Topological States and Phase Transitions in Strongly Correlated Systems July 4, 2017 KITS, Beijing, China

Lattice model constructions for gapless domain walls of topological phases

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Collaborators

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Chenfeng Bao, Shuo Yang, Chen-jie Wang, and Zheng-Cheng Gu, in preparation.

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Outline

1. Criterion for gapless domain wall
2. A simple example of gapless domain wall
3. Numerical method: Loop tensor network renormalization
4. Numerical results
5. Conclusion and discussion

Two ways to generate CFT from topological phases of matter

Quantum phase transitions   Fine tune

g_c

Topological phase 1   Topological phase 2

Domain wall / edges   Potential general and (perturbatively) stable

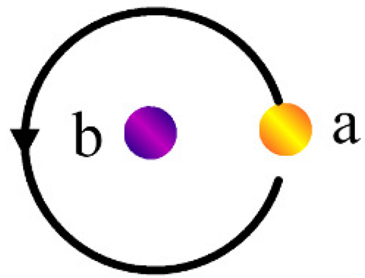
Topological phase 1   Topological phase 2

Universal data and gapless domain wall of chiral topological phases

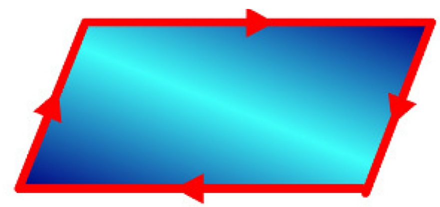
Braiding T, S matrices and chiral central charge as the universal data of topological order (no symmetry).

T_a :

S_{ab} :



c :



A Lagrangian subset is defined as:

(1) All the quasiparticles in M have trivial mutual statistics

(2) Every quasiparticle that is not in M has nontrivial mutual statistics with at least one quasiparticle in M.

(Michael Levin, 2013)

A gapped edge is possible if and only if there exists a nontrivial Lagrangian subset M.

Example

(1) All the quasiparticles in M have trivial mutual statistics

(2) Every quasiparticle that is not in M has nontrivial mutual statistics with at least one quasiparticle in M.

Toric code

$$\{1, e, m, f\}$$

$$\mathcal{T} = \mathrm{Diag}(1, 1, 1, -1),$$

$$S_{TC} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & 1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix}$$

subset: $\{1, e\}$ or $\{1, m\}$

Double semion

$$\{1, s, \bar{s}, b\}$$

$$\mathcal{T} = \mathrm{Diag}(1, \mathrm{i}, -\mathrm{i}, 1),$$

$$S_{DS} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix}$$

subset: $\{1, b\}$

Domain walls between non-chiral topological phases

Gapped domain walls of non-chiral topological phases are easily understood via S and T matrices in 2D.
(Tian Lan et al, Phys. Rev. Lett. 114, 076402 (2015))

Lattice model of gapless domain walls can be systematically constructed in 2D, and can be possibly generalized into higher dimensions.

A simple example

Toric code model and double semion model

$$H_{\mathrm{t.c.}} = -\sum_v Q_v - \sum_p \left(\prod_{l \in p} \tau_l^x \right) P_p$$

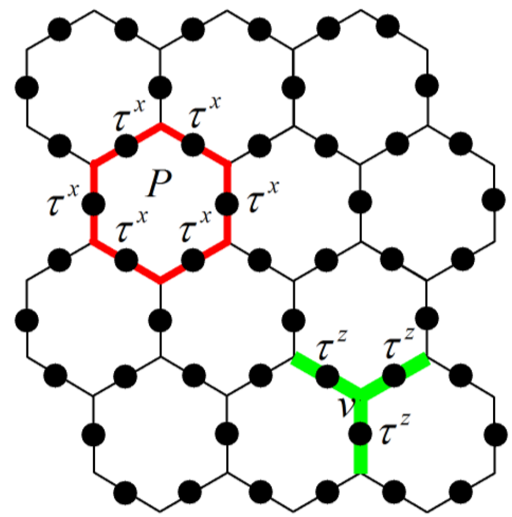
$$H_{\mathrm{d.s.}} = -\sum_v Q_v - \sum_p \left(\prod_{l \in p} \tau_l^x \prod_{l \in \text{legs of } p} i^{\frac{1+\tau_l^z}{2}} \right) P_p$$

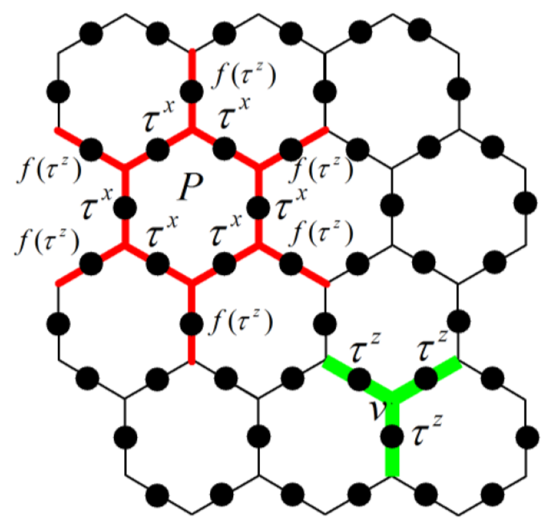
$$Q_v = \prod_{l \in v} \tau_l^z$$

$$P_p = \prod_{v \in p} \frac{1+Q_v}{2}$$

$$|\Psi_{\mathrm{t.c.}}\rangle = \sum_{X \,\mathrm{closed}} |X\rangle$$

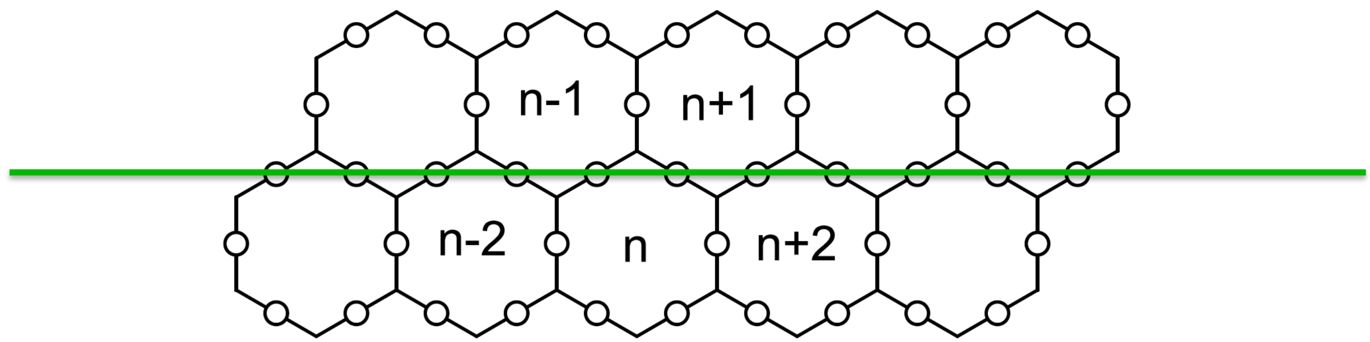
$$|\Psi_{\mathrm{d.s.}}\rangle = \sum_{X \,\mathrm{closed}} (-)^{n(X)} |X\rangle$$





Hamiltonian algebra of domain wall between toric code and double semion

Double semion



Toric code

On the boundary the plaquette terms do not commute.

Hamiltonian algebra:

$$\left(B_n^{t.c.}\right)^2 = 1, \quad \left(B_n^{d.s.}\right)^2 = 1$$

$$\left[B_m^{t.c.}, B_n^{d.s.}\right] = 0, \quad \text{when } |m-n| > 1$$

$$B_n^{t.c.} B_{n\pm1}^{d.s.} = -B_{n\pm1}^{d.s.} B_n^{t.c.} \tau_{n-1,n+1}^z \tau_{n\pm1,n\pm2}^z$$

Next: duality map

Duality and Z2 gauge models

Duality transformation

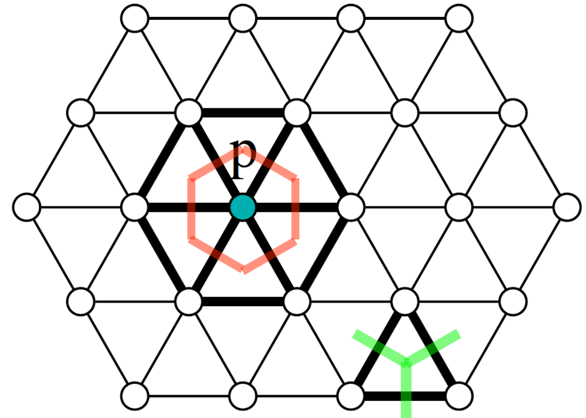
$$\tau_l^z = \sigma_p^z \sigma_q^z \mu_{pq}^z, \ \tau_l^x = \mu_{pq}^x$$

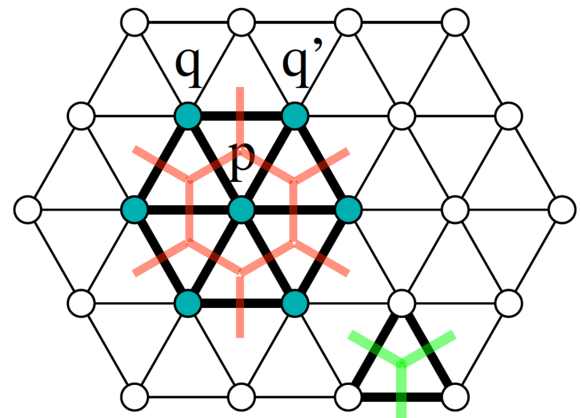
$$\widetilde{H}_0 = -\sum_{\langle pqr \rangle} \mu_{pq}^z \mu_{qr}^z \mu_{rp}^z - \sum_p \sigma_p^x O_p$$

$$\widetilde{H}_1 = -\sum_{\langle pqr \rangle} \mu_{pq}^z \mu_{qr}^z \mu_{rp}^z - \sum_p \widetilde{B}_p O_p$$

$$O_p = \prod_{\langle pqr \rangle} \frac{1 + \mu_{pq}^z \mu_{qr}^z \mu_{rq}^z}{2}$$

$$\widetilde{B}_p = \sigma_p^x \prod_{\langle pqq' \rangle} i^{\frac{1+\sigma_q^z \mu_{qq'}^z \sigma_{q'}^z}{2}}$$



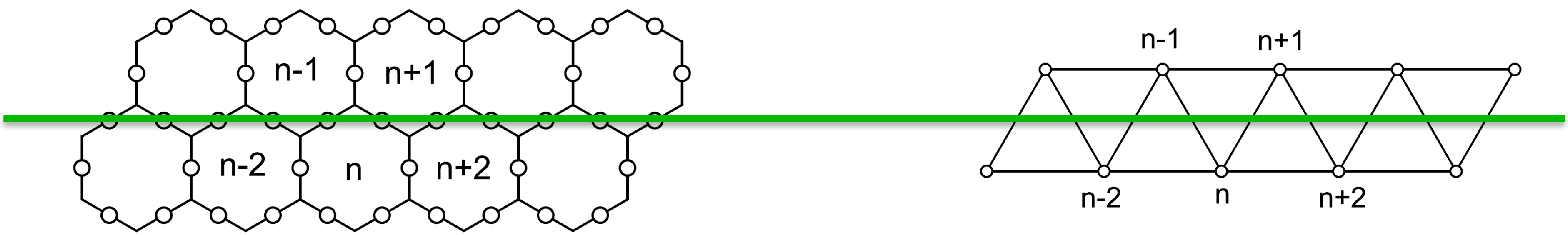


Z2 Gauge invariant subspace satisfy: $\prod_q \mu_{pq}^x = \sigma_p^x$

Next: simplify the domain wall

Hamiltonian algebra after duality transformation

Twisted Z2 gauge theory



Z2 gauge theory

$$\left(B_n^{t.c.}\right)^2 = 1, \quad \left(B_n^{d.s.}\right)^2 = 1$$

$$\left[B_m^{t.c.}, B_n^{d.s.}\right] = 0, \quad \text{when } \; |m-n| > 1$$

$$B_n^{t.c.} B_{n\pm 1}^{d.s.} = -B_{n\pm 1}^{d.s.} B_n^{t.c.} \tau_{n-1,n+1}^z \tau_{n\pm 1,n\pm 2}^z$$

$$(\sigma_n^x)^2 = 1, \quad (\widetilde{B}_n)^2 = 1$$

$$\left[\sigma_m^x, \widetilde{B}_n\right] = 0, \quad \text{when } \; |m-n| > 1$$

$$\sigma_n^x \widetilde{B}_{n\pm 1} = -\widetilde{B}_{n\pm 1} \sigma_n^x \sigma_{n\mp 1}^z \sigma_{n\pm 2}^z \mu_{n-1,n+1}^z \mu_{n\pm 1,n\pm 2}^z$$

Gauge fixing and simplified Hamiltonian algebra

Low energy sector with zero Z2 flux admits a uniform choice of Z2 gauge field

$$(\sigma_n^x)^2 = 1, \quad (\widetilde{B}_n)^2 = 1$$

$$\left[\sigma_m^x, \widetilde{B}_n\right] = 0, \quad \text{when } |m-n| > 1$$

$$\sigma_n^x B_{n\pm1} = -B_{n\pm1}\sigma_n^x \sigma_{n\mp1}^z \sigma_{n\pm2}^z$$

$$B_p = \sigma_p^x \prod_{\langle pqq'\rangle} i^{\frac{1+\sigma_q^z \sigma_{q'}^z}{2}}.$$

Next: solve this algebra

1D model that realizes the same Hamiltonian algebra

Introduce spin degrees of freedom, reproduce the previous Hamiltonian algebra

$$\sigma_n^x = \frac{1}{\sqrt{2}}\left(\bar{\tau}_n^y + \bar{\tau}_{n-1}^z \bar{\tau}_n^x \bar{\tau}_{n+1}^z\right)$$
$$B_n = \frac{1}{\sqrt{2}}\left(\bar{\tau}_n^y - \bar{\tau}_{n-1}^z \bar{\tau}_n^x \bar{\tau}_{n+1}^z\right)$$
$$\sigma_n^z = \bar{\tau}_n^z$$

After basis change, we can make a uniform Hamiltonian

$$\sigma_n^x = B_n = \frac{1}{\sqrt{2}}\left(\bar{\tau}_n^y + \bar{\tau}_{n-1}^z \bar{\tau}_n^x \bar{\tau}_{n+1}^z\right)$$

$$H_{\text{domain wall}} = -\frac{1}{\sqrt{2}}\sum_n \left(\bar{\tau}_n^y + \bar{\tau}_{n-1}^z \bar{\tau}_n^x \bar{\tau}_{n+1}^z\right)$$

Evidence of gapless domain walls

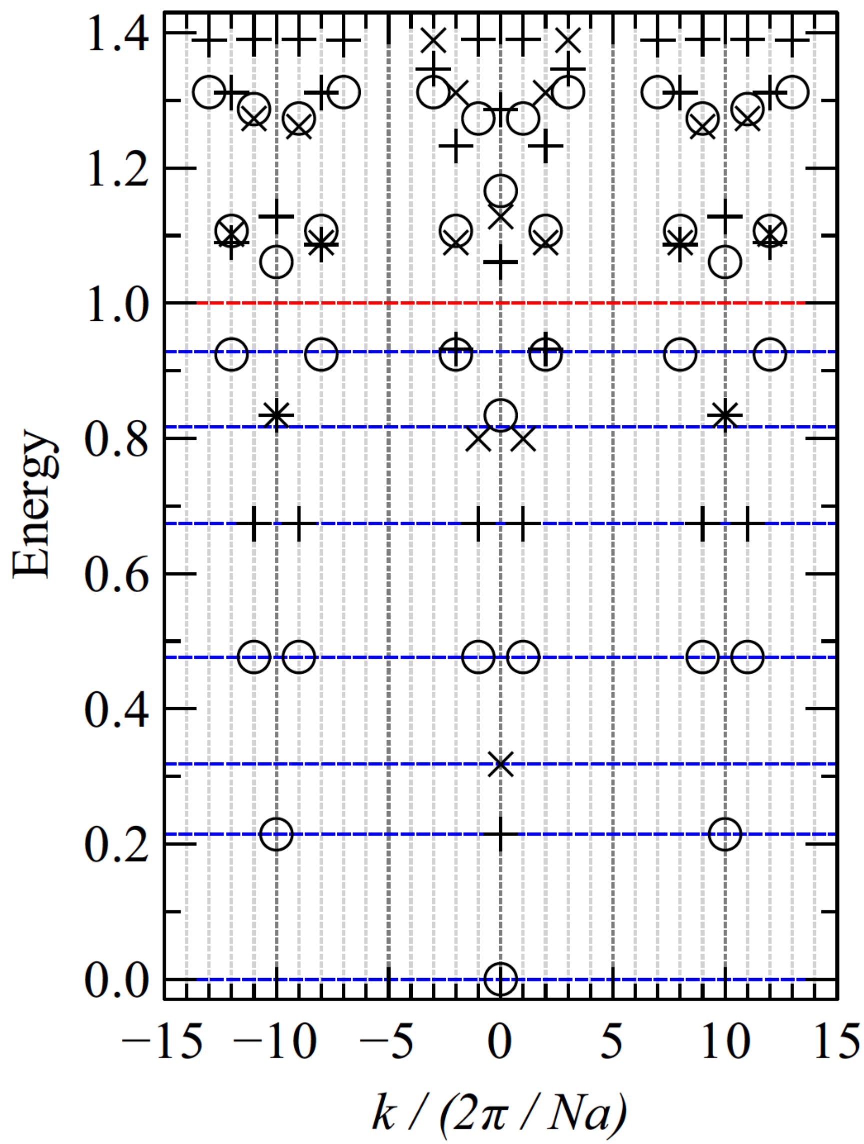
Self-duality
$$H(g) = -\frac{1}{\sqrt{2}}\sum_p [g\tau_p^y + \tau_{p-1}^z\tau_p^x\tau_{p+1}^z]$$

$$\boxed{\begin{aligned} \bar{\tau}_p^x &\to \tau_{p-1}^z\tau_p^y\tau_{p+1}^z, \\ \bar{\tau}_p^y &\to \tau_{p-1}^z\tau_p^x\tau_{p+1}^z, \\ \bar{\tau}_p^z &\to -\tau_p^z. \end{aligned}}$$

$$H(g) = \frac{1}{\sqrt{2}}\sum_p [g\bar{\tau}_{p-1}^z\bar{\tau}_p^x\bar{\tau}_{p+1}^z + \bar{\tau}_p^y] = gH(1/g)$$

Is it a CFT? If yes, what kind of CFT it is?

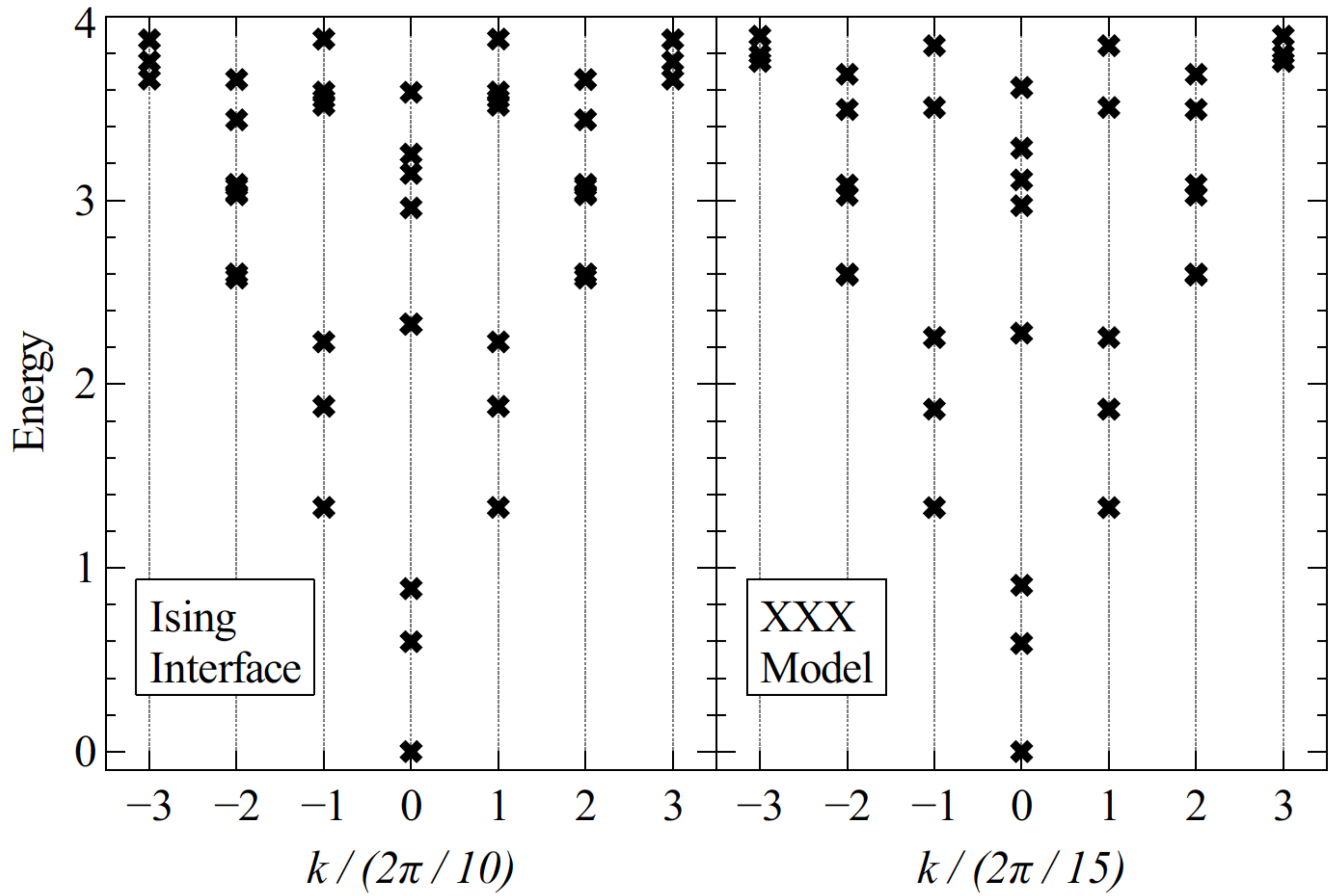
Low energy spectrum



Three minimal are observed in momentum space. (with N=30)

Starting from the ground state, degeneracies of these seven energy levels are respectively 1,3,1,6,6,7,8. This feature is consistent with the XXX model (spin-1/2 AF Heisenberg model).

A comparison of low-energy spectra after folding



A comparison of low-energy spectra after folding

Although exact diagonalization provides some evidences for the similarity to XXX model, it is not sufficient to ping down the underlying CFT due to finite size effect.

Unbiased numerical methods are very desired.

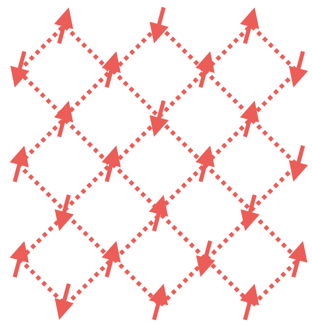
We developed the state-of-art non-perturbative real space renormalization algorithm to study the effective domain wall Hamiltonian.

Shuo Yang, Zheng-Cheng Gu, Xiao-Gang Wen, PRL 118, 110504 (2017).

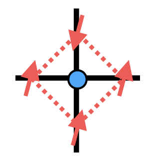
Overview

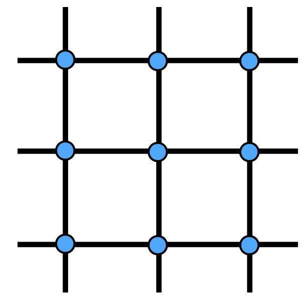
tensor network + renormalization group = tensor network renormalization

Partition function of a classical system

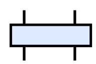


$$Z = \sum_{\{\sigma\}} \exp(\beta \sum_{\langle ij \rangle} \sigma_i \sigma_j)$$

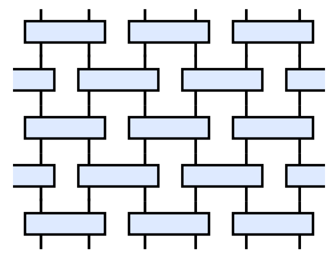


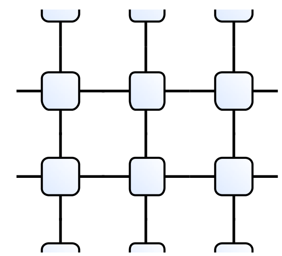


Euclidean path integral of a 1D quantum system

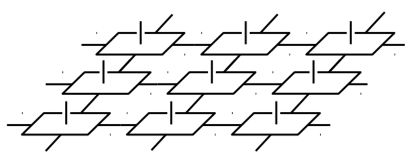


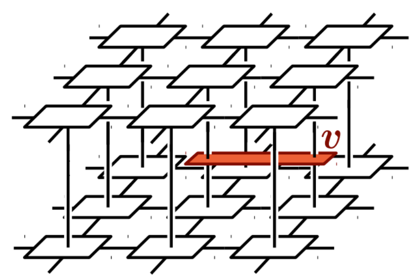
$e^{-\tau h}$

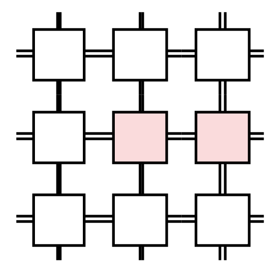




Physical observables of a 2D quantum system



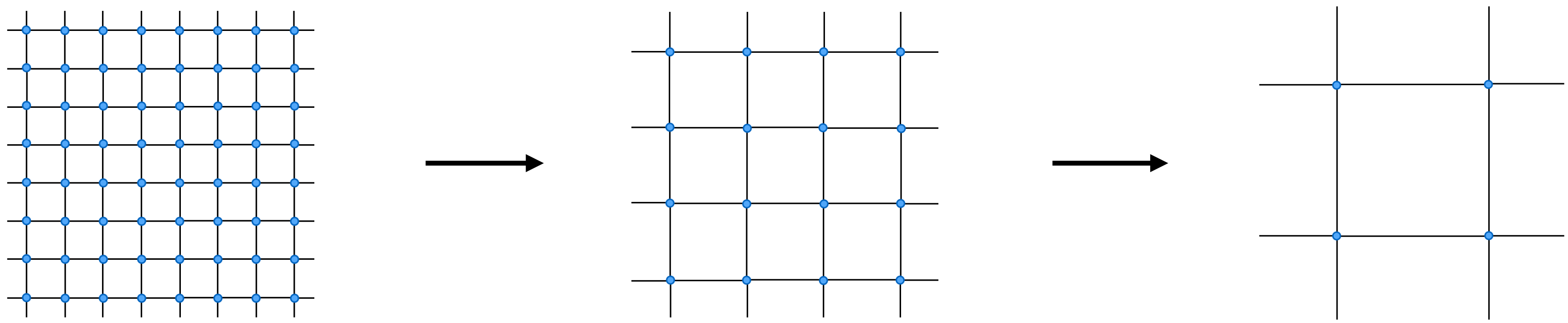




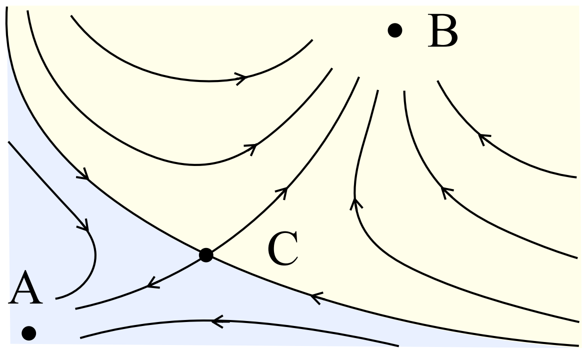
Overview

tensor network + renormalization group = tensor network renormalization

- Real space coarse-graining transformation



RG flow

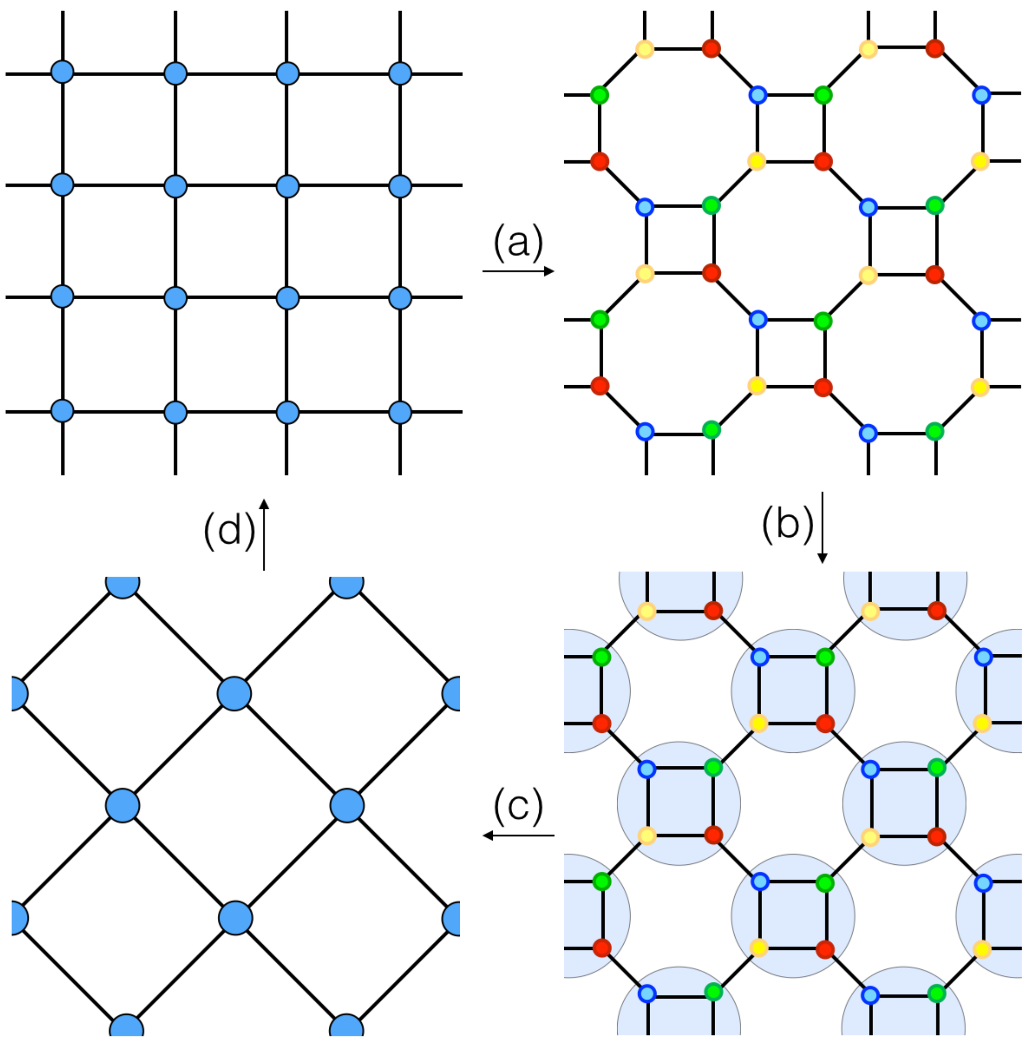


Aim

- Remove short-range entanglement / correlations
- Generate proper RG flow & correct fixed points
- Recover scale invariance at criticality

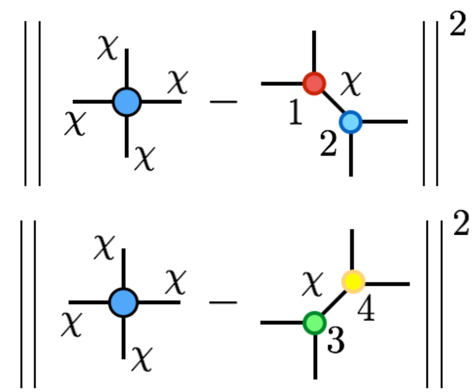
Tensor renormalization group

Levin & Nave (2007) TRG (LN-TNR)

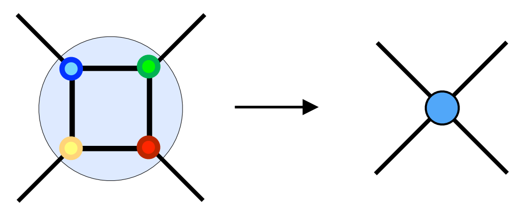


Three steps

1. Deform tensors, make a truncation minimizing local cost functions by SVD

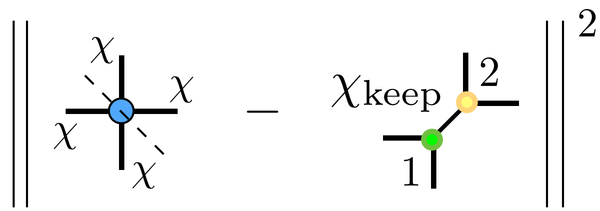


2. Coarse graining



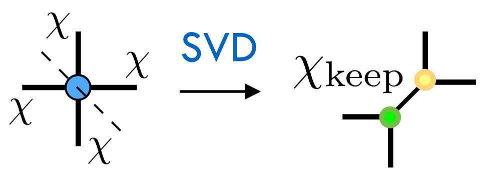
3. Renormalize tensors
(multiply tensor by a constant factor)

Tensor renormalization group



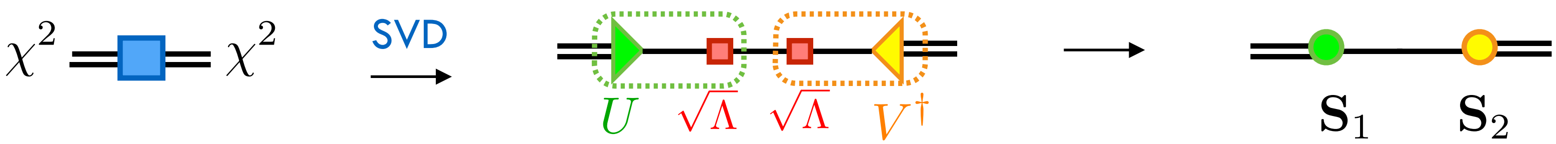
cost function

$||\mathbf{T} - \mathbf{S}_1 \cdot \mathbf{S}_2||^2$



singular value decomposition (SVD)

only keep the largest χ_{keep} singular values



original

$\chi_{\text{keep}} = \chi^2$

$\chi_{\text{keep}} = 2\chi$

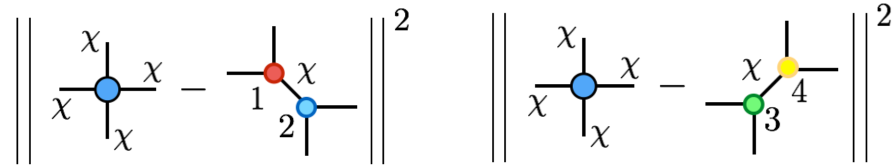
$\chi_{\text{keep}} = \chi$

$\chi_{\text{keep}} = \chi/4$

Tensor renormalization group

Merits of LN-TNR

- simple & efficient, widely used
- can provide OK results



Drawbacks of LN-TNR

- accumulation of short-range entanglement
- cannot give the correct structure of non-critical fixed point
- cannot explicitly recover scale invariance at criticality

How to improve LN-TNR

- LN-TNR is only exact for tree tensor networks
- We need loops!

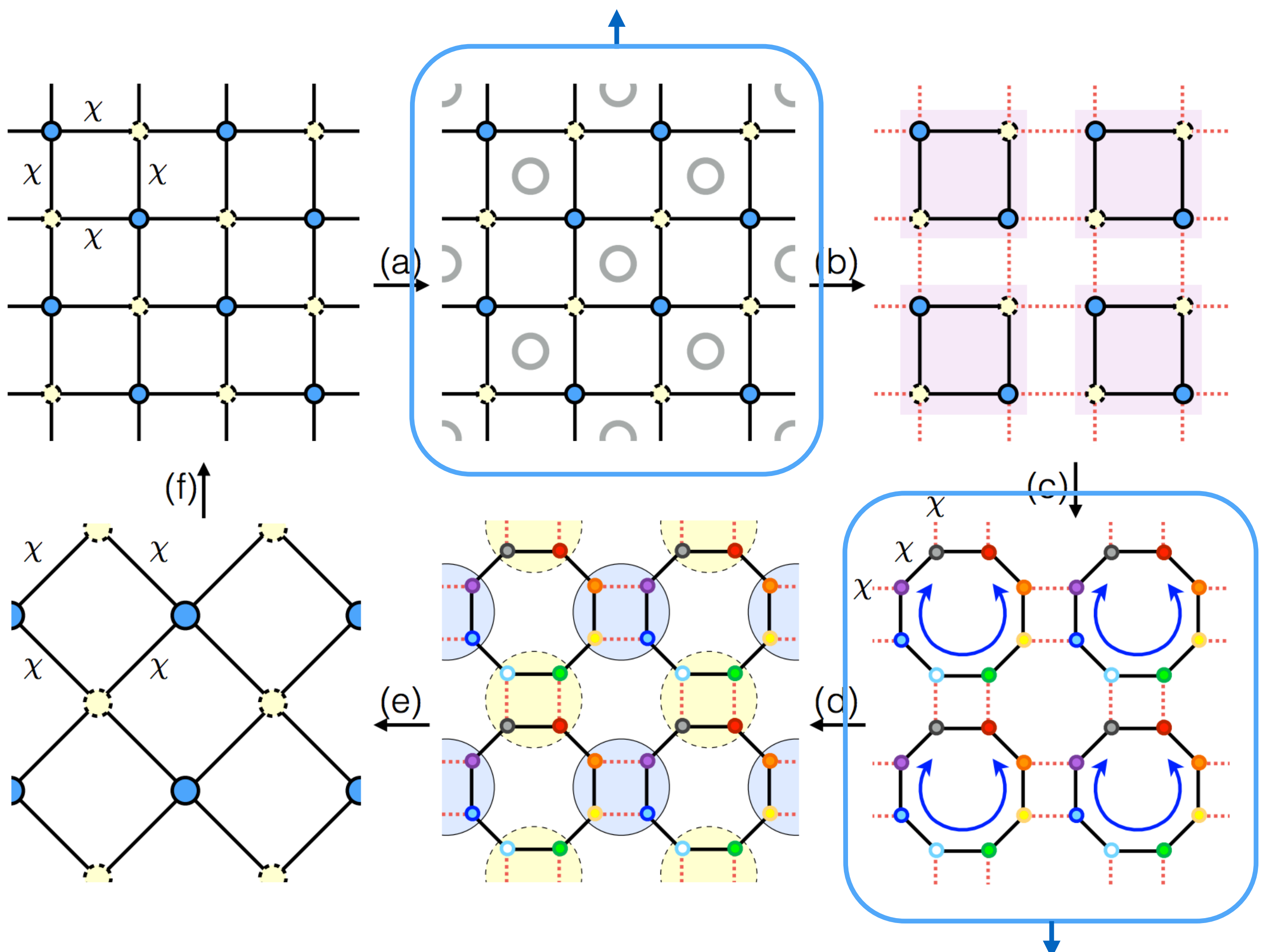
LN-TNR + loop optimization

further remove short-range entanglement inside a loop

This work !

Algorithms of Loop-TNR

Part One: Entanglement filtering



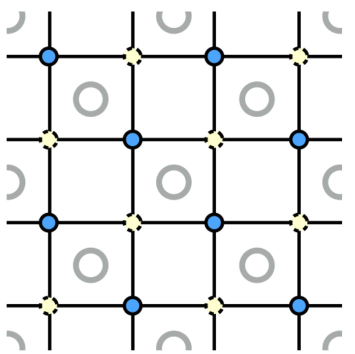
Part Two: Optimizing tensors on a loop

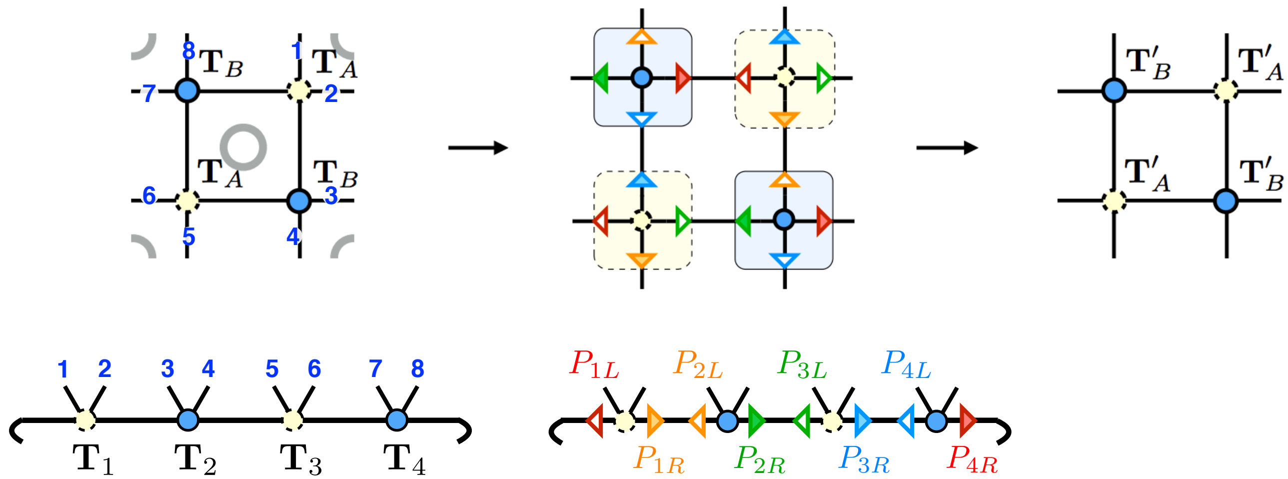
Together: Complete remove short-range entanglement

Part One — Entanglement filtering

How

1. Find & insert projectors
2. Define new tensors



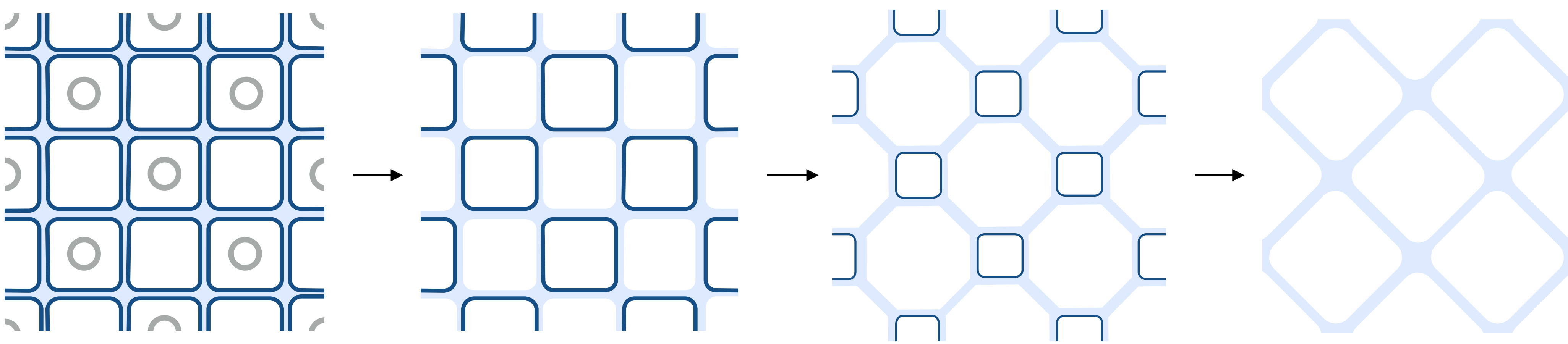


Aim

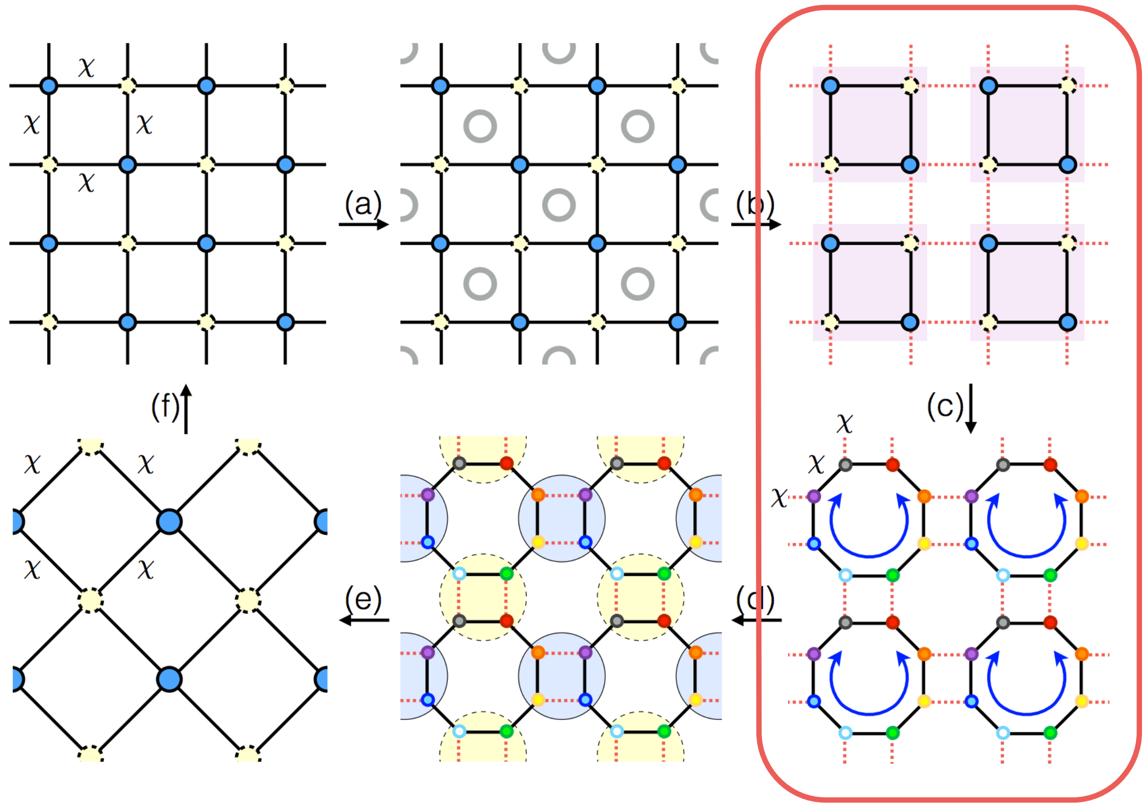
Remove conner double line (CDL) tensors

Generate local canonical gauge

Zheng-Cheng Gu and Xiao-Gang Wen, Phys. Rev. B **80**, 155131 (2009).

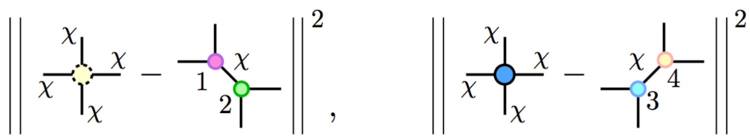


Part Two — Optimizing tensors on a loop

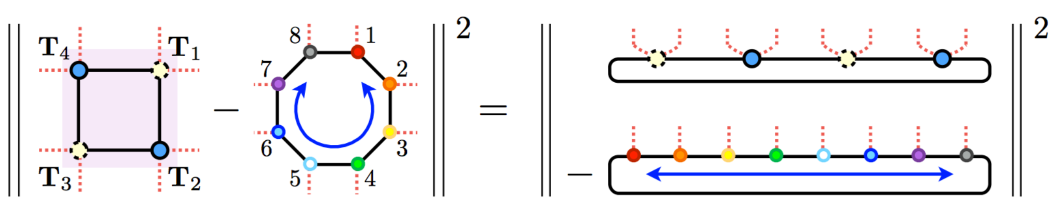


cost function

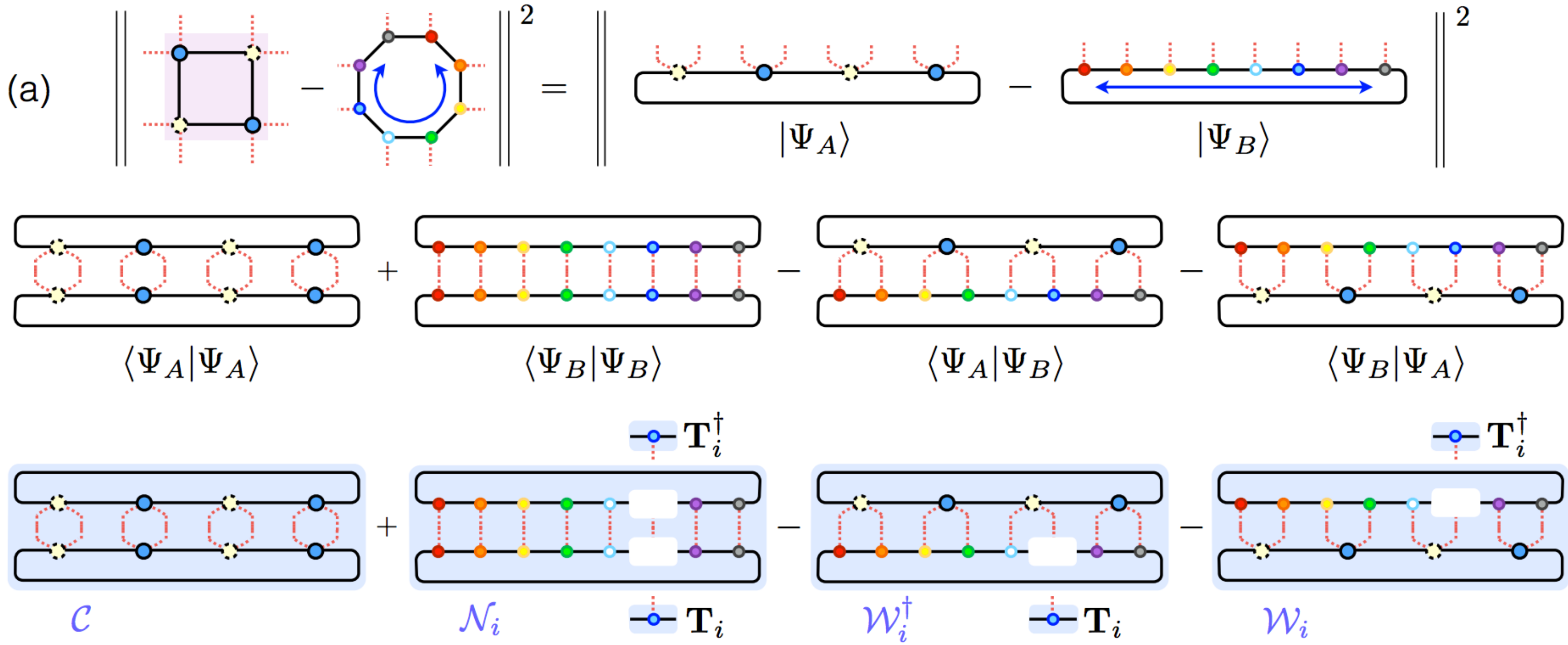
- LN-TNR



- Loop-TNR



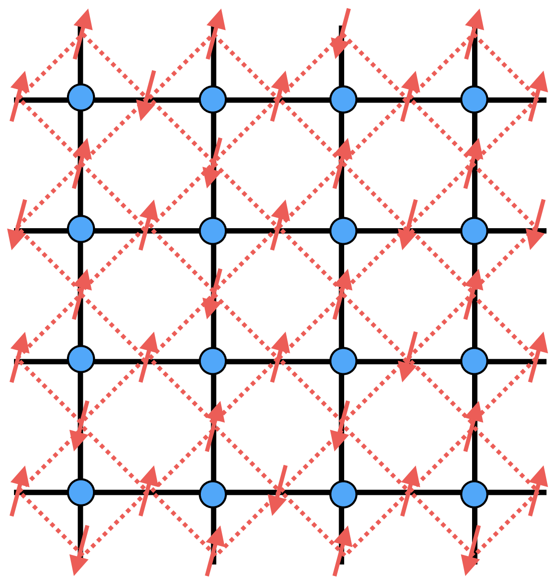
Part Two — Optimizing tensors on a loop



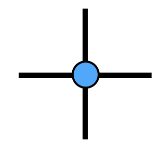
$$f(\mathbf{T}_i) = \||\Psi_A\rangle - |\Psi_B\rangle\| = \langle\Psi_A|\Psi_A\rangle + \langle\Psi_B|\Psi_B\rangle - \langle\Psi_A|\Psi_B\rangle - \langle\Psi_B|\Psi_A\rangle$$
$$= \mathcal{C} + \mathbf{T}_i^\dagger \mathcal{N}_i \mathbf{T}_i - \mathcal{W}_i^\dagger \mathbf{T}_i - \mathbf{T}_i^\dagger \mathcal{W}_i,$$

solve the linear equation $\quad \mathcal{N}_i \mathbf{T}_i = \mathcal{W}_i.$

Example: classical Ising model



partition function $Z = \sum_{\{\sigma\}} \exp(\beta \sum_{\langle ij \rangle} \sigma_i \sigma_j)$

local tensor  $\mathbf{T} = T^{\text{Ising}}_{u,l,d,r}$

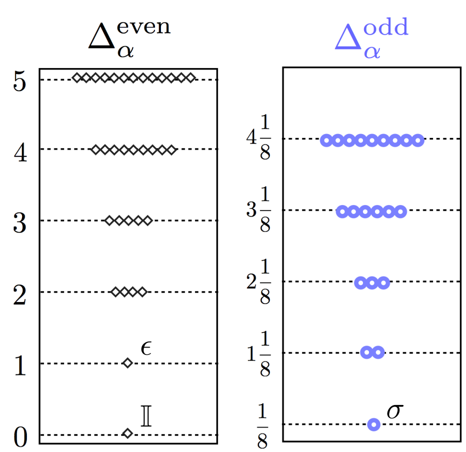
$$T^{\text{Ising}}_{1,2,1,2} = e^{-4\beta},\ T^{\text{Ising}}_{2,1,2,1} = e^{-4\beta},$$
$$T^{\text{Ising}}_{1,1,1,1} = e^{4\beta},\ T^{\text{Ising}}_{2,2,2,2} = e^{4\beta},$$
$$\text{others} = 1.$$

Ising CFT

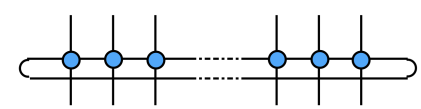
central charge

$c = 1/2$

scaling dimensions $\longrightarrow$



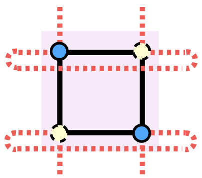
Calculate scaling dimensions

transfer matrix 

eigenvalues of the transfer matrix $\longrightarrow c, \Delta_\alpha$

Zheng-Cheng Gu and Xiao-Gang Wen,
Phys. Rev. B **80**, 155131 (2009).

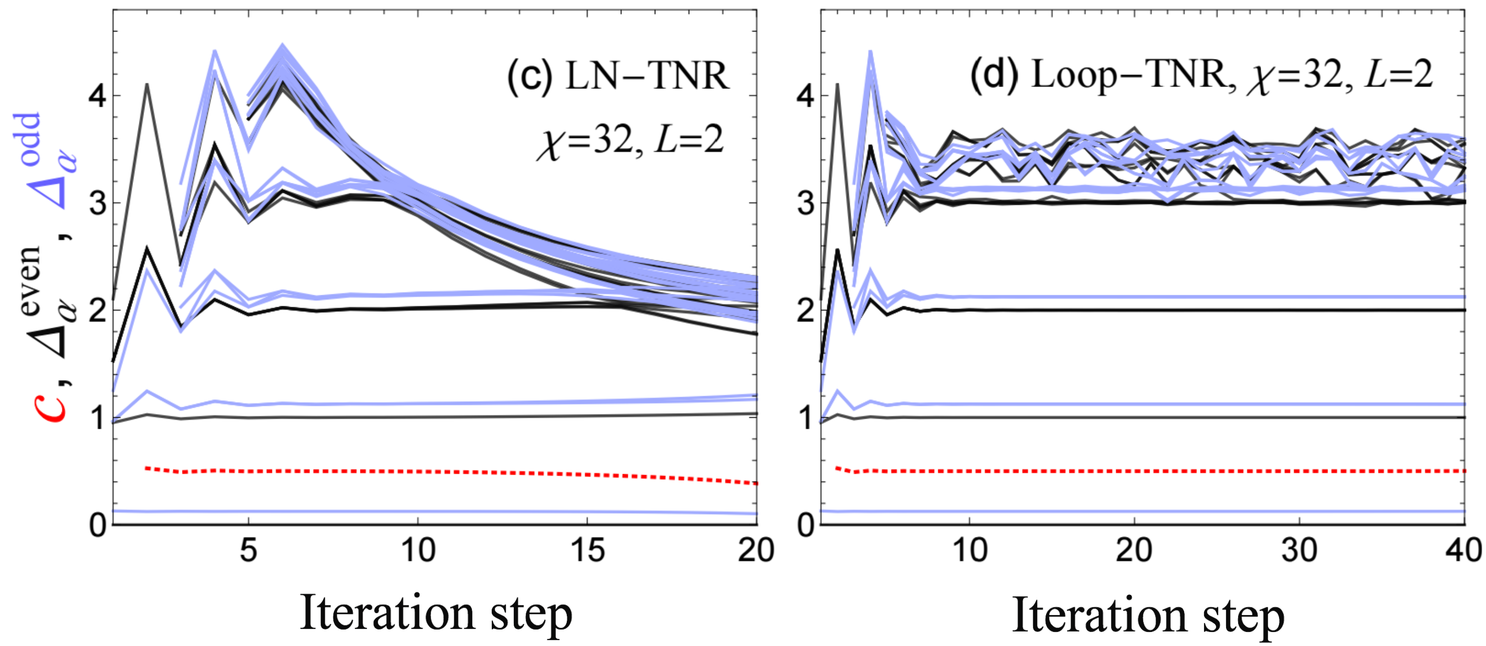
Comparison of results



$L = 2$

LN-TNR Results

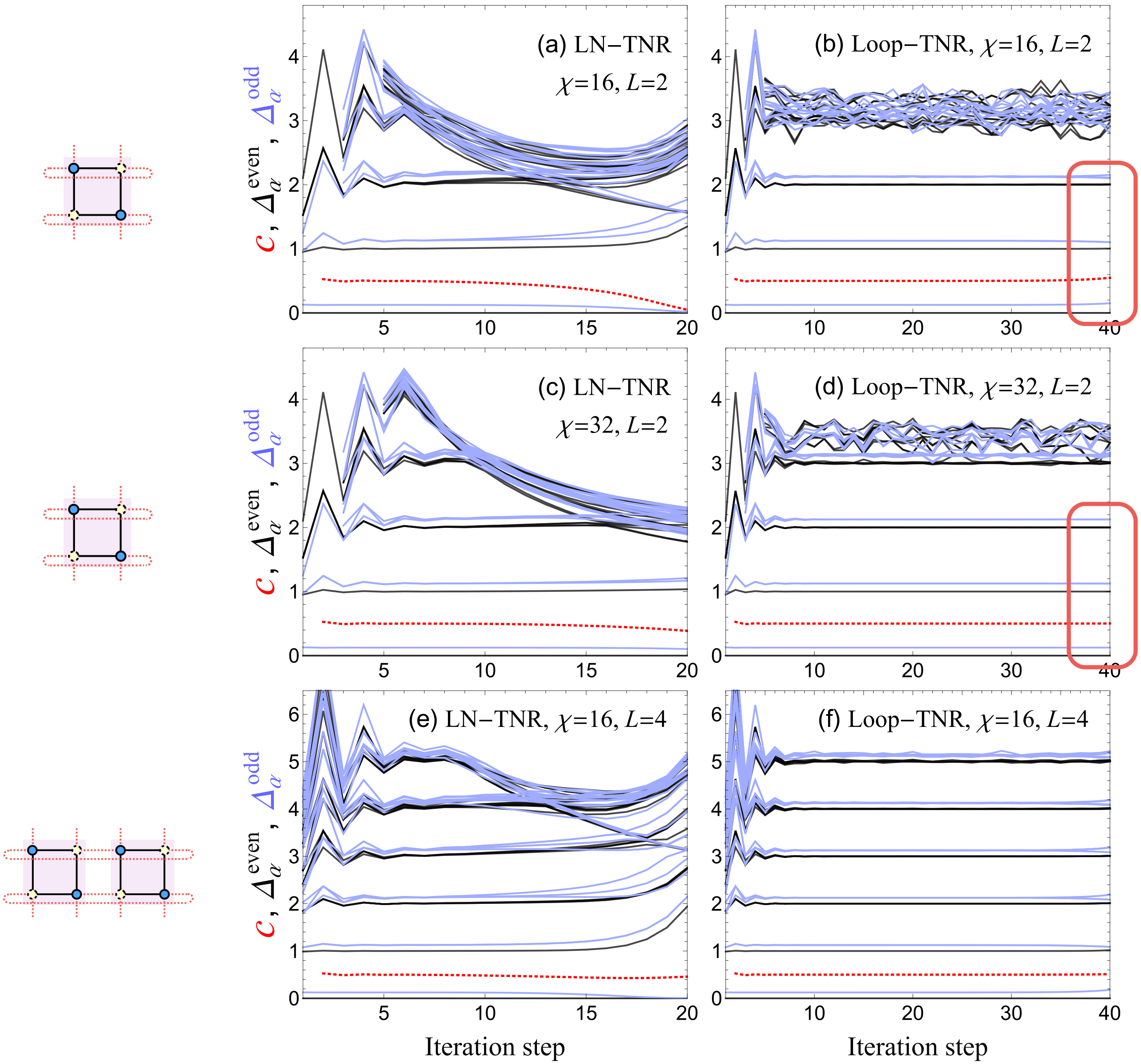
Loop-TNR Results



- scaling dimensions change with scale ~ cannot recover scale invariance
- high-index parts will destroy low-index parts

- scaling dimensions does not change with scale ~ scale invariance
- a clear gap between high-level parts and low-level parts

Stability



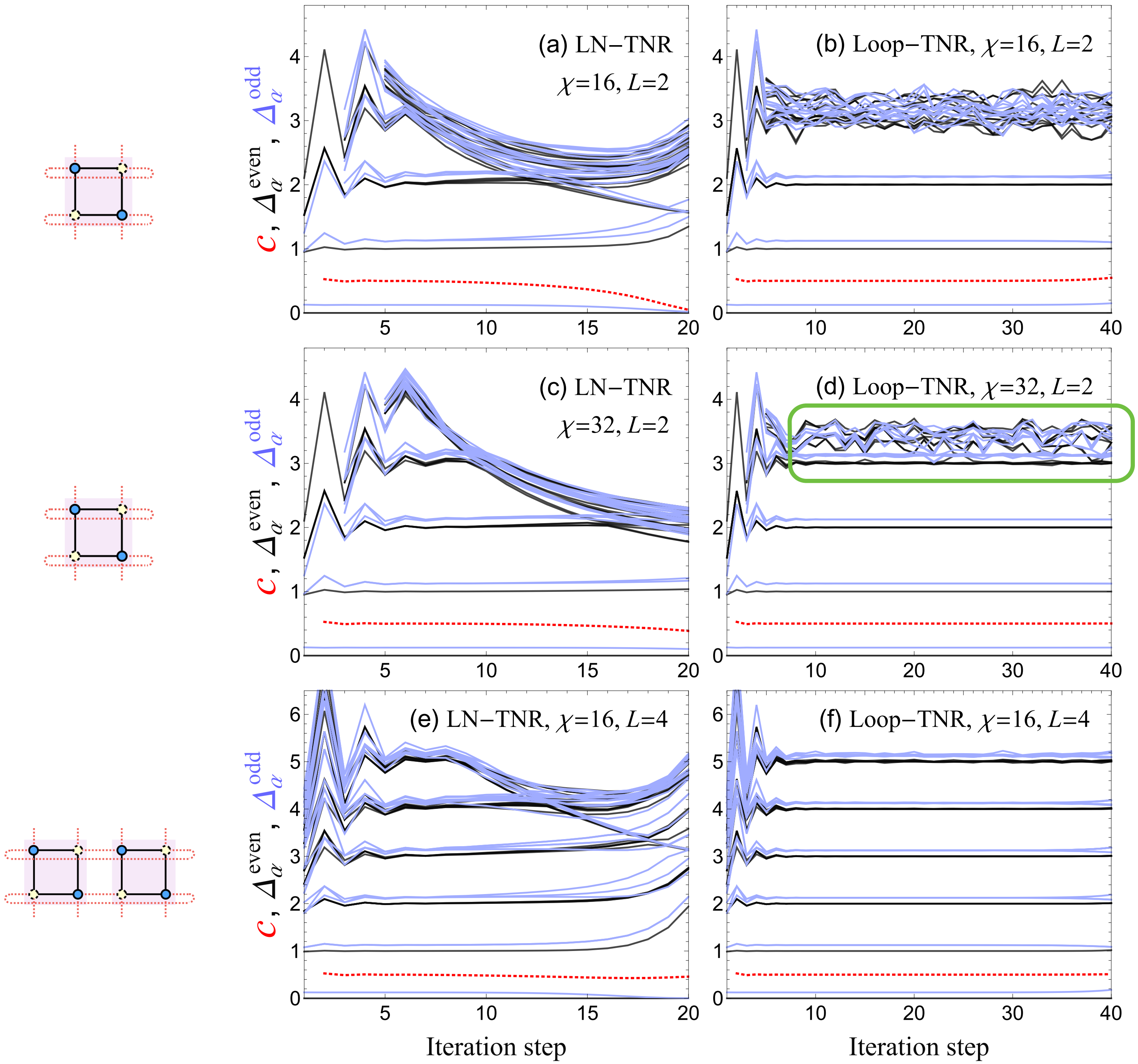
$\chi = 16$

remain accurate up to 40 iteration steps

even longer for $\chi = 32$

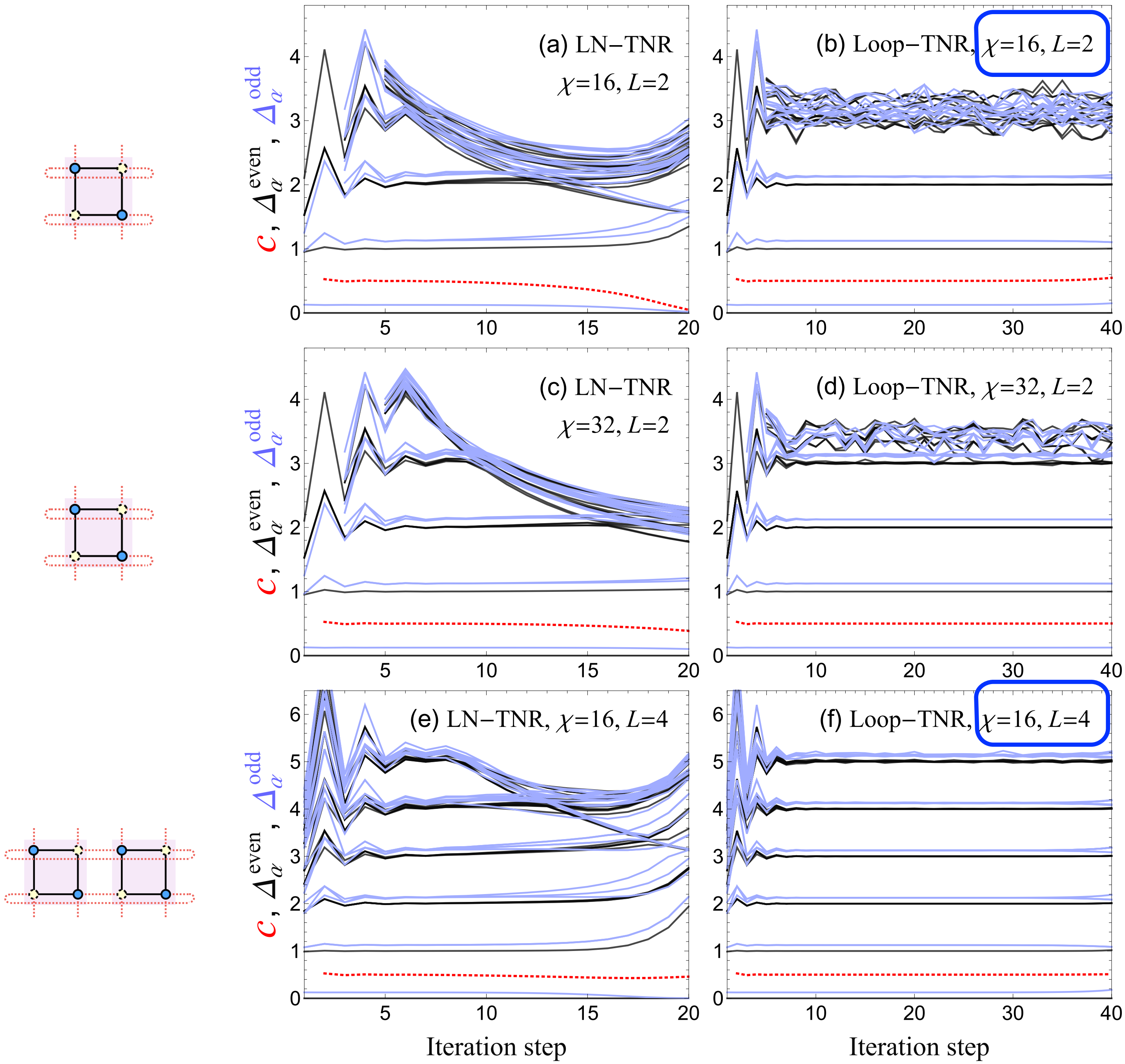
the proper RG flow last longer for larger χ

Stability



increasing χ, more scaling dimensions can be resolved

Stability



effectively, $\chi = 16$

effectively, $\chi = 16^2 = 256$

much more scaling

dimensions can be read off

accuracy is higher

infinite χ ~ infinite

dimensional fixed point tensor

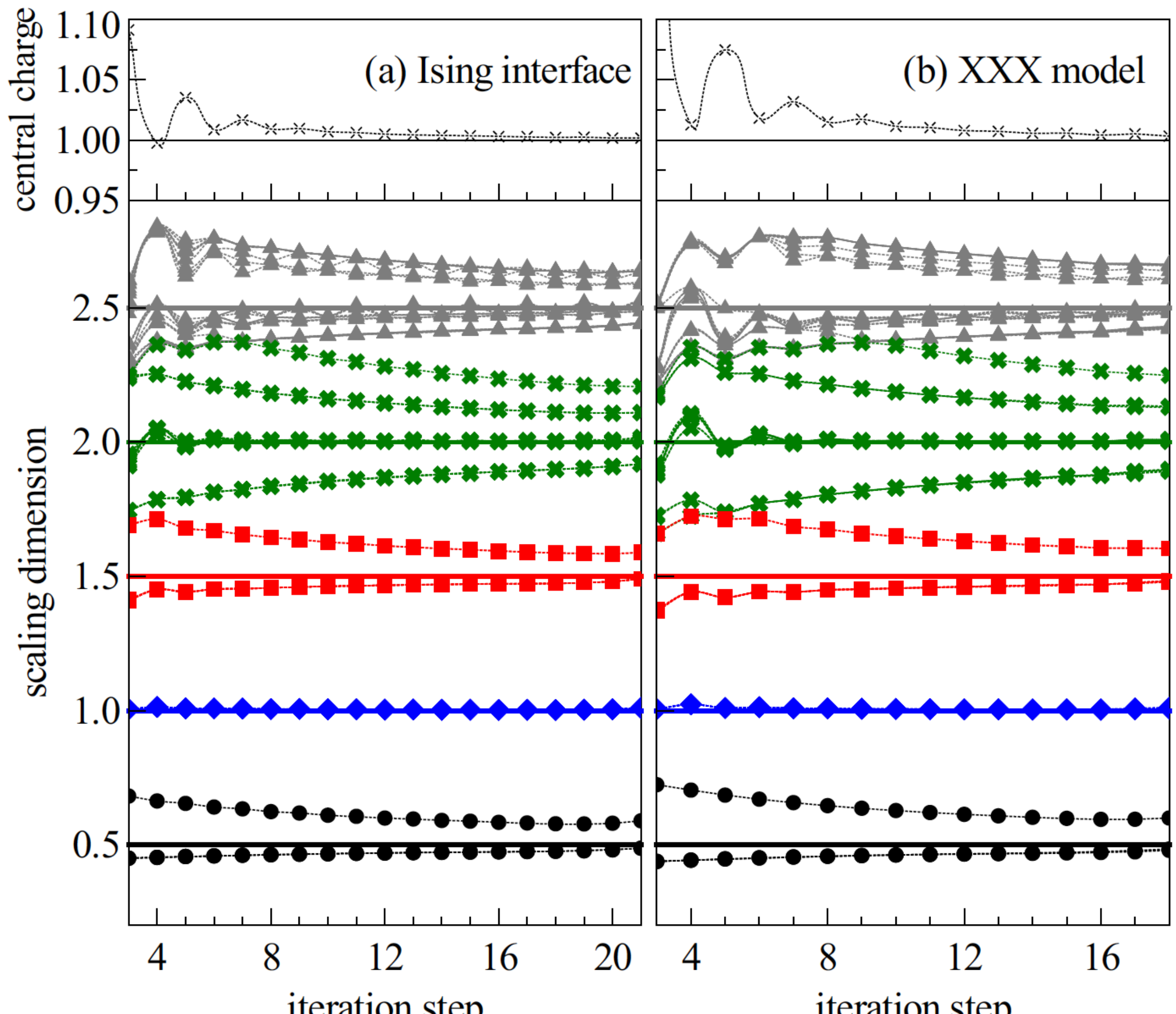
described by Ising CFT

Accuracy

	Exact	LN-TNR $\chi = 64$ $L = 1$ 2^{11} spins	LN-TNR $\chi = 64$ $L = 2$ 2^{11} spins	Loop-TNR $\chi = 16$ $L = 2$ 2^{18} spins	Loop-TNR $\chi = 24$ $L = 2$ 2^{18} spins	Loop-TNR $\chi = 16$ $L = 4$ 2^{18} spins	Loop-TNR $\chi = 24$ $L = 4$ 2^{18} spins	EV-TNR [50] $\chi = 24$ $L = 2$ 2^{18} spins
c	0.5	0.49946958	0.49970058	0.50001491	0.50000165	0.50009255	0.50008794	0.50001
σ	0.125	0.12504027	0.12500837	0.12500528	0.12500011	0.12501117	0.12499789	0.1250004
ϵ	1	1.00028269	0.99996784	1.00000566	1.00000601	0.99999403	1.00000507	1.00009
	1.125	1.12368834	1.12444247	1.12495187	1.12499400	1.12498755	1.12500559	1.12492
	1.125	1.12394625	1.12450246	1.12510600	1.12500464	1.12498755	1.12500559	1.12510
	2	1.92334948	1.99811859	2.00000743	1.99970911	1.99999517	2.00000985	1.99922
	2	1.96264143	1.99815644	2.00066117	2.00016629	1.99999517	2.00000985	1.99986
	2	1.97496787	1.99868822	2.00066117	2.00031103	2.00002744	2.00001690	2.00006
	2	2.00274974	1.99948966	2.00586886	2.00131384	2.00006203	2.00002745	2.00168

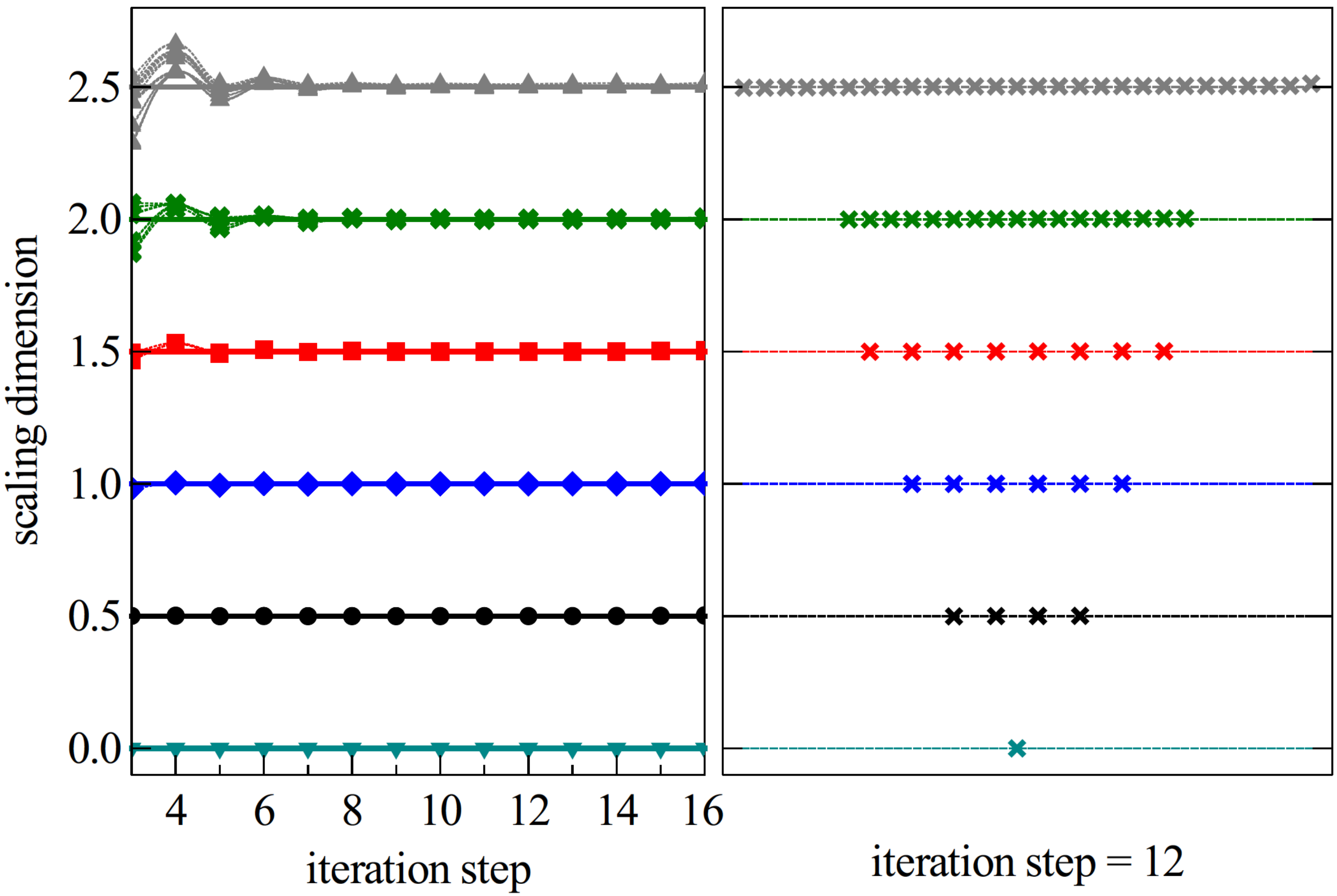
Universal CFT data

Perform the loop-TNR algorithm (Yang, Gu ,Wen 2017)



Remove the marginally irrelevant operator in XXX model

$$H'_{\mathrm{XXX}} = \sum_n (\mathbf{S}_n \cdot \mathbf{S}_{n+1} + J_2 \mathbf{S}_n \cdot \mathbf{S}_{n+2})$$



Anyon exchange symmetry and gapless nature of domain wall

The domain wall model can be regarded as the boundary of a stacking system

$$S_{TC} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & 1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix}$$

$$S_{DS} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix}$$

	1 s	1 $\bar{s}$	e 1	m 1	f $\bar{s}$	f s	m b	e b	1 1	1 b	e s	e $\bar{s}$	m s	m $\bar{s}$	f 1	f b
1 s	-1	1	1	1	1	-1	-1	-1	1	-1	-1	1	-1	1	1	-1
1 $\bar{s}$	1	-1	1	1	-1	1	-1	-1	1	-1	1	-1	1	-1	1	-1
e 1	1	1	1	-1	-1	-1	-1	1	1	1	1	1	-1	-1	-1	-1
m 1	1	1	-1	1	-1	-1	1	-1	1	1	-1	-1	1	1	-1	-1
f $\bar{s}$	1	-1	-1	-1	-1	1	1	1	1	-1	-1	1	-1	1	1	-1
f s	-1	1	-1	-1	1	-1	1	1	1	-1	1	-1	1	-1	1	-1
m b	-1	-1	-1	1	1	1	1	-1	1	1	1	1	-1	-1	-1	-1
e b	-1	-1	1	-1	1	1	-1	1	1	1	-1	-1	1	1	-1	-1
1 1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
1 b	-1	-1	1	1	-1	-1	1	1	1	1	-1	-1	-1	-1	1	1
e s	-1	1	1	-1	-1	1	1	-1	1	-1	-1	1	1	-1	-1	1
e $\bar{s}$	1	-1	1	-1	1	-1	1	-1	1	-1	1	-1	-1	1	-1	1
m s	-1	1	-1	1	-1	1	-1	1	1	-1	1	-1	-1	1	-1	1
m $\bar{s}$	1	-1	-1	1	1	-1	-1	1	1	-1	-1	1	1	-1	-1	1
f 1	1	1	-1	-1	1	1	-1	-1	1	1	-1	-1	-1	-1	1	1
f b	-1	-1	-1	-1	-1	-1	-1	-1	1	1	1	1	1	1	1	1

subset: $\{1, e, eb, b\}$ or $\{1, m, mb, b\}$

Conclusions and discussions

Gapless domains of non-chiral topological phases are constructed systematically in 2D.

We use the state-of-art loop-TNR algorithm to test the Z2 example and a su(2)_1 WZW CFT is found even in the absence of global SU(2) symmetry.

CFT and geometric degrees of freedom naturally emerge on gapless domain walls.

Our constructions can be potentially generalized into higher dimensions.

Thank you!