# Landau versus topological phase transitions


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Lokman Tsui et al, Nucl. Phys. B **896**, 330 (2015).
Lokman Tsui et al, arXiv 1701.00834 (Nucl. Phys. B accepted)

Collaborators

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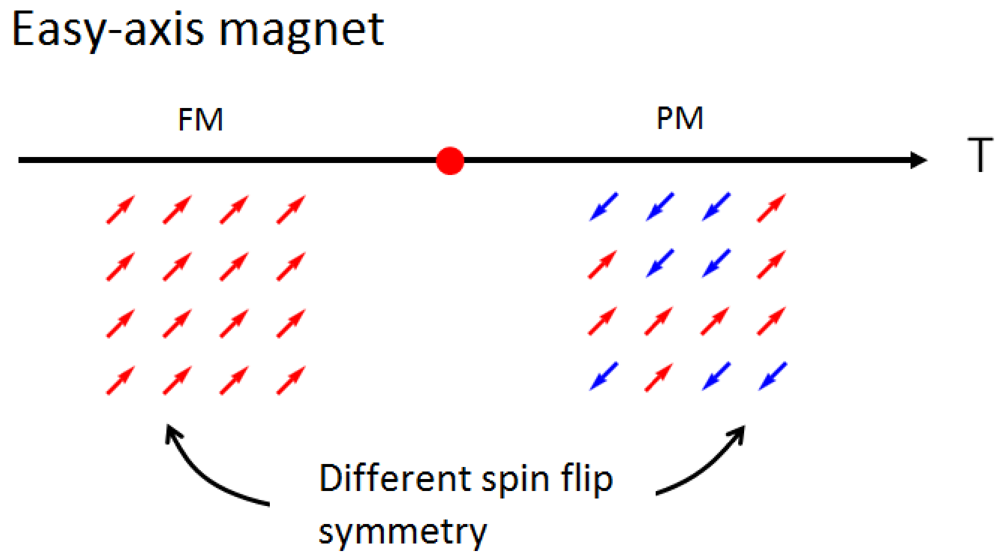
Congratulations to KITS, and Fu-chun !!!

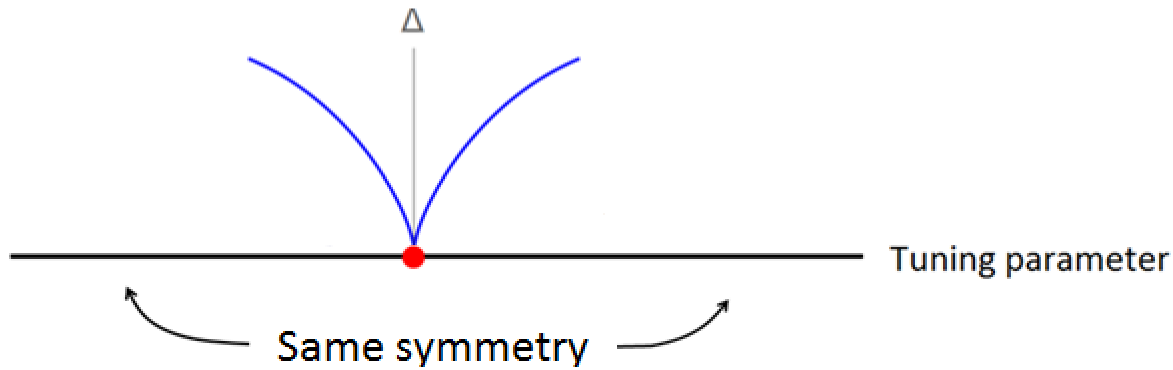
Landau transition: phase transition between phases with different symmetries

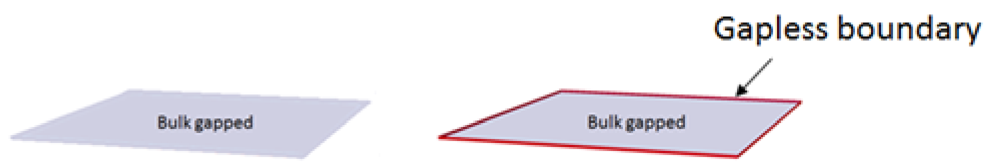



The study of Landau phase transitions has revolutionized physics.

- Conformal symmetry
- Renormalization group
- Physical understanding of continuum quantum field theories

Recently a new type of phase transition comes into focus. These transitions occur between phases with the same symmetry.




The difference between the phases reside on their boundaries

These phases are known as the symmetry protected topological states (SPTs). X. Chen, Z-C Gu, Z-X Liu, X-G Wen, Science 338, 1604 (2012)

Examples include:

- Bosonic: the Haldane phase
- Fermionic: topological insulators

Question:
What are the difference (if any) between Landau symmetry breaking phase transitions and the phase transitions between bosonic SPTs?

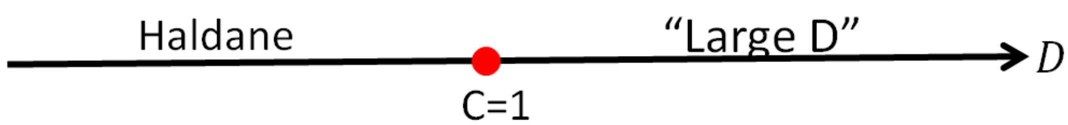
The Landau phase transitions are described by space-time fluctuating order parameter, i.e. bosonic field theories.

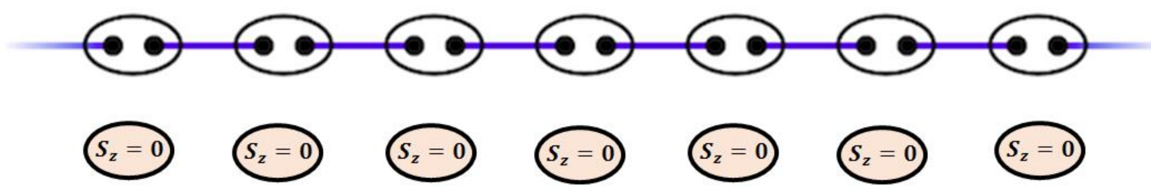
To avoid obvious differences we focus on phase transition between bosonic SPTs.

# Example: topological phase transition in spin-1 chain

W. Chen *et al*, Phys. Rev. B 67 104401 (2003)

$$H = \sum_i \vec{S}_i \cdot \vec{S}_{i+1} + D \sum_i S_{iz}^2$$



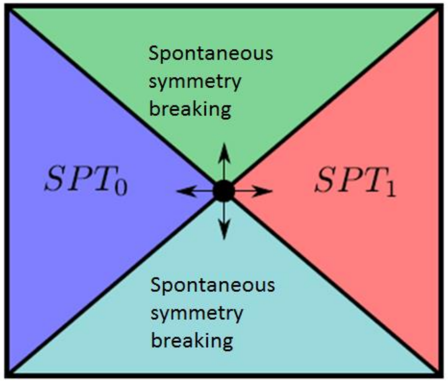

Symmetry ($U(1) \times Z_2$) protected topological phases

# Main results

- A theorem about topological phase transition in 1D

Lokman Tsui et al, Nucl. Phys. B accepted

1. Theorem:
In 1 D bosonic SPT transitions are described by conformal field theories with central charge $\geq 1$




2. SPT transition are Multi-critical point of Landau forbidden transition and SPT phase transition.

- A holographic theory of bosonic SPT phase transition valid for any $d$.

Lokman Tsui et al, Nucl. Phys. B **896**, 330 (2015).

1. The critical state possesses
- 1D: fractionalized particles
- 2D: fluctuating loops (with gapless excitations)
- 3D:fluctuating membranes (with gapless excitations)

2. Critical point has emerging duality symmetry.

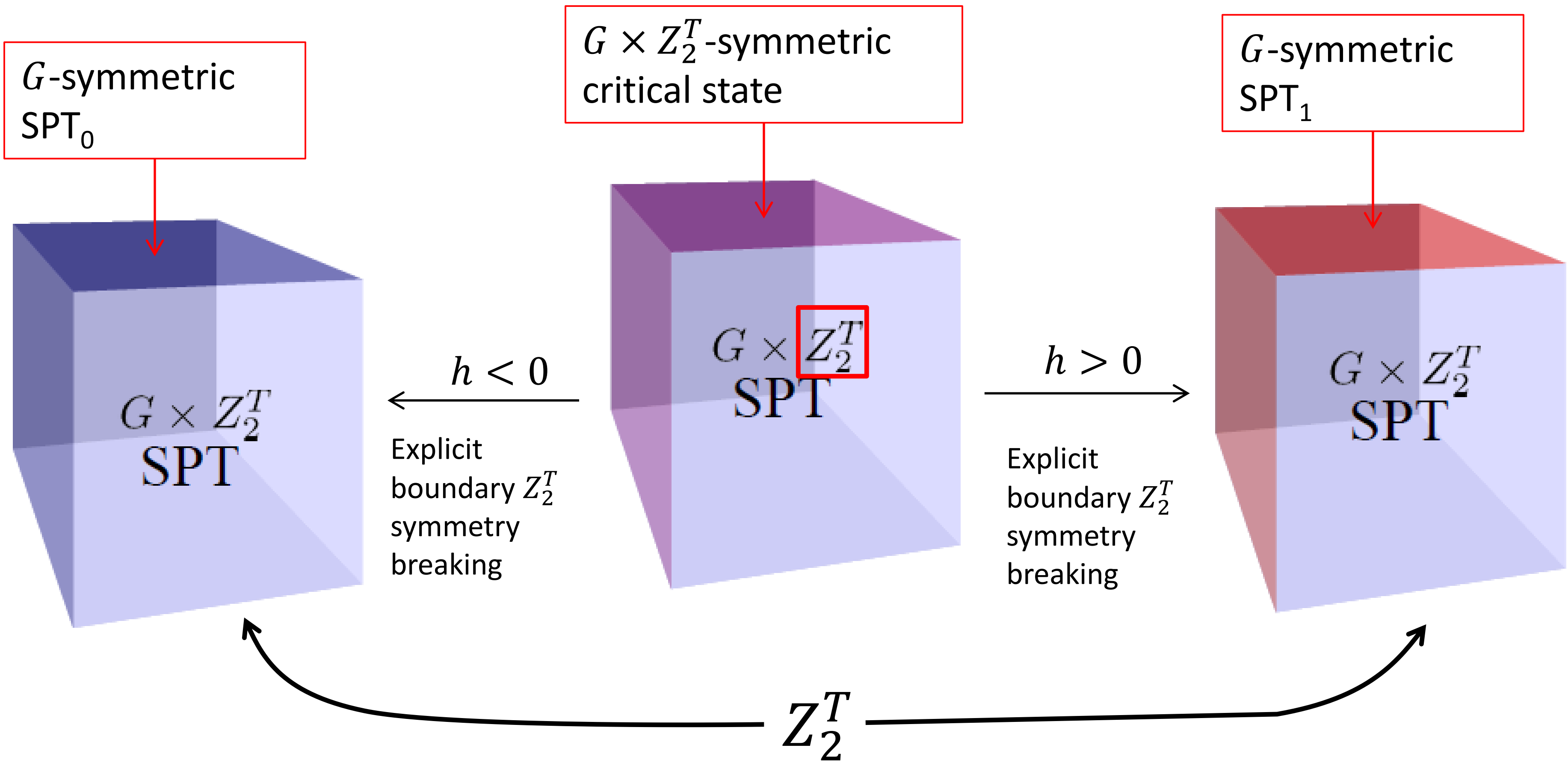
# A holographic theory of topological phase transitions in any dimensions

Applies to phase transitions between the trivial and the non-trivial phases of SPTs whose group cohomology classification contains a $Z_{2m}$ factor.

Viewing the critical state between two $G$ symmetric SPTs as the boundary state of a non-trivial SPT protected by $G \times Z_2^T$.

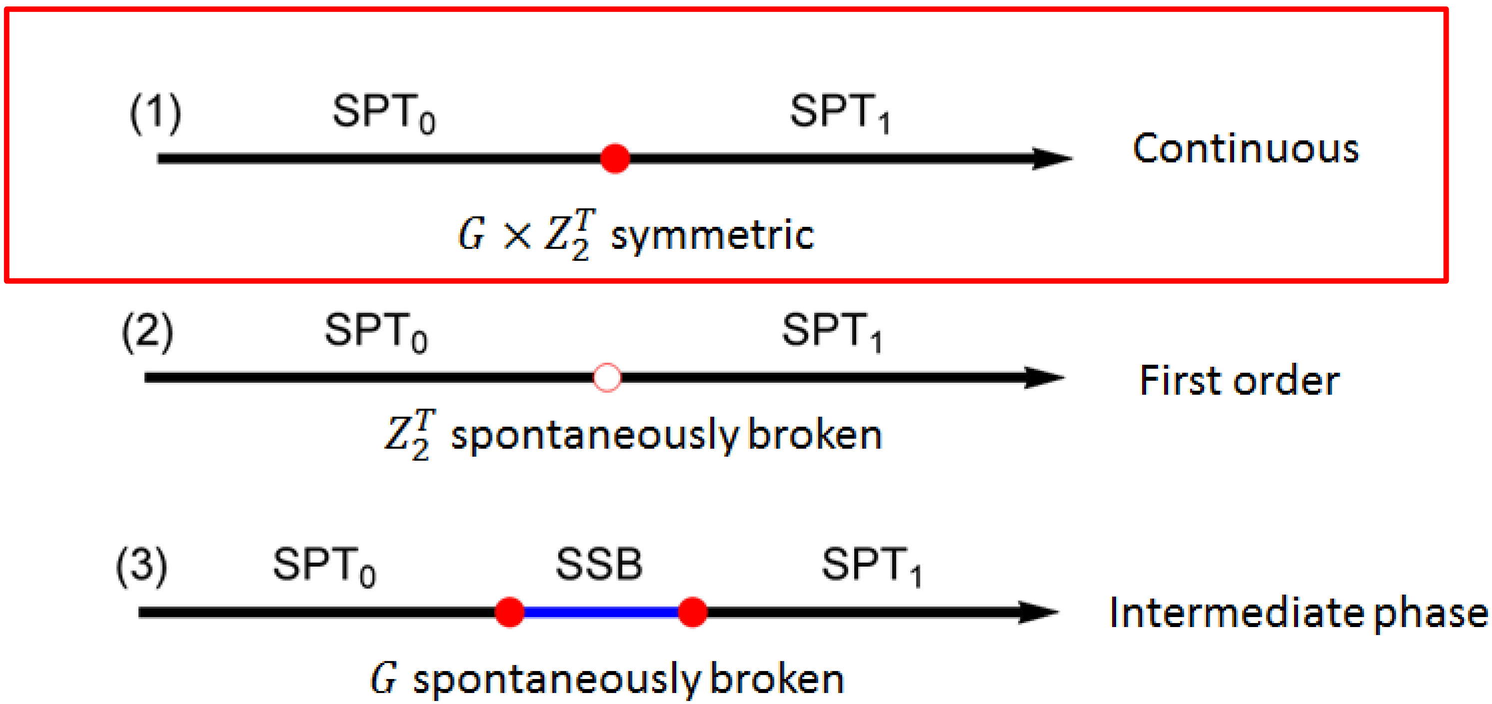
Lokman Tsui et al, Nucl. Phys. B **896**, 330 (2015).

i.e. the phase transition between $G$-symmetric SPTs occurs on the boundary of a $d+1$ dimensional $G \times Z_2^T$ symmetric SPT



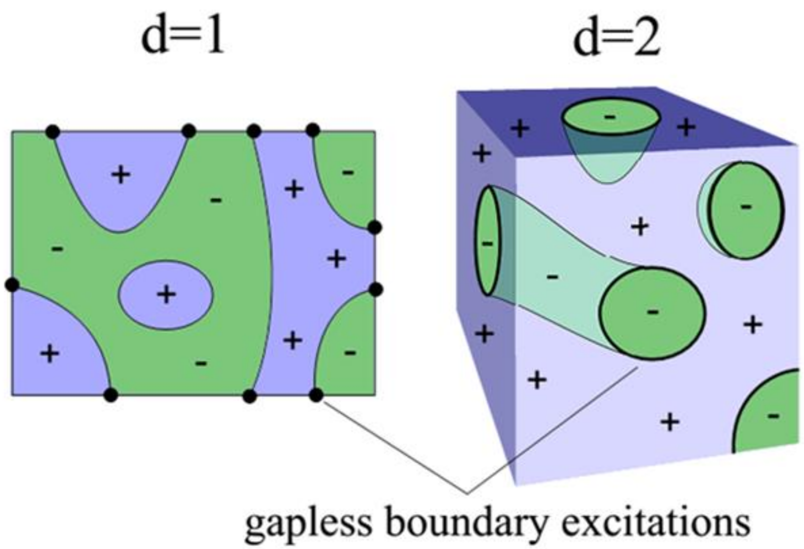

$Z_2^T$ transformation exchanges SPT$_0$ and SPT$_1$ hence acting as a duality transformation !

# Three phase transition scenarios



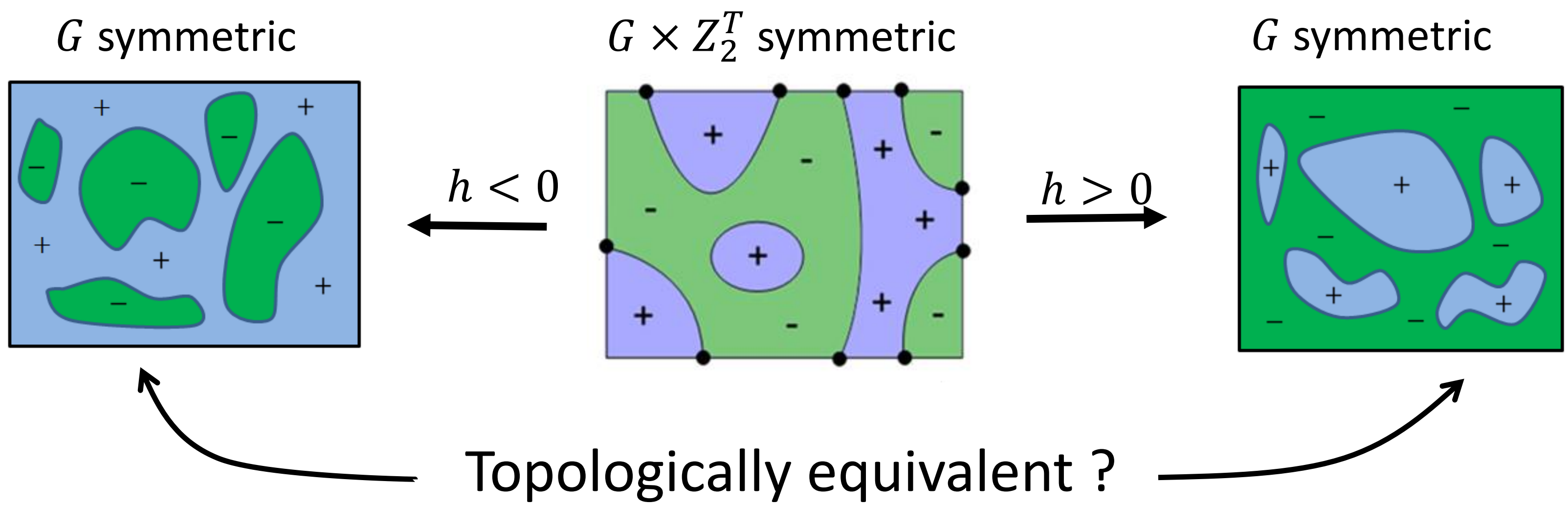
# How to construct the $d+1$ dimensional SPT ?

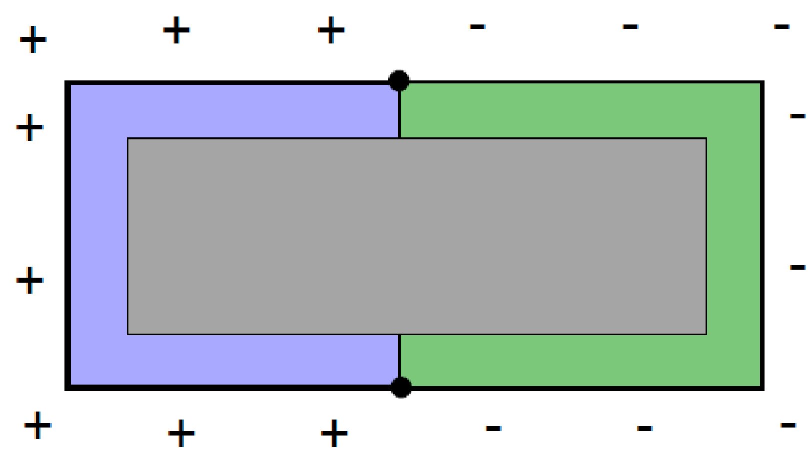
The d+1 dimensional $G \times Z_2^T$ symmetric SPT is obtained by proliferating $Z_2^T$ domain walls each "decorated" with a non-trivial $G$-symmetric SPT.




$d$=1 "fractionalized" particles.
$d=2$ fluctuating loops with gapless excitation
$d=3$ fluctuating membranes with gapless excitation

# Boundary phase transition



Topologically inequivalent

The critical state of bosonic SPT transition possesses

$d = 1$ delocalized “fractionalized” particles.
$d = 2$ fluctuating loops with gapless excitation
$d = 3$ fluctuating membranes with gapless excitation

Ordinary Landau transition can also have fluctuating surfaces, e.g., the domain walls of the Ising model.

However for topological phase transitions these surfaces are infested with gapless excitations.

# Bosonic SPT phase transitions in 1D

Q: Which conformal field theory can describe topological phase transitions between bosonic SPTs ?

Theorem

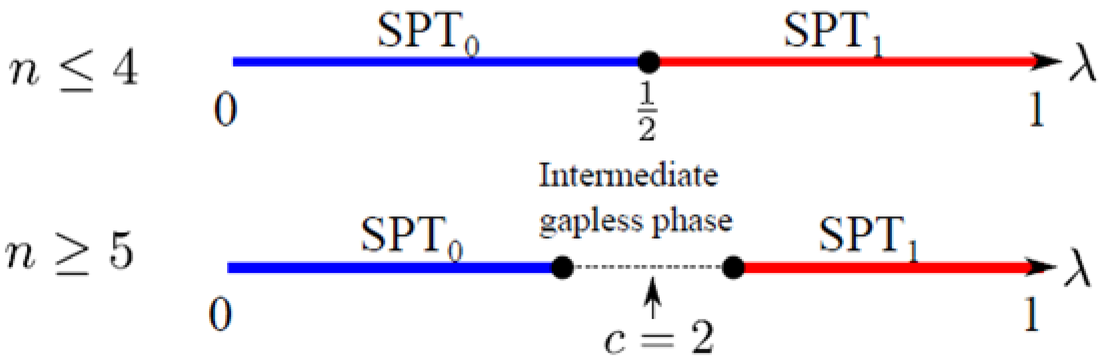
For minimal models (central charge <1) the maximum on-site symmetry (including the emergent symmetry) is either $Z_2$ or $S_3$.

Ruelle and Verhoeven, Nucl. Phys. B 535,650 (1998)

However neither $S_3$ nor $Z_2$ can protect non-trivial SPT phases in 1D.

- The minimal models can not be the critical theory of the bosonic SPT phase transitions.
- The CFT of bosonic SPT transition must have $c \geq 1$

# Examples: Phase transitions between $Z_n \times Z_n$ protected bosonic SPTs




| Symmetry group | Central charge |
|---|---|
| $Z_2 \times Z_2$ | 1 |
| $Z_3 \times Z_3$ | $8/5$ |
| $Z_4 \times Z_4$ | 2 |

| $n$ | $h+\bar{h}$ | $h-\bar{h}$ | Multiplicity |
|---|---|---|---|
| 2 | 0 | 0 | 1 |
| | $1/4$ | 0 | 2 |
| | 1 | 0 | 2+2 |
| | 1 | $\pm1$ | 2 |
| | $5/4$ | $\pm1$ | 8 |

| $n$ | $h+\bar{h}$ | $h-\bar{h}$ | Multiplicity |
|---|---|---|---|
| 3 | 0 | 0 | 1 |
| | $4/15$ | 0 | 4 |
| | $4/5$ | 0 | 2 |
| | $14/15$ | 0 | 4 |
| | $17/15$ | $\pm1$ | 8 |
| | $19/15$ | $\pm1$ | 16 |
| | $4/3$ | 0 | 4 |
| | $22/15$ | 0 | 8 |
| | $8/5$ | 0 | 1 |

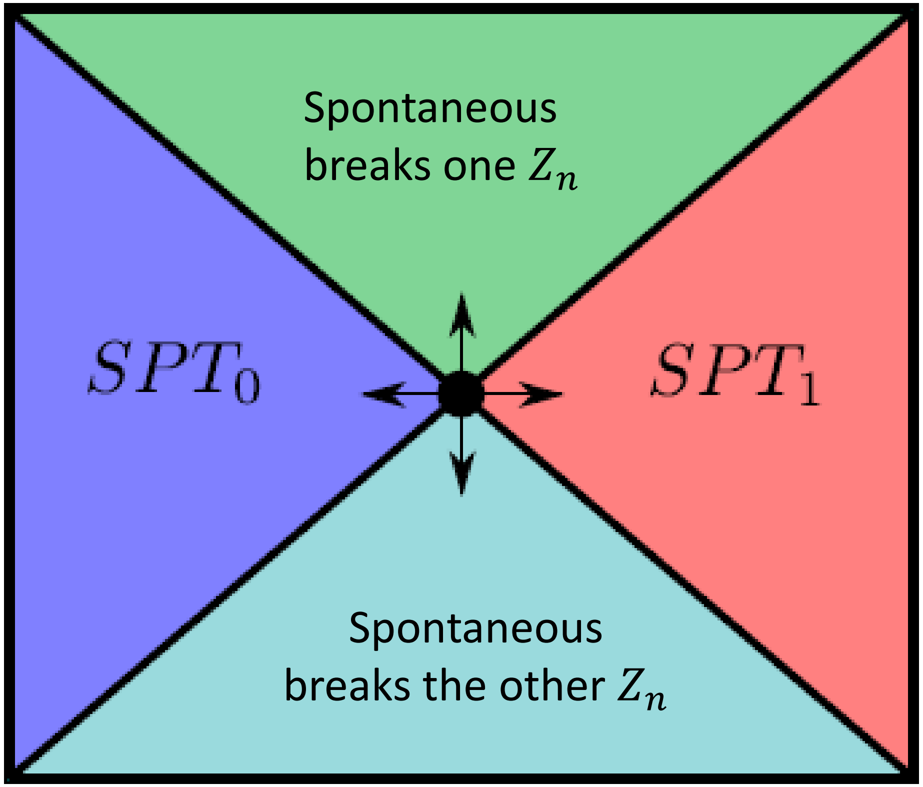
| $n$ | $h+\bar{h}$ | $h-\bar{h}$ | Multiplicity |
|---|---|---|---|
| 4 | 0 | 0 | 1 |
| | $1/4$ | 0 | 4 |
| | $1/2$ | 0 | 2 |
| | $5/8$ | 0 | 4 |
| | 1 | 0 | 2 |
| | $9/8$ | $\pm1$ | 8 |
| | $5/4$ | 0 | 20 |
| | $5/4$ | $\pm1$ | 12 |
| | $5/4$ | $\pm1$ | 16 |
| | $3/2$ | $\pm1$ | 8 |
| | $13/8$ | 0 | 4 |
| | $13/8$ | $\pm1$ | 16 |

Entries in blue are invariant under $Z_n \times Z_n$

Blue entries in red box are gap-opening operators

But why are there two gap opening operators ?

# Two gap opening operators




SPT phase transitions are also Landau-forbidden transitions, i.e., they are multi-critical points.

# Conclusions

- The critical state of bosonic SPT phase possesses fractionalized particles in 1D and fluctuating loops (2D) and fluctuating membranes (3D) with gapless excitations residing on them. The critical points are self-dual.
- In d=1 the CFT describing SPT phase transitions must have $c \geq 1$. For the models studied they are multi-critical point.

## Open problems

- Among $c > 1$ CFT in 1+1 dimensions which ones can describe SPT phase transitions?
- Holographic understanding of the critical phenomenon of SPT phase transitions?
- The effect of disorder on SPT transitions.
- And many more ….

Thank you !