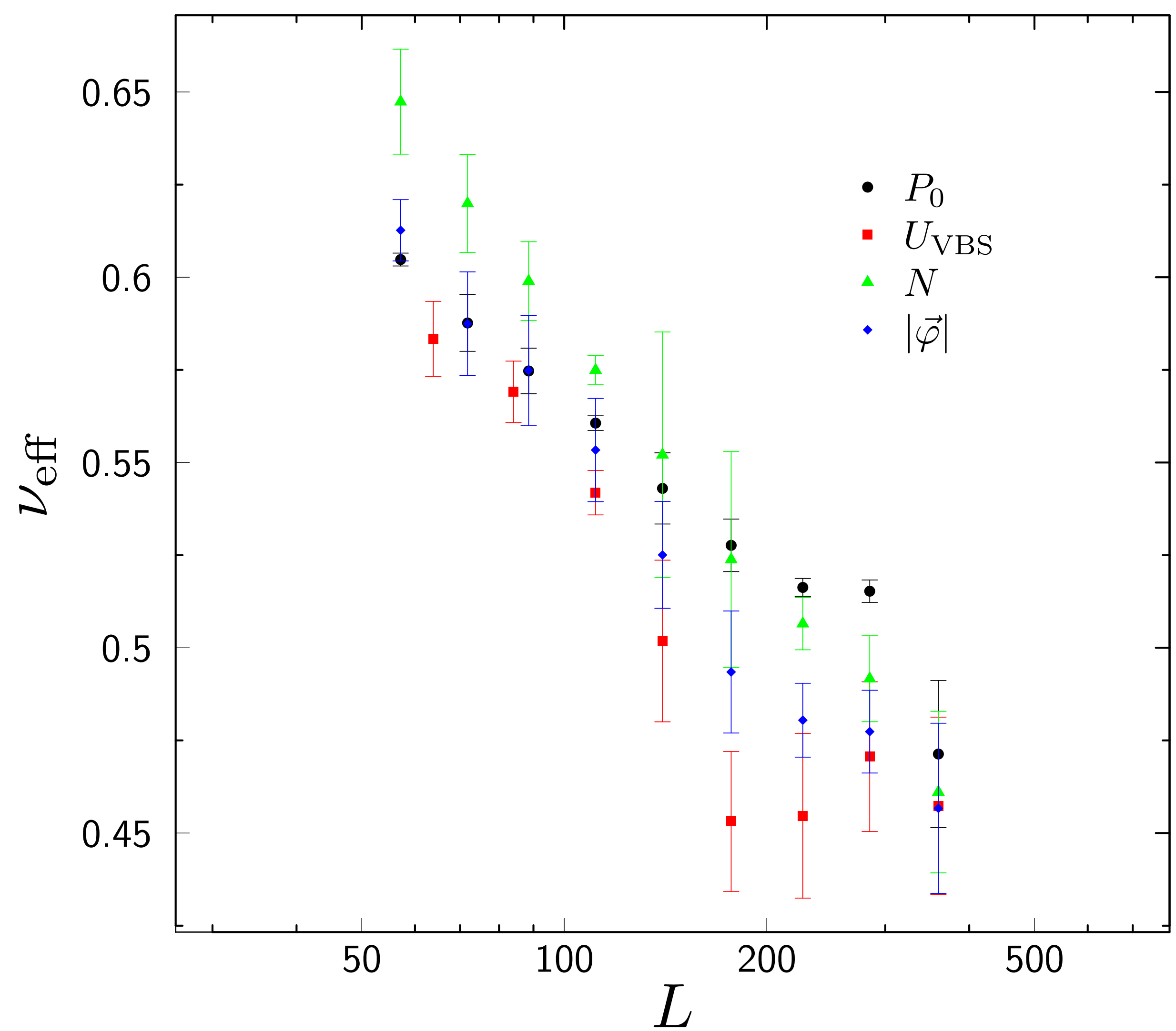
Scaling collapse fails

(e.g. order parameters, heat capacity.....)

Diverging Neel stiffness



Critical exponents drift

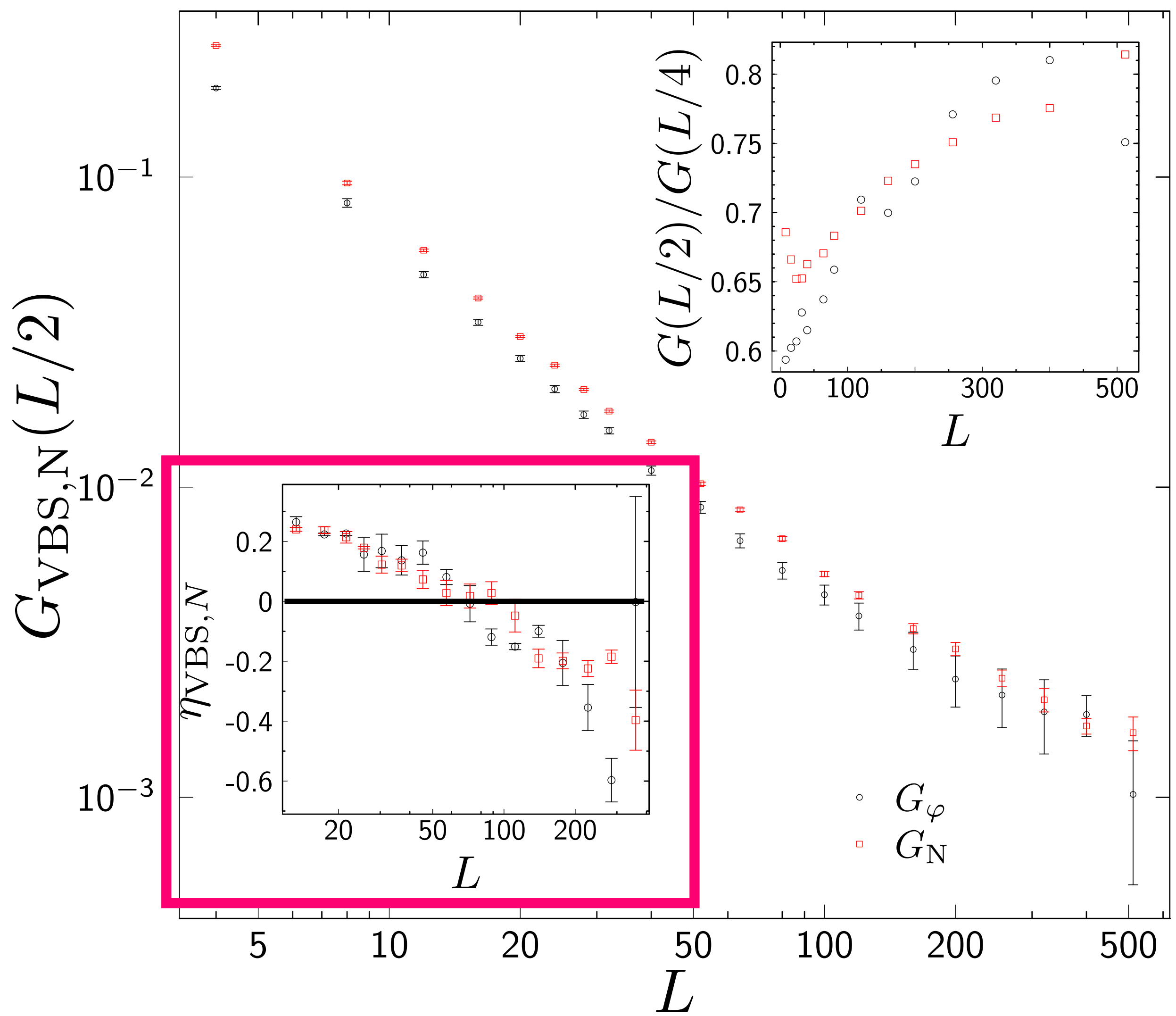


Power law divergence, not log (cf Sandvik '10)

Violation of unitarity bounds

Assuming $G(r, L) = L^{-(1+\eta)} f(r/L)$,

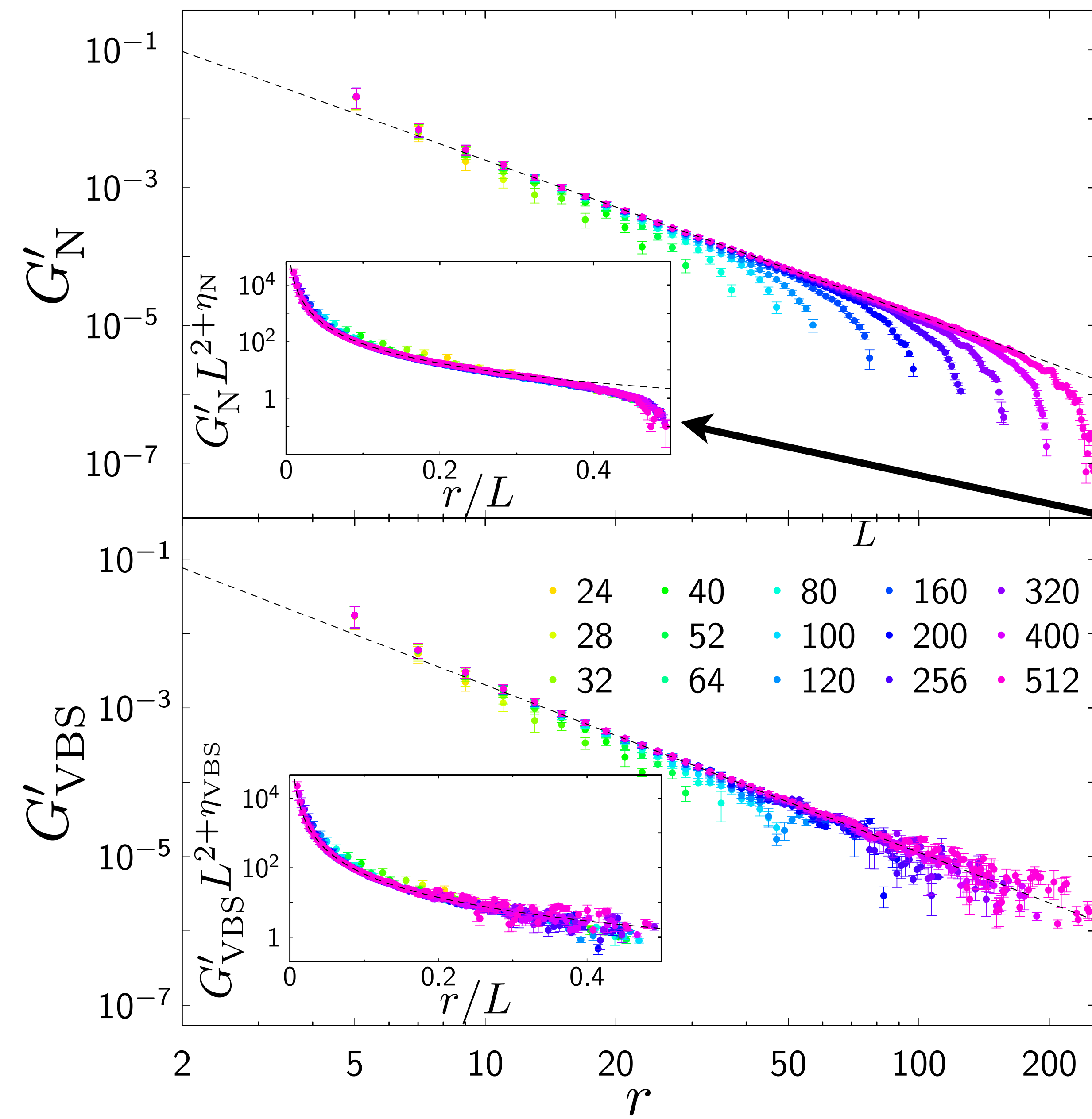
anomalous dimensions for ***Neel/VBS*** *become negative at large size*



But $\eta \geq 0$

for any unitary CFT

extra slides



scaling collapse!

$\eta_{\mathrm{Neel}} = 0.259(6)$

$\eta_{\mathrm{VBS}} = 0.25(3)$