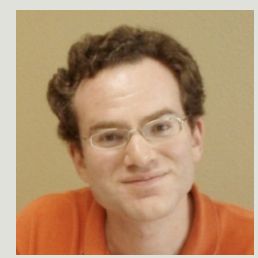
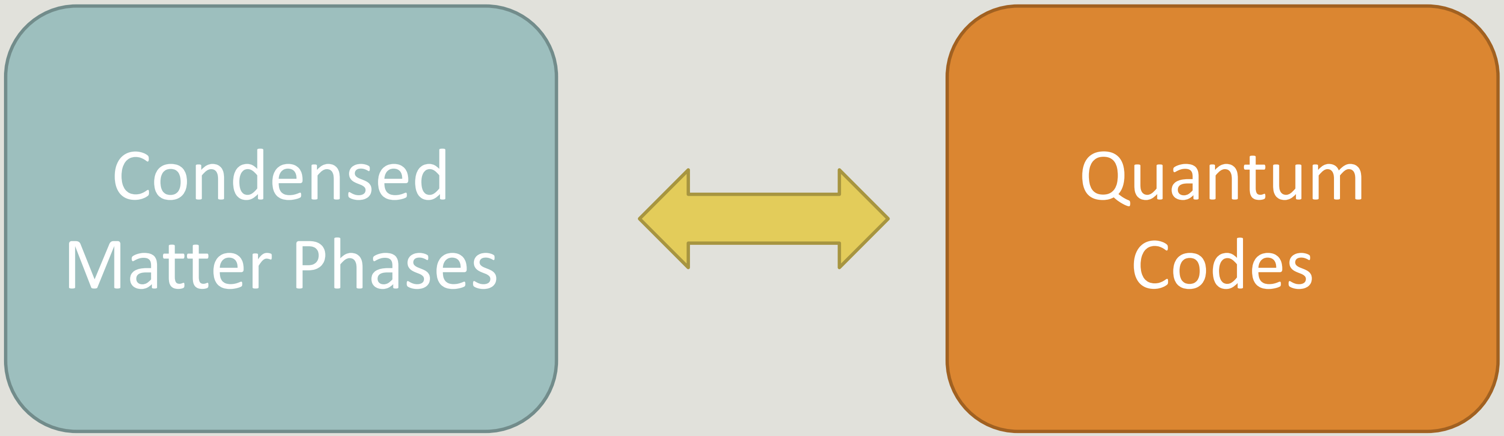
Coupled layer construction of Fracton models

XIE CHEN, CALTECH

KITS, JULY, 2017



arXiv:1701.00747, Han Ma, Ethan Lake, XC, Michael Hermele
Han Ma, Michael Hermele, XC, to appear


Condensed
Matter Phases
Quantum
Codes

Ferromagnetic phase

0

↓

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1

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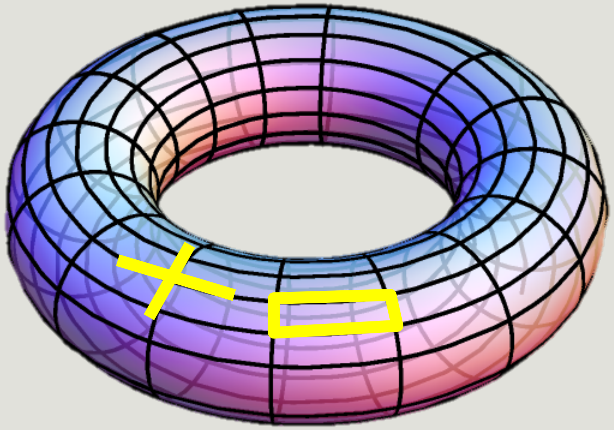
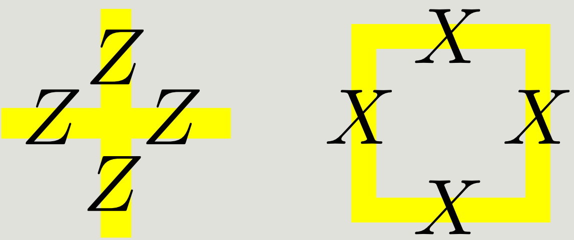
1111111
1111111

Classical repetition code

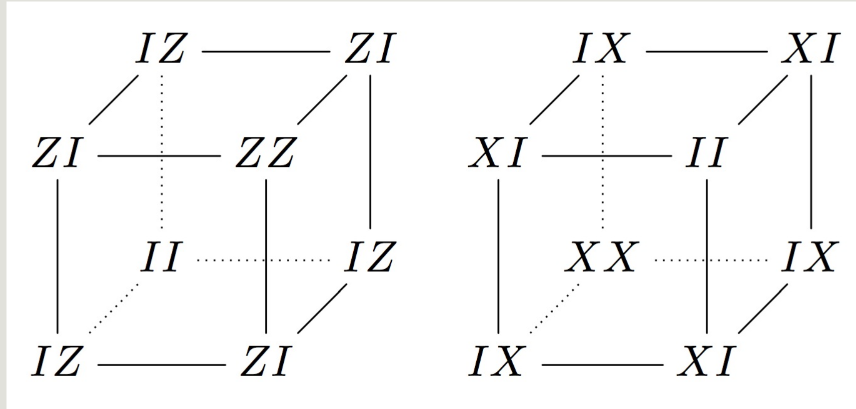
Toric Code
Topological
phase





Quantum
topological
code

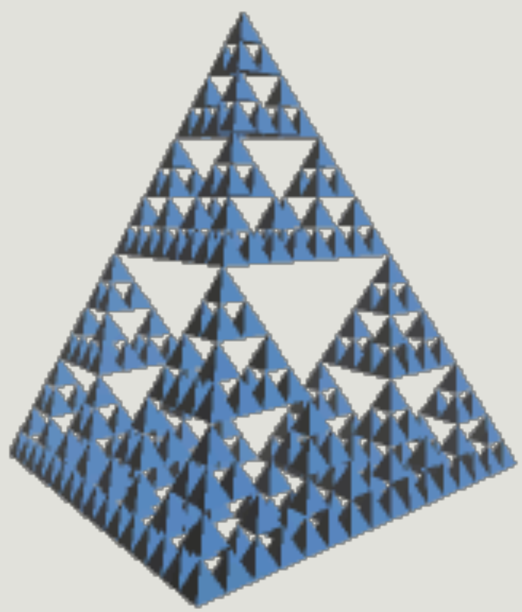
Kitaev, 1997



Haah’s code

(Haah, 2011)

Quantum memory



What kind of phase is it?!

Haah's code

- Are there other phases like it?
- Is it possible to relate its property to that of the known phases?
- Are there other phases in between Haah's code and known phases?

Known phases in 3D

Haah's code

Fracton I

Fracton II

Fracton III

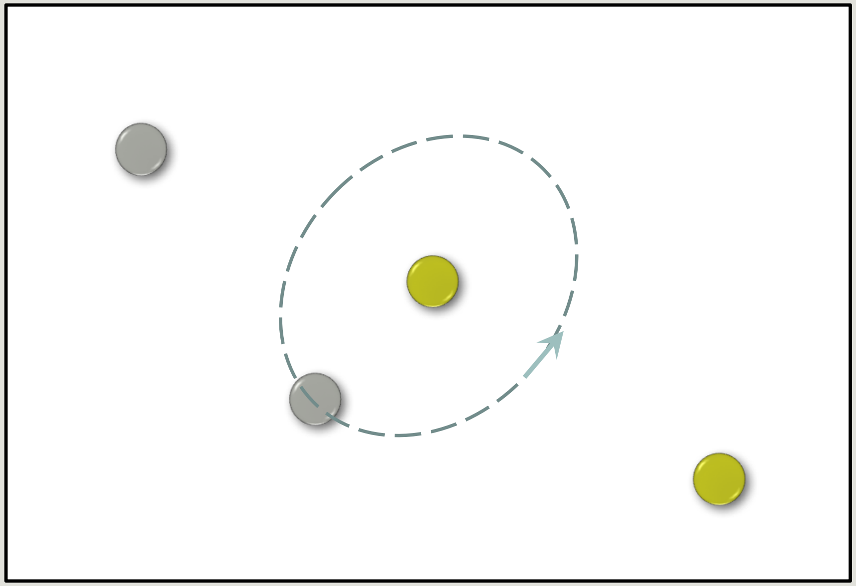
Vijay, Haah, Fu, (2015, 2016)

- ❑ More systematic understanding of 3D topological phase diagram
- ❑ Fractons beyond stabilizers or exactly solvable models
- ❑ More practical proposal for realizing quantum memory

Known phases in 3D

Anyons

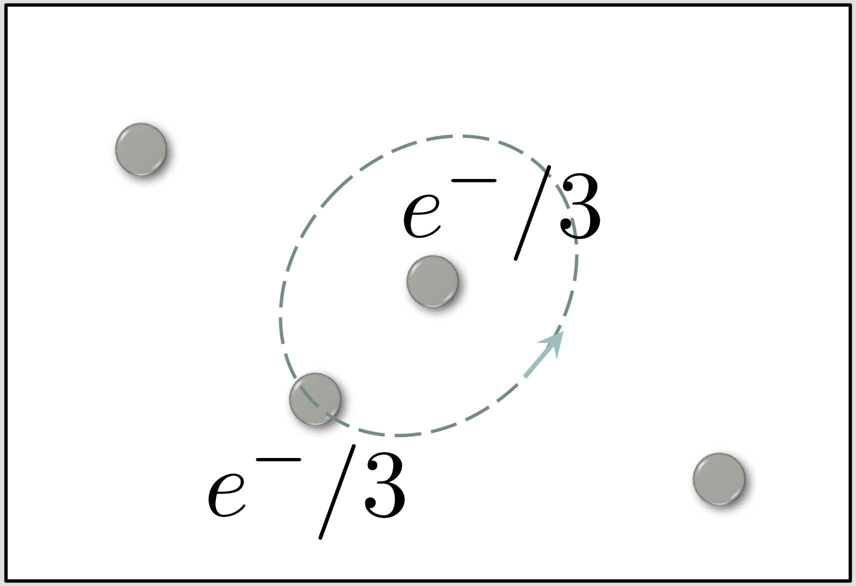
- Point like excitations in 2D topological phases
- Free to move
- Exchange or Braiding processes generates topological phase factors
- Exchange Bosons -- +1
- Exchange Fermions -- -1
- Exchange anyons – other phase factors



Fractional quantum Hall

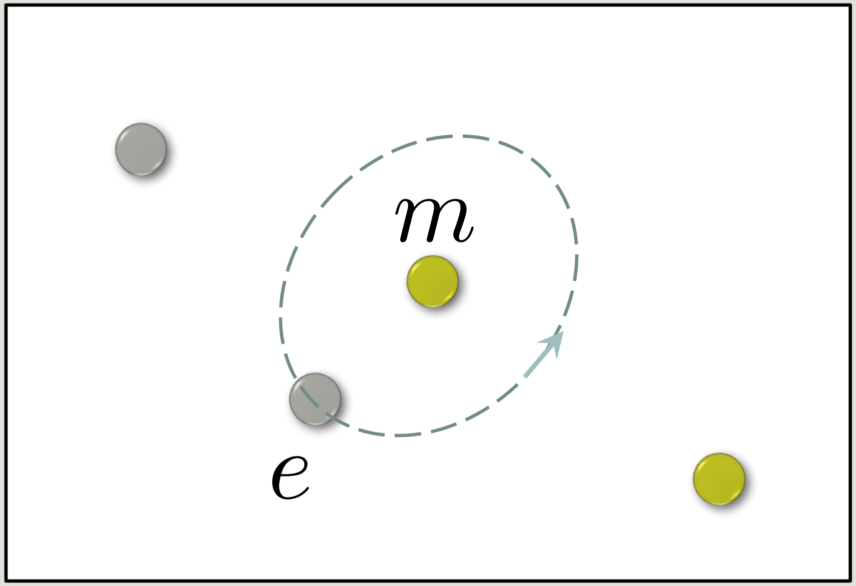
- Anyons – 1/3 of electron
- Braiding statistics –

$$2\pi/3$$



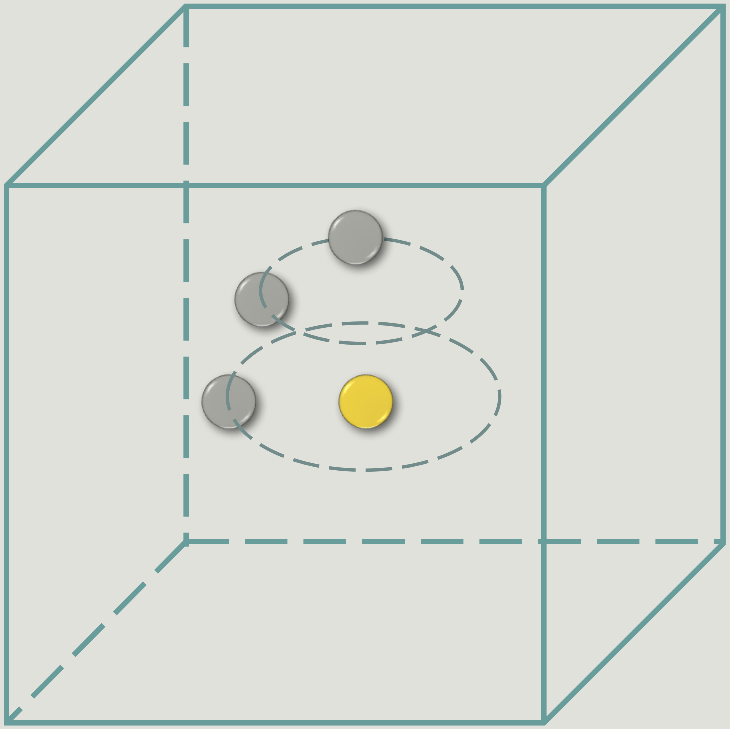
Toric Code

- electric anyon e
- magnetic anyon m
- Braiding statistics – π



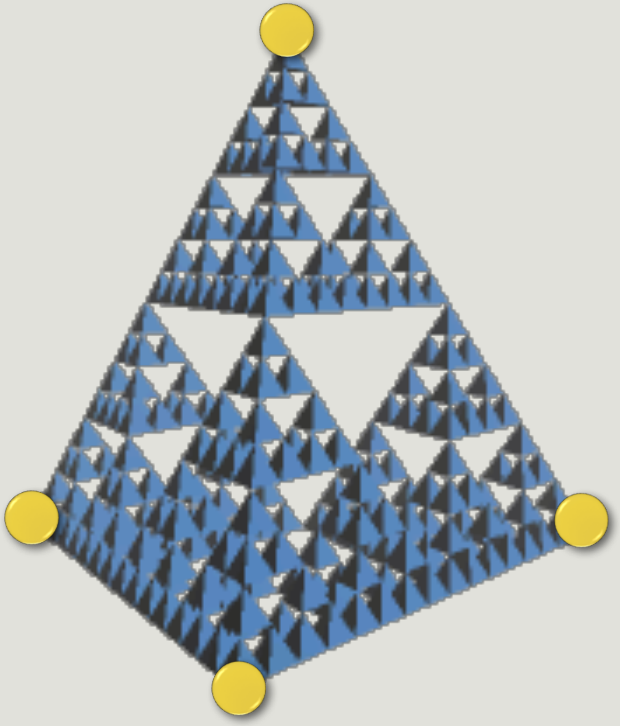
Point excitations in 3D

- A familiar type is one that is free to move
- Trivial braiding
- Exchange statistics +1 / -1
- Either boson / fermion
- No other anyons



Fractons

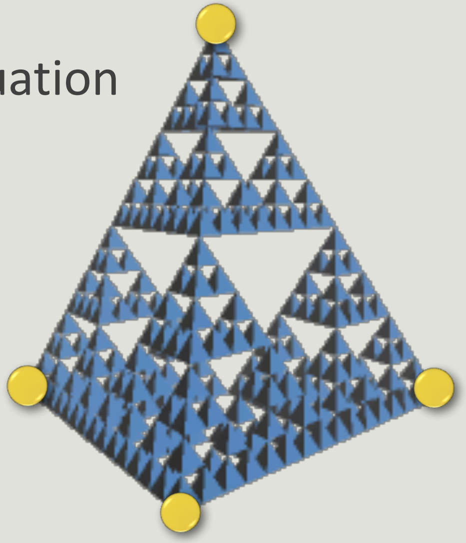
- Point excitations in 3D which cannot move freely
- Haah's code
- Point excitations appearing in set of four
- Located at vertices of rigid tetrahedral
- Each one cannot move freely
- Four move away from each other with a fractal structured operator
- Essential for quantum memory!



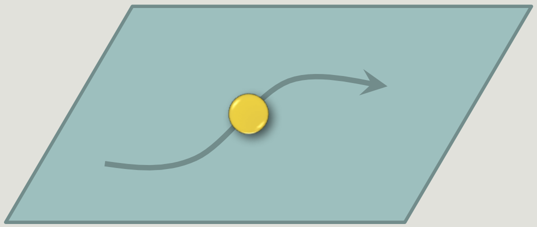
Haah’s code vs. 3D anyons

- Free to move
- Appearing at ends of string
- Constant energy cost for generation and separation
- Significant thermal fluctuation

- Not free to move
- Appearing at corners of tetrahedral
- Log energy cost for generation and separation
- Thermal fluctuation suppressed

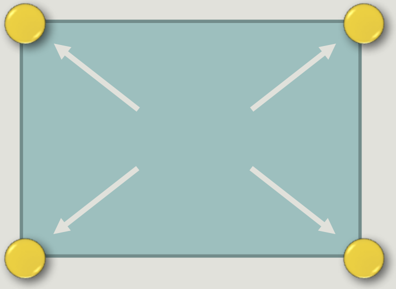


Fractons – new examples



Restricted to move in a 2D plane

Restricted to move along a 1D line



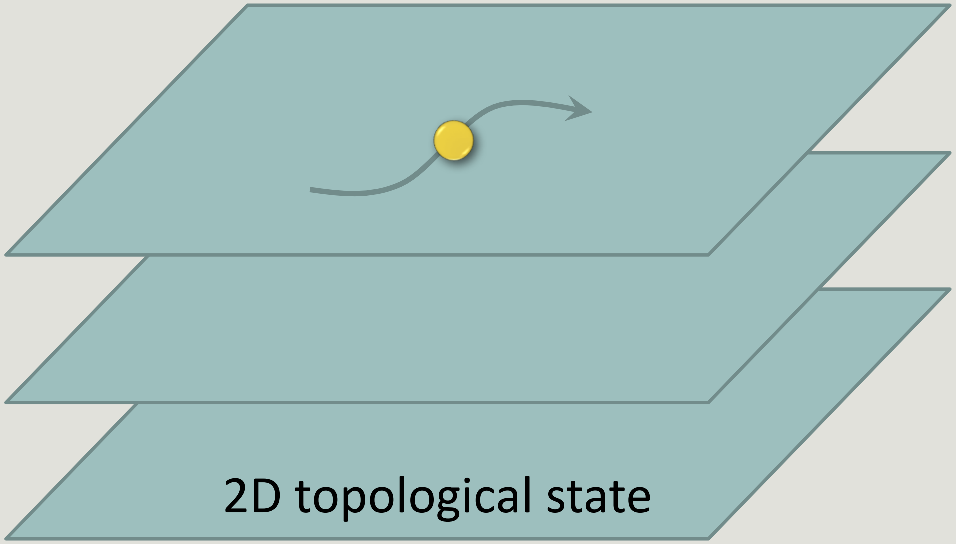
Restricted to be at the corners of a rectangle

- ❑ Do not give better quantum memories
- ❑ But do represent new topological orders

Vijay, Haah, Fu (2015)

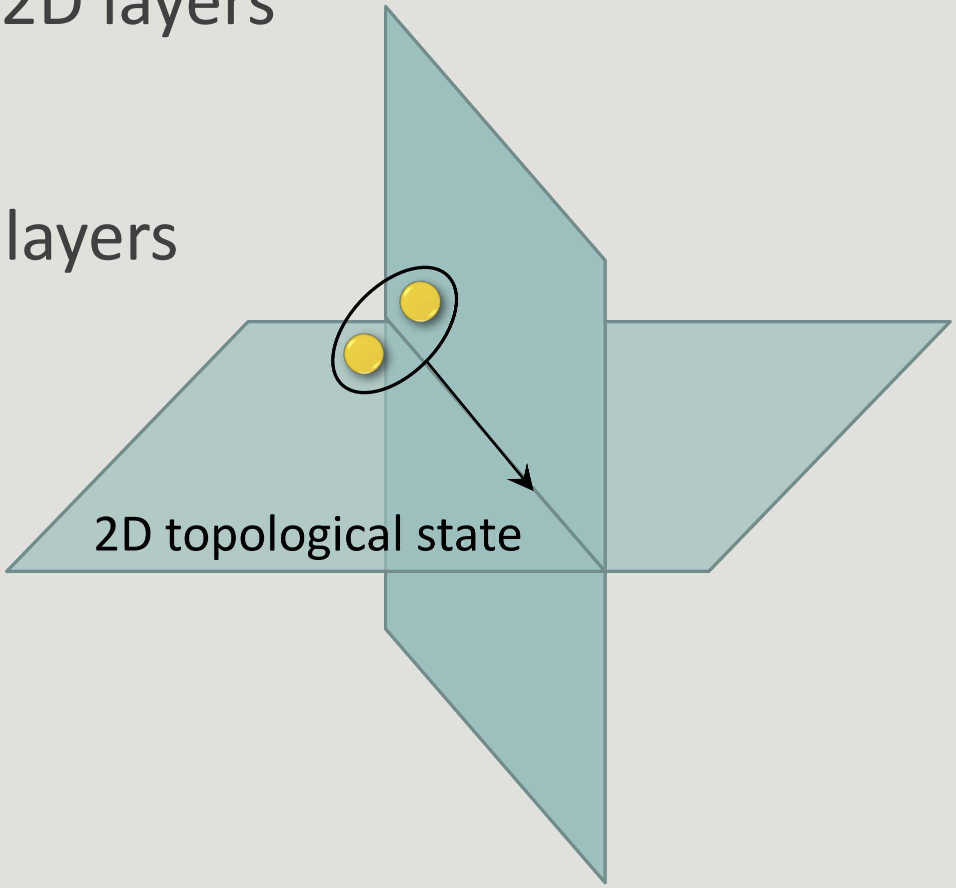
How surprising are they?

- Stack of 2D layers
- No coupling between layers
- Point excitations restricted to move in 2D



How surprising are they?

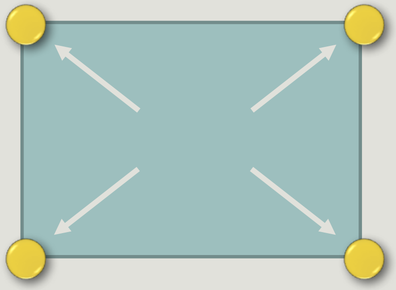
- Intersecting Stacks of 2D layers
- No coupling between layers
- Composite Point excitations restricted to move along 1D line



But things are not so simple

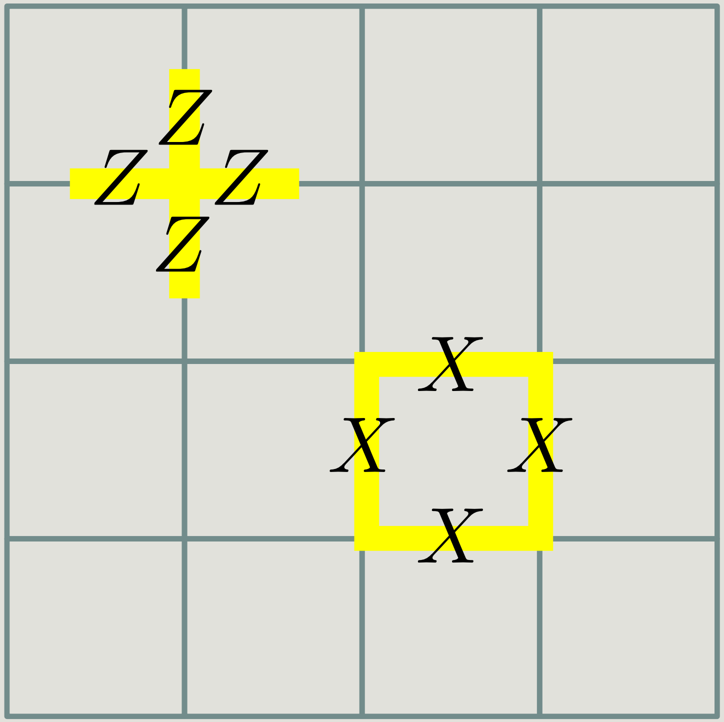
X-Cube model Vijay, Haah, Fu (2015)

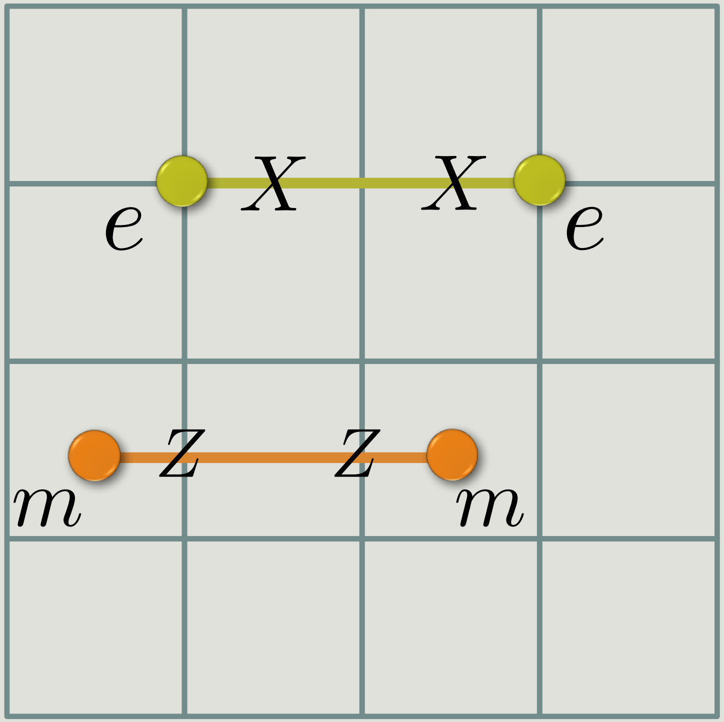
- Two types of point excitations



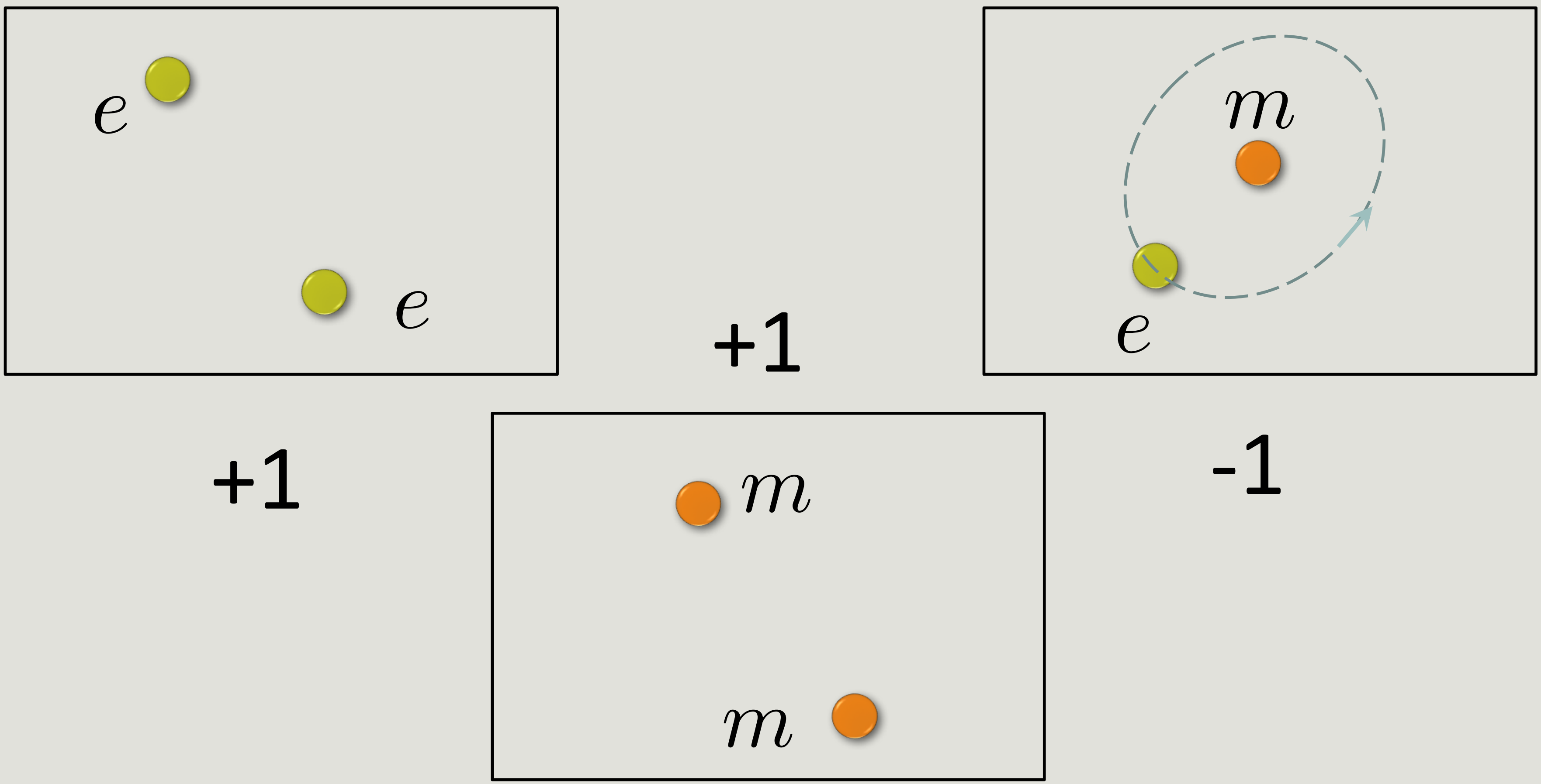
- No point excitation moving freely in 2D
- Definitely not pure stack of 2D layers
- Coupled layers?
- Couple 2D Toric Code layers with ‘p-loop’ condensation

Couple 2D Toric Codes into 3D Toric Codes

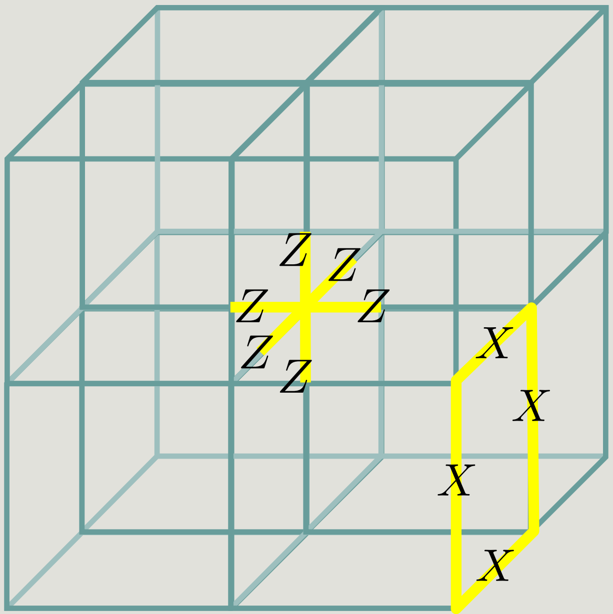


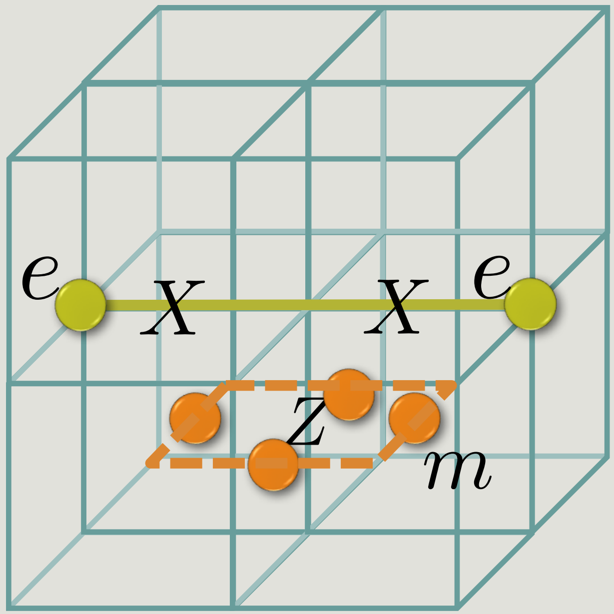


2D Toric Code

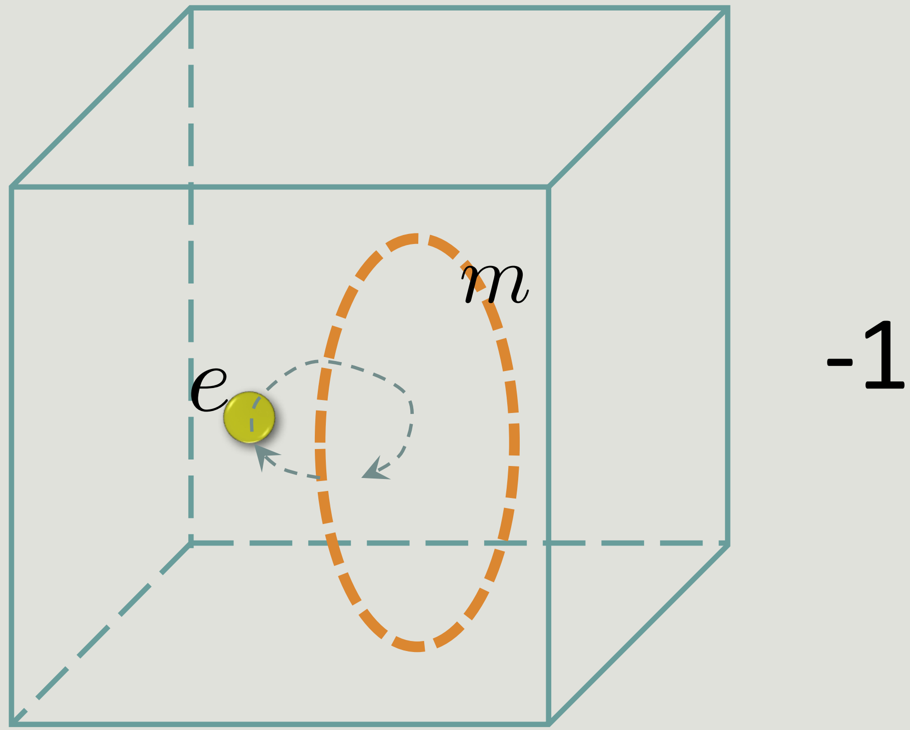


3D Toric Code





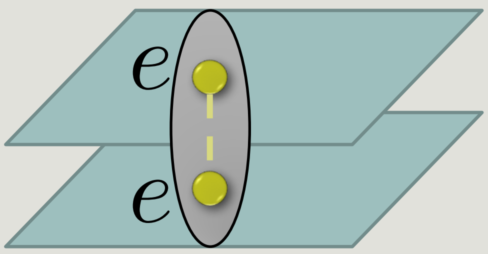
3D Toric Code



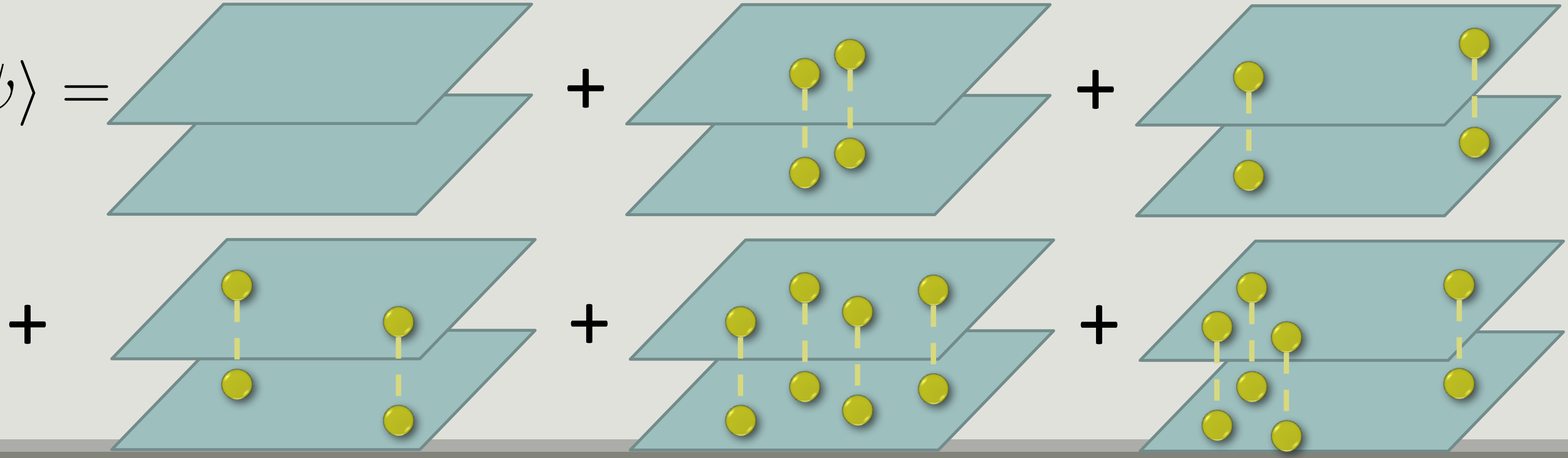
2D TC layers to 3D TC

"Condensing" e-e pairs: Lower the energy cost of e-e pairs such that

- ❑ They are everywhere
- ❑ Move freely around
- ❑ Can appear and vanish

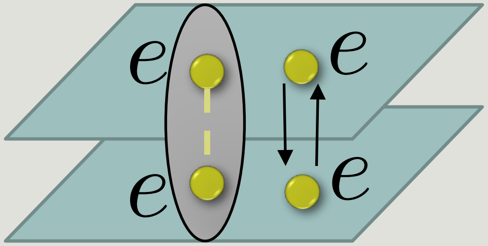


$|\psi\rangle =$



2D TC layers to 3D TC

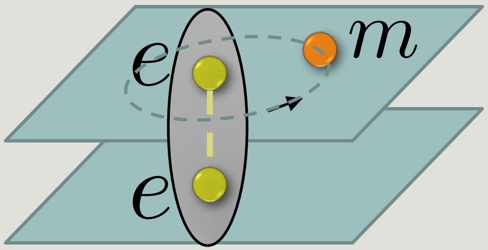
"Condensing" e-e pairs



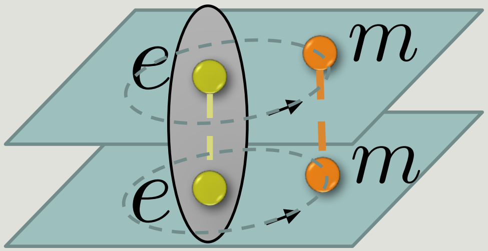
- e particle in the two layers become identical to each other
- e particle can hop from one layer to another

2D TC layers to 3D TC

"Condensing" e-e pairs



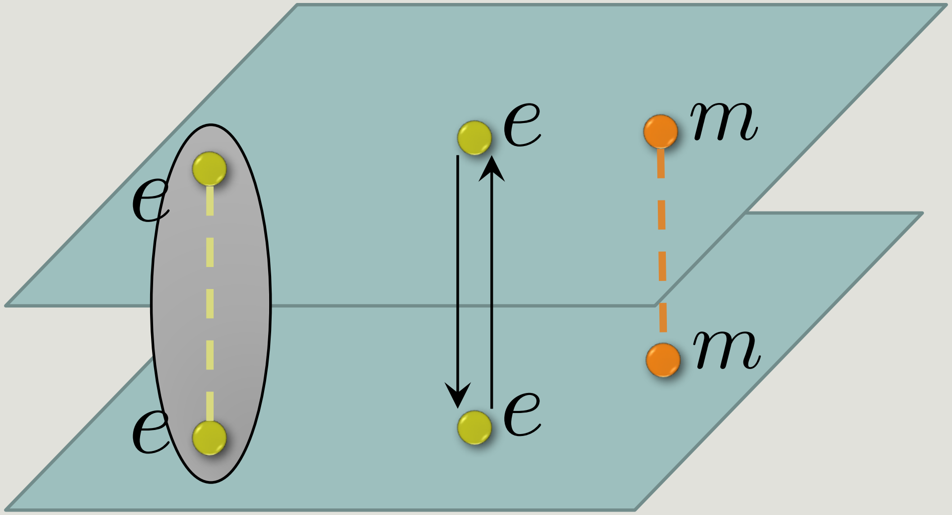
- ❑ m particles interfere with the condensate
- ❑ m particles cannot exist any more
- ❑ they get confined



- ❑ m-m pairs do not interfere with the condensate
- ❑ They still exist

2D TC layers to 3D TC

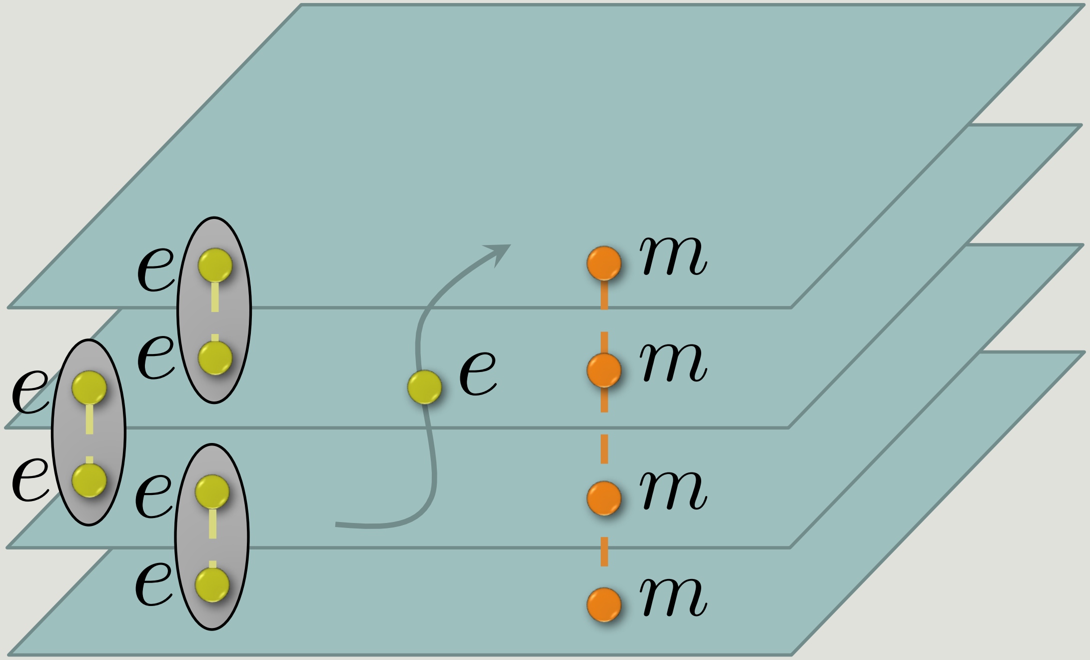
"Condensing" e-e pairs



- e particle free to hop between planes
- m particle connect across the layers

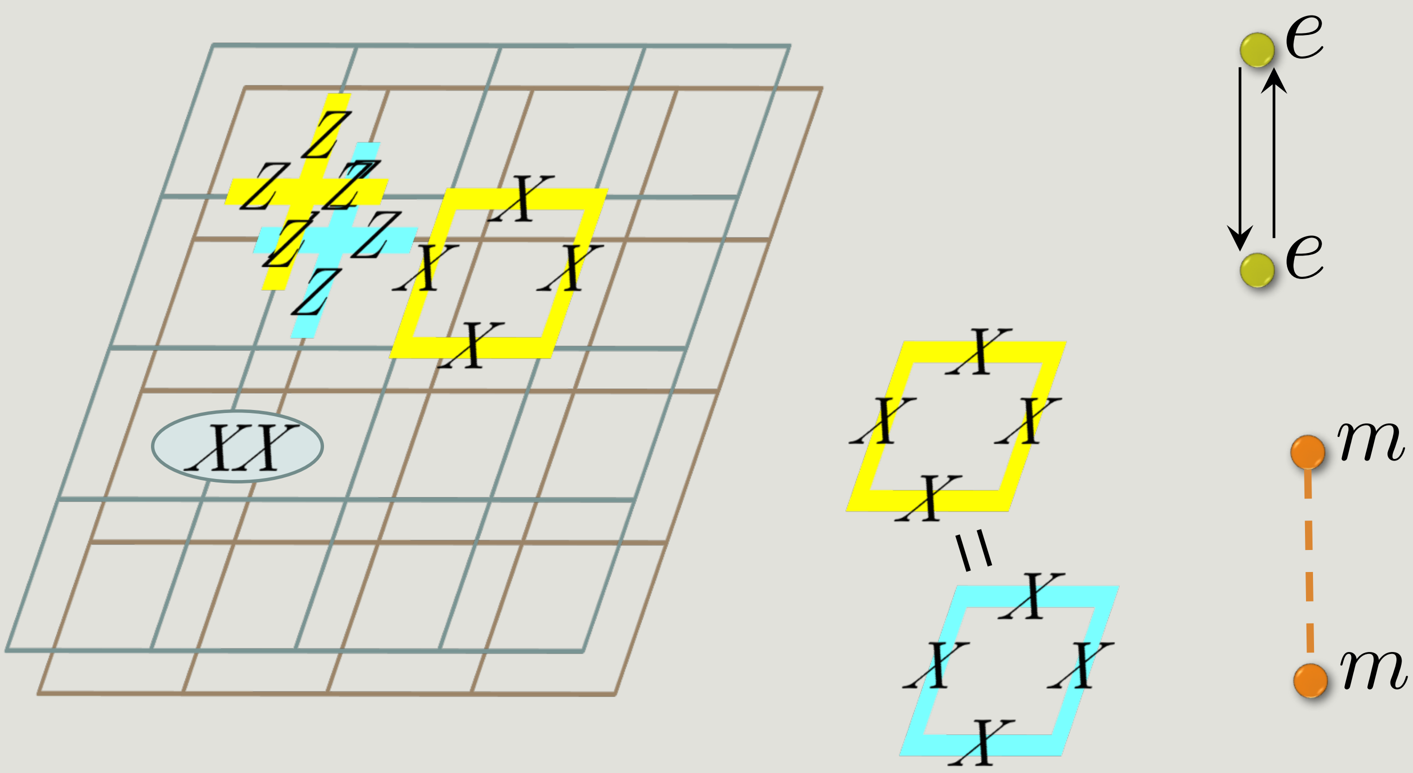
2D TC layers to 3D TC

"Condensing" e-e pairs between neighboring layers couples 2D Toric Code layers into a 3D Toric Code

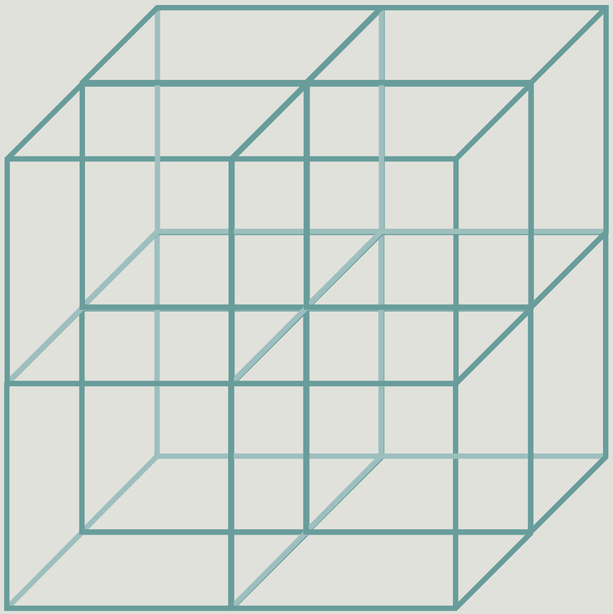


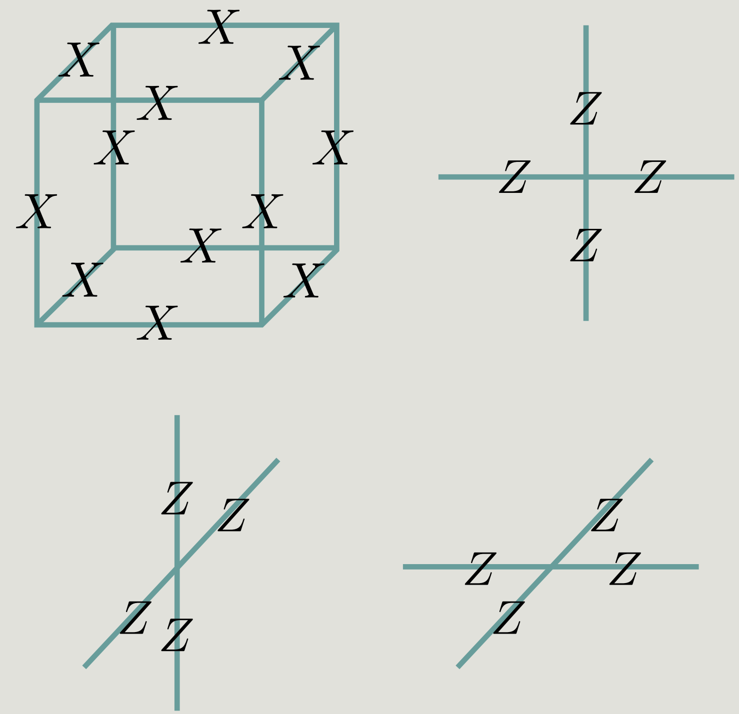
- e particle moving freely in 3D
- m particles connect into loops.

2D TC layers to 3D TC



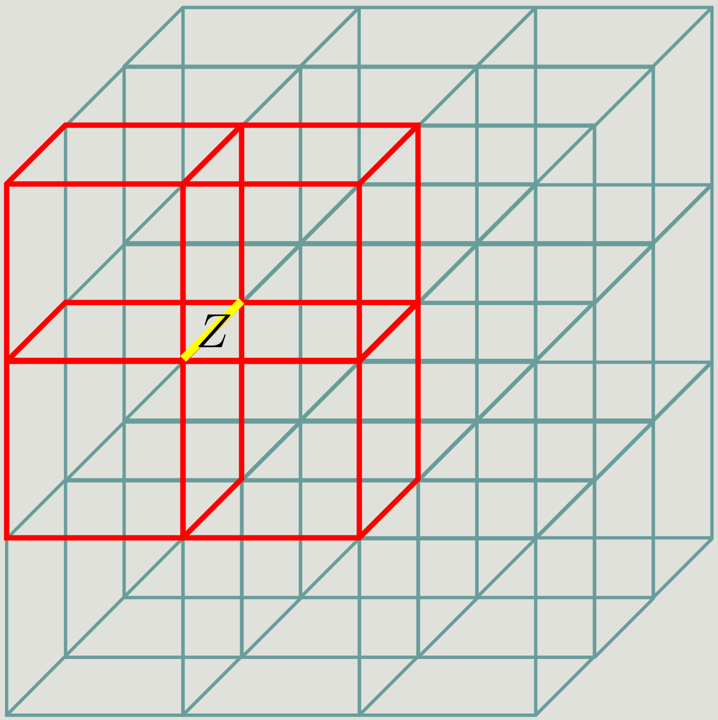
X-cube model

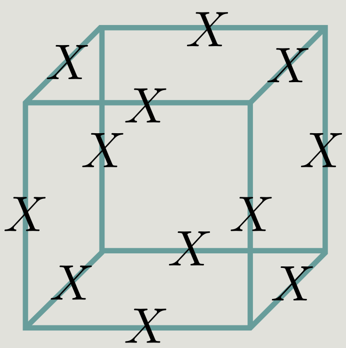




Vijay, Haah, Fu (2015)

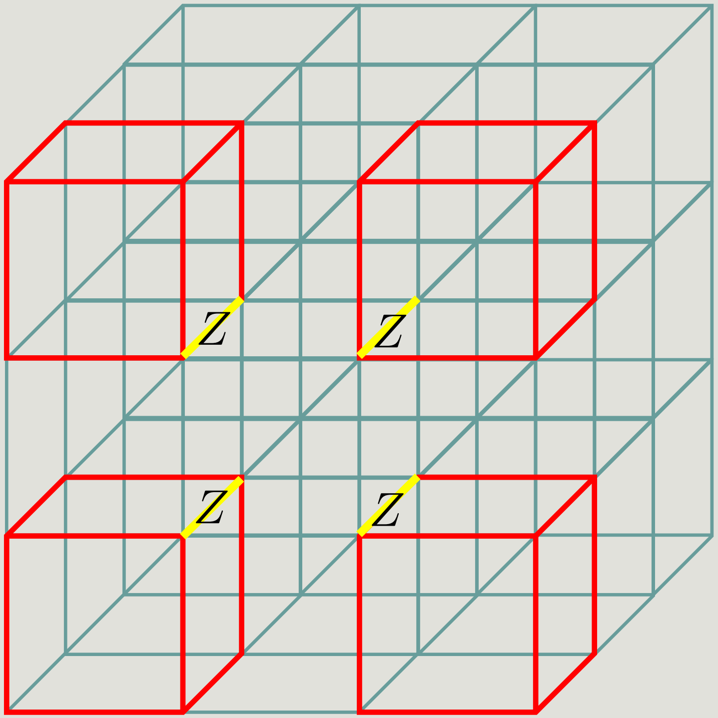
X-cube model

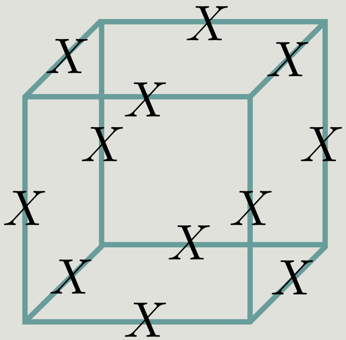




Fracton excitation appearing at the corner of a rectangle

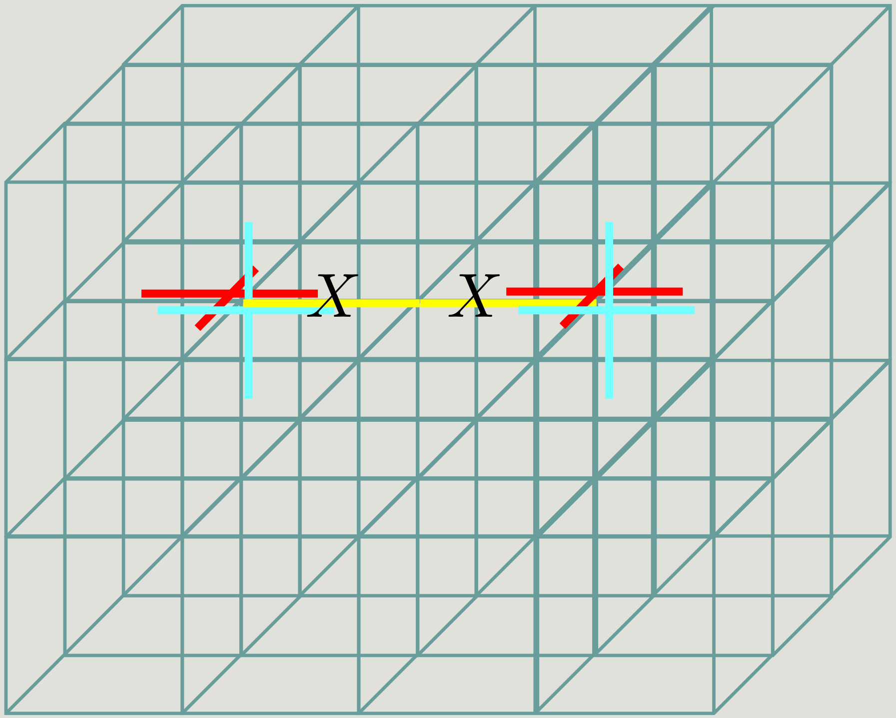
X-cube model

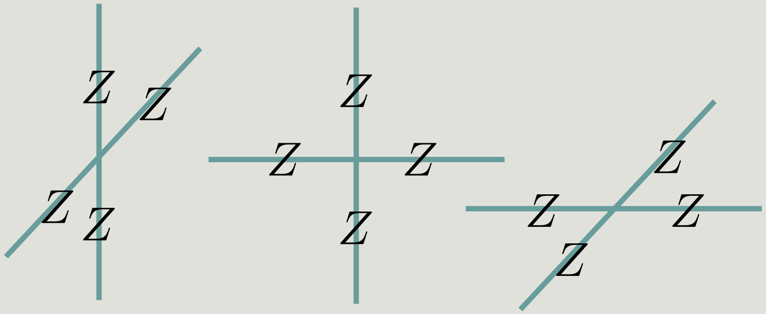




Fracton excitation appearing at the corner of a rectangle

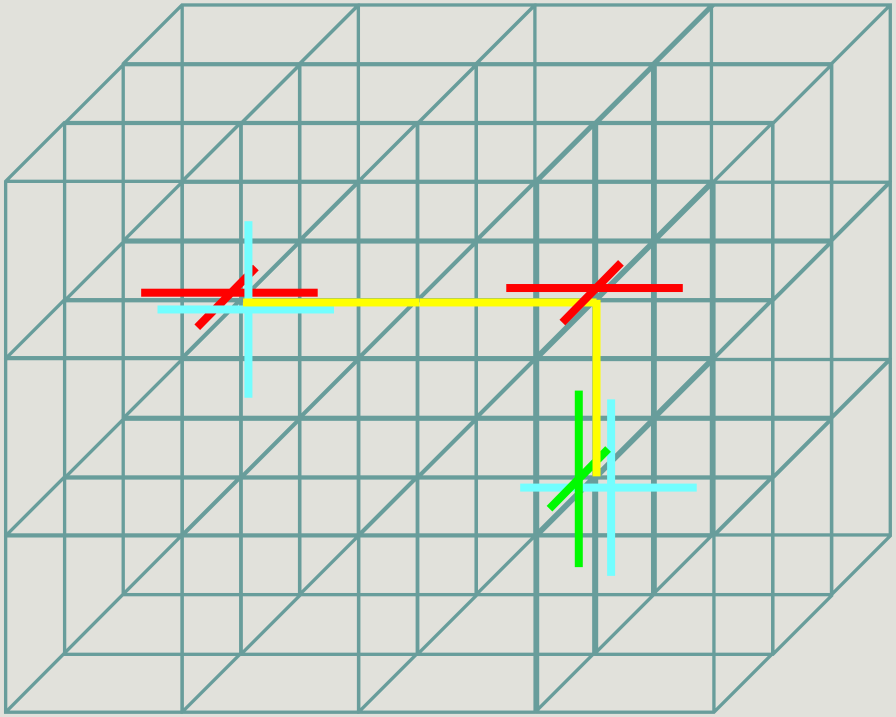
X-cube model

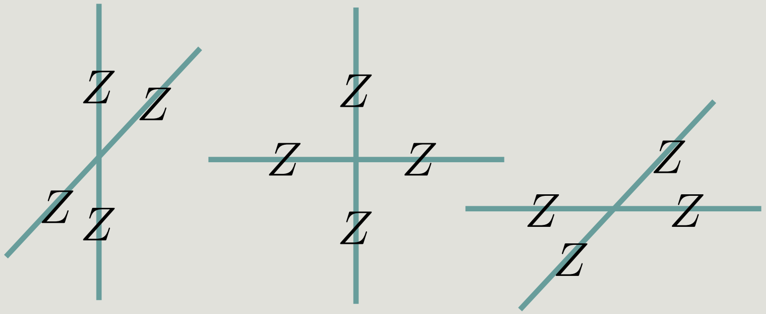




Fracton excitations restricted to move along a line

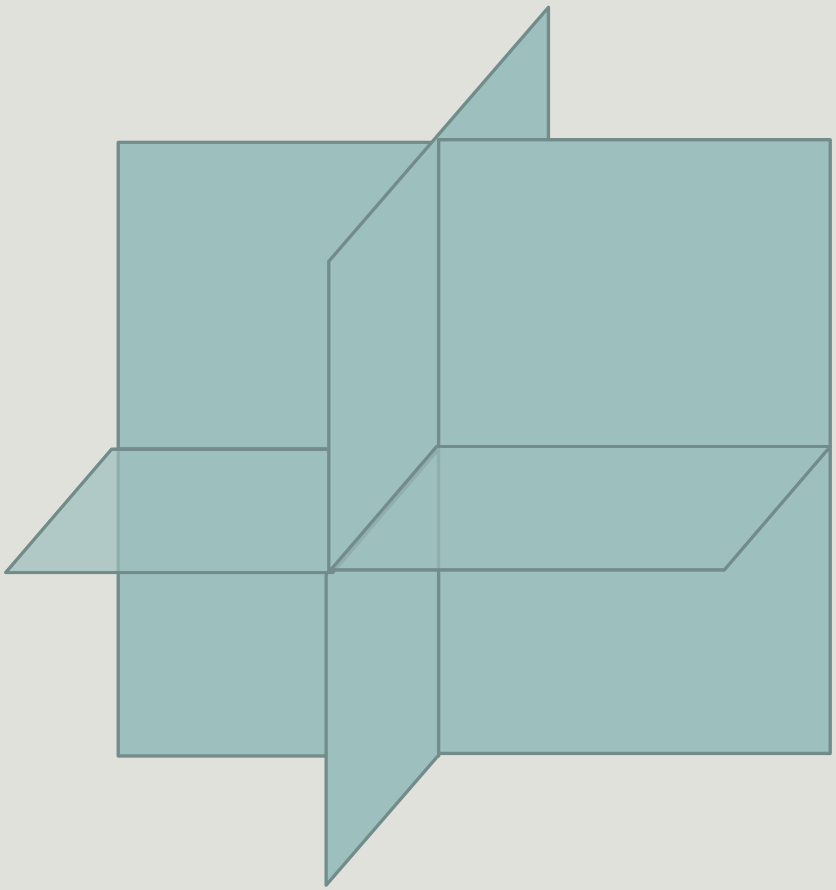
X-cube model

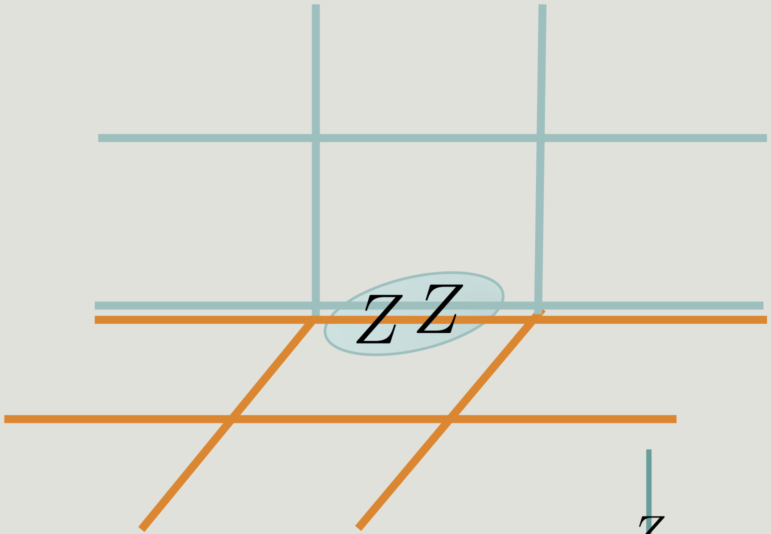


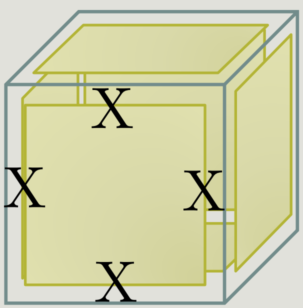


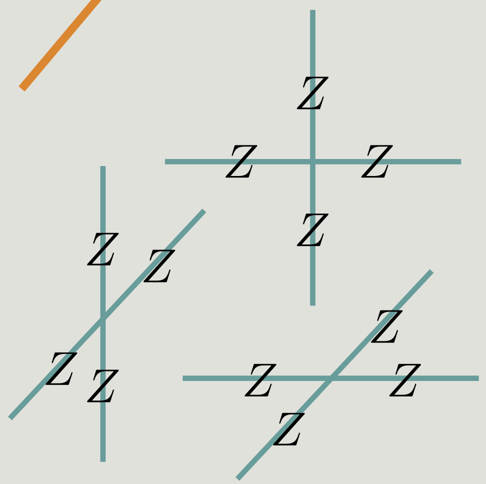
Changing direction leaves residues at corner

2D TC layers to 3D X-cube

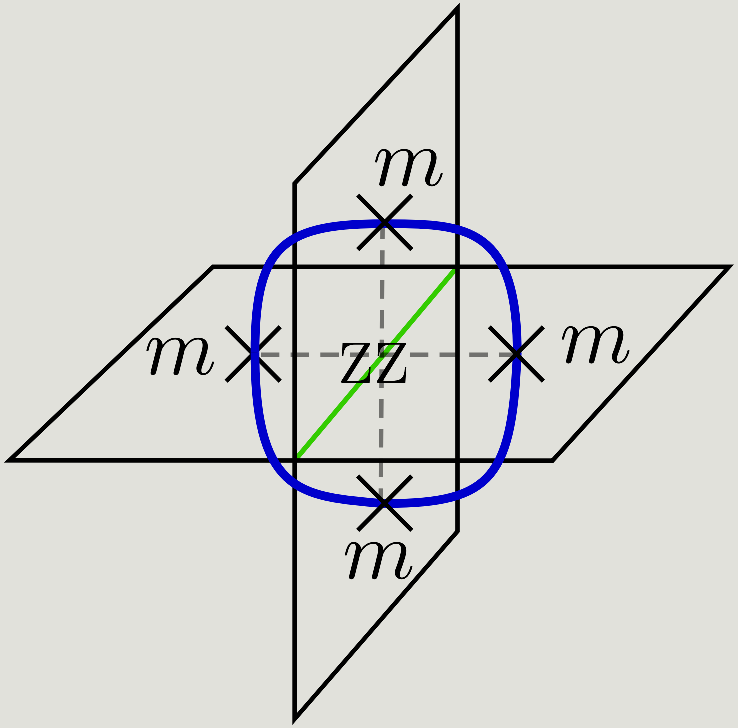






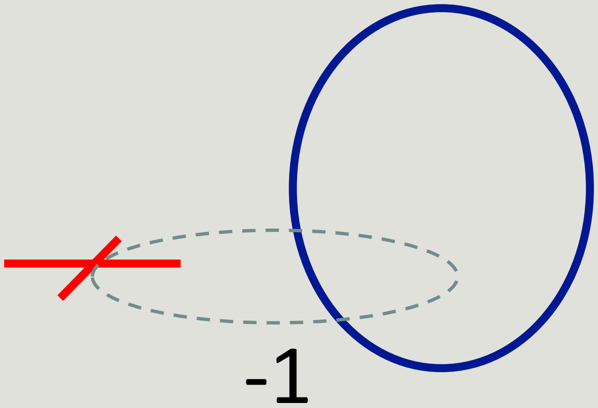


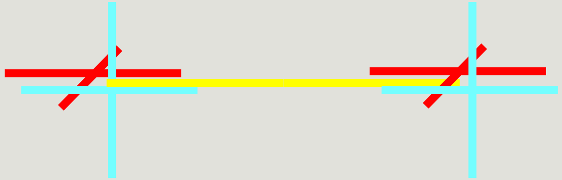
2D TC layers to 3D X-cube



- ❑ Applying ZZ on one pair of links creates a loop of m excitations
- ❑ Enforcing ZZ on every pair of link condenses loops of m excitations
- ❑ Ground states contains all possible m loop configurations

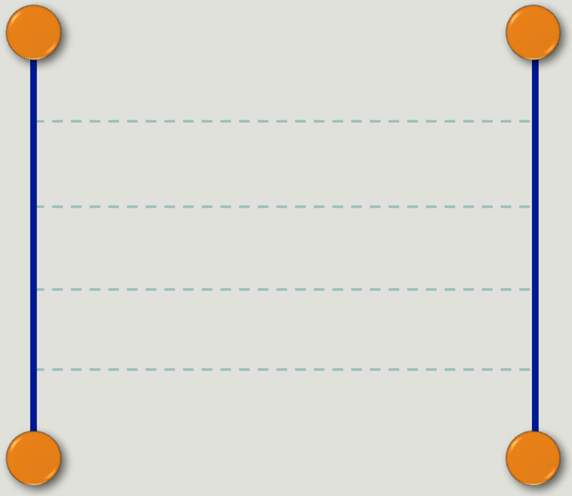
2D TC layers to 3D X-cube





- A single e excitation interferes with the condensate – single e is confined
- A composite of e excitations from intersecting planes can exist
- The composite e excitation moves along a line

2D TC layers to 3D X-cube



- In a condensate of m flux loop, excitations are ends of flux strings
- Flux string are created in pairs
- Ends of flux strings appear at corner of rectangle

Haah’s code

Fracton I

Fracton II

Fracton III

- Better understanding of topological phases
- Simpler model with fracton order

Kevin Slagle, Yong Baek Kim, arXiv: 1704.03870

Known phases in 3D

Haah's code

A different perspective from higher rank gauge theory

Fracton I

Pretko, 1604.05329, 1606.08857, 1702.07613, 1706.01899

Han Ma, Mike Hermele, XC, to appear

Fracton II

Fracton III

Known phases in 3D

Rank 1 (normal) gauge theory $E_i, \ A_i$

Gauss' Law

$$\partial_i E_i = \rho$$

Conservation Law

$$\int \rho \ d^3\mathbf{x} = 0$$

Charge Conservation

Rank 2 gauge theory $E_{ij}, \ A_{ij}$

Gauss' Law

$$\partial_i \partial_j E_{ij} = \rho$$

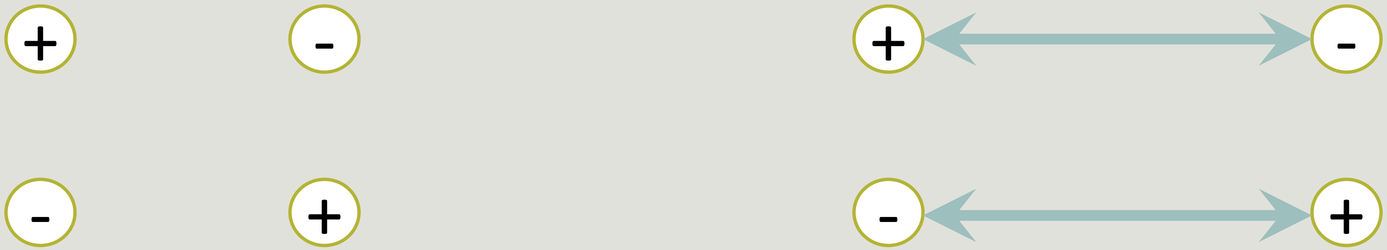
Conservation Law

$$\int \rho \ d^3\mathbf{x} = 0$$

Charge Conservation

$$\int \rho \vec{x} \ d^3\mathbf{x} = 0$$

Dipole Conservation



- Charges in a rank 2 gauge theory are fractons!
- There are also gapless photon modes

  Rasmussen, You, Xu, 2016
- Gapped fracton phases by Higgsing to discrete gauge theory?

Rank 2 U(1) gauge theory → (Higgs) → Rank 2 Z_2 gauge theory

$\vec{x}$ Continuous space	Discrete lattice
ρ Charge density	Z_2 charge $\rho_2 = 0, 1$

Charge Conservation

$$\int \rho \; d^3\mathbf{x} = 0 \qquad \left(\sum \rho_2\right) \bmod 2 = 0$$

Dipole Conservation

$$\int \rho \vec{x} \; d^3\mathbf{x} = 0 \qquad \left(\sum \rho_2 x_i\right) \bmod 2 = 0$$

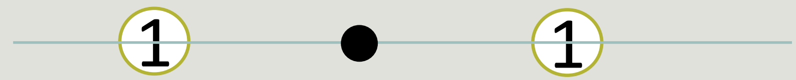
Rank 2 U(1) gauge theory → (Higgs) Rank 2 Z_2 gauge theory

Fracton

$$\int \rho \, d^3\mathbf{x} = 0$$

$$\int \rho\vec{x} \, d^3\mathbf{x} = 0$$

No Fracton!



$$\left(\sum \rho_2\right) \bmod 2 = 0$$

$$\left(\sum \rho_2 x_i\right) \bmod 2 = 0$$

Gauge charges can hop, with step size 2

Rank 2 Z_2 gauge theory = Four copies of Rank 1 Z_2 gauge theory

Fractons can remain after Higgsing if more constraints are added to the rank 2 U(1) gauge theory

Rank 2 gauge theory $E_{ij},\ A_{ij}$

Gauss' Law

$$\partial_i \partial_j E_{ij} = \rho$$

Conservation Law

$$\int \rho \ d^3\mathbf{x} = 0$$ Charge Conservation

$$\int \rho\vec{x} \ d^3\mathbf{x} = 0$$ Dipole Conservation

Extra constraint $E_{ii} = 0$

Extra conservation law

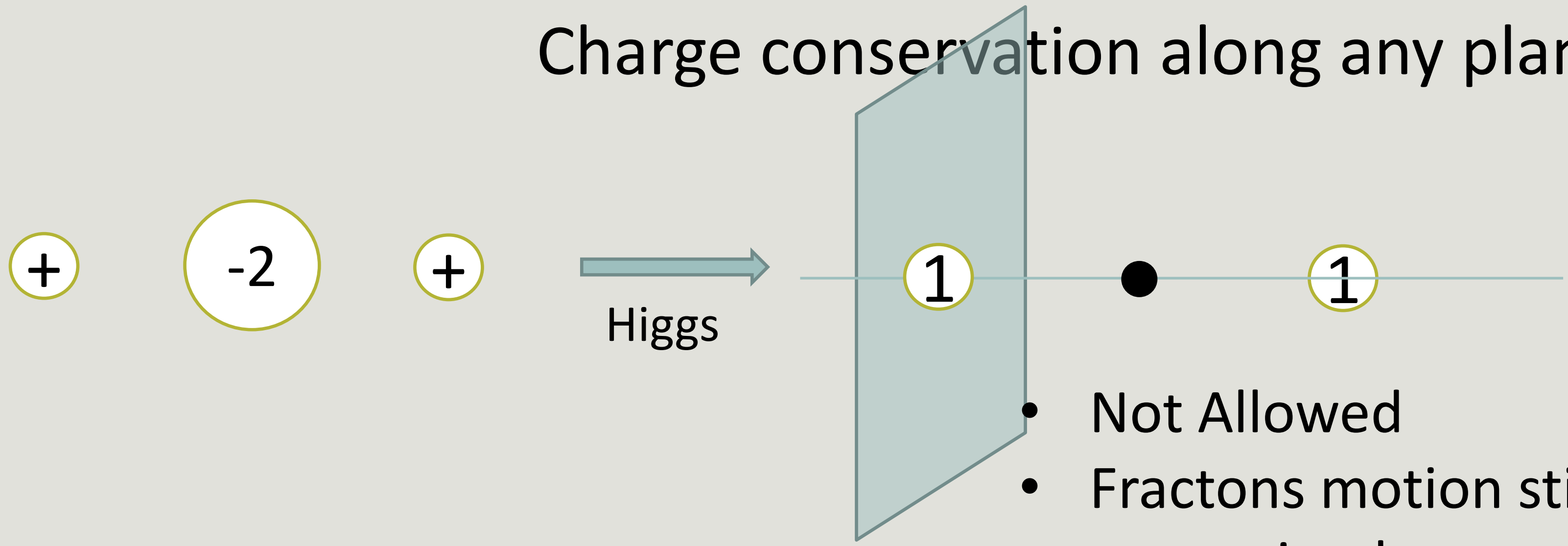
$$\int \rho f(x_i) d^3\mathbf{x} = 0$$

$$\int \rho f(x_i) d^3\mathbf{x} = 0$$

→ Higgs

$$\left(\sum \rho_2 \delta_{x_i, x_i^0}\right) \bmod 2 = 0$$

Charge conservation along any plane



- Not Allowed
- Fractons motion still constrained
- X cube model

More General Questions

- What is a fracton topological phase?
- When are two fracton models in the same phase?
- Is there always a ’field theory’ description?
- What is the physical mechanism for generating fractons?

- Isotropic Layer Construction and Phase Diagram for Fracton Topological Phases
  Sagar Vijay, arXiv:1701.00762
- A Generalization of Non-Abelian Anyons in Three Dimensions
  Sagar Vijay, Liang Fu, arXiv:1706.07070
- Fracton topological order, generalized lattice gauge theory, and duality
  Sagar Vijay, Jeongwan Haah, and Liang Fu, Phys. Rev. B 94, 235157
- Generalized Electromagnetism of Subdimensional Particles: A Spin Liquid Story
  Michael Pretko, arXiv:1606.08857
  Subdimensional Particle Structure of Higher Rank U(1) Spin Liquids
  Michael Pretko, arXiv:1604.05329