

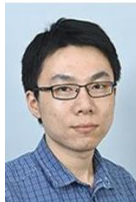
The Transformation of the Superconducting Gap to an Insulating Pseudogap with Decreasing Hole Density in the Cuprates

T. Maurice Rice
ETH Zurich & Brookhaven National Lab.

A priori treatment of strong correlations easier in \mathbf{r} – space.

BUT the Pseudogap is in \mathbf{k} - space.

\mathbf{k} - space analysis requires postulating low energy effective interactions
which leads to qualitative results only

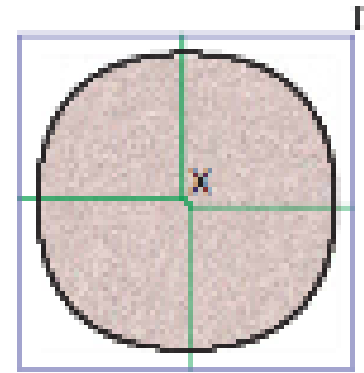
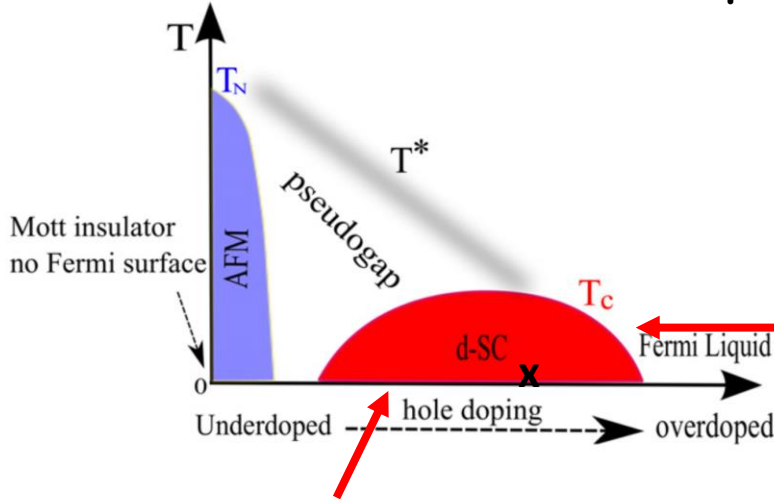


Y.-H. Liu, W.-S. Wang, Q.-H. Wang, F.C. Zhang & T. M. Rice arXiv1703.03695

Also Y. H. Liu, R. Konik, T.M. Rice & F.C. Zhang Nature Commun. **7** 10378 (2016)
Giant Phonon Anomalies associated with the Pseudogap

Phase diagram of cuprates :

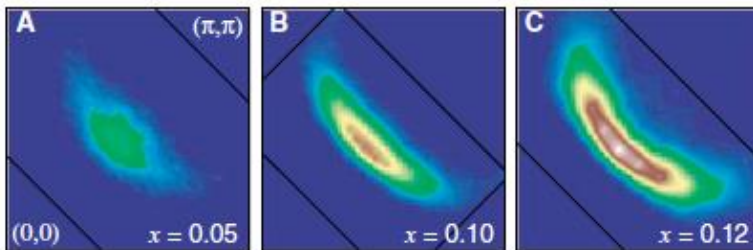
Fermi Surface Evolution : Overdoped to Underdoped : Full Fermi surface in overdoped region



Fermi Surface from Quantum Oscillations
B. Vignolle et al Nature 2008

Pseudogapped Fermi surface in underdoped region : $T < T^*$

- Breakup of Fermi Surface \rightarrow 4 disconnected Pockets(Arcs)
- Centered on nodal direction ARPES : Norman ... Hong Ding (1998)



ARPES on $\text{Ca}_{2-x}\text{Na}_x\text{CuO}_2\text{Cl}_2$

Kyle M. Shen et al Science 2005

Pseudogap is located at antinodal region

Precursor of the Mott Insulator ?

Pseudogap Phase only Short Range Order

Mott Insulating State viewed in k -space

Real space

k -space

Underlying lattice

Umklapp scattering processes allowed
→ Momentum conserved modulo $\{G\}$

Band filling 1 el./site

Surface in k -space enclosing
an area of $\frac{1}{2}$ - Brillouin zone.

Conclusion ; Examine growing Umklapp processes as x decreases
connecting k - points on a U-surface :

U -surface a) spanned by elastic U- scattering processes

b) enclosing an area of $\frac{1}{2}$ - Brillouin zone.

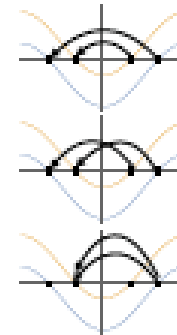
How to describe a Short Range Ordered State ?

- Use a Wave- Packet Formalism localized in both \mathbf{k} & \mathbf{r} - Space
Approximate as a Lattice of Strongly Correlated Supercells
- Ossadnik arXiv 1305.2534 & 1603.04041
- Energy Spectrum of Single Supercells Decisive:

e.g $\frac{1}{2}$ -filled 2-Leg Hubbard Ladder with U-scattering processes

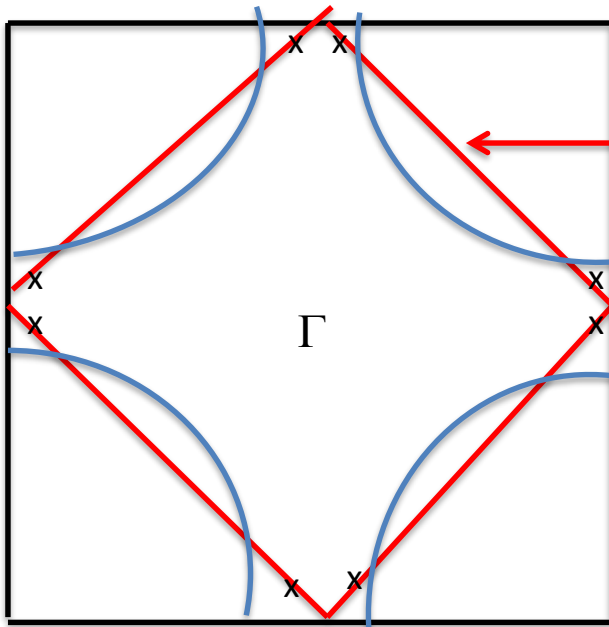
Isolated Unique Insulating Groundstate
with Charge Gap and Spin Gap
1- & 2- particle gaps & $S = 1$ gap

SRO : d-wave pairing & AF (π, π)
=> D-Mott Insulator Balents - Fisher '96 & '98
Continuous Crossover between weak and Strong Coupling.



Extend to 2D Square Lattice near $\frac{1}{2}$ - filling ?

Umklapp Scattering Surface in 2D



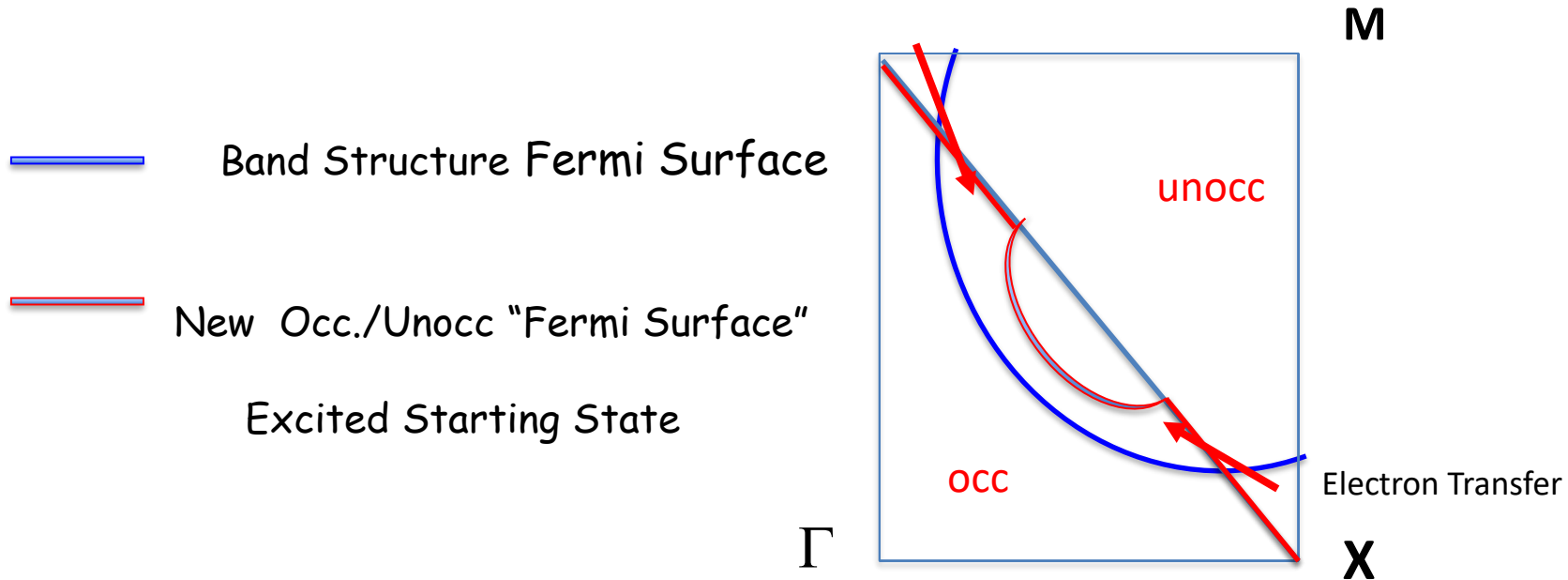
Umklapp Scattering Surface
Not at constant energy with nnn hopping
BUT 8 x-points are degenerate and
are connected by U-processes

Scattering in 1-Loop FRG gives a low energy effective model :

Strong p-h Umklapp - AF (π, π) & d-wave Cooper pair scatterings
[Honerkamp et al '01, Q. H. Wang et al]

Modifying the Starting Occ / Unocc Surface to Maximize Umklapp Scattering

Maximize Overlap Occ/Unocc Surface with U-Surface with fixed electron count



Turn on Interactions \Rightarrow Large SRO Pairing & AF Gap fixed on U-surface

\Rightarrow Real Gap Function with Insulating Character opens on the U-surface

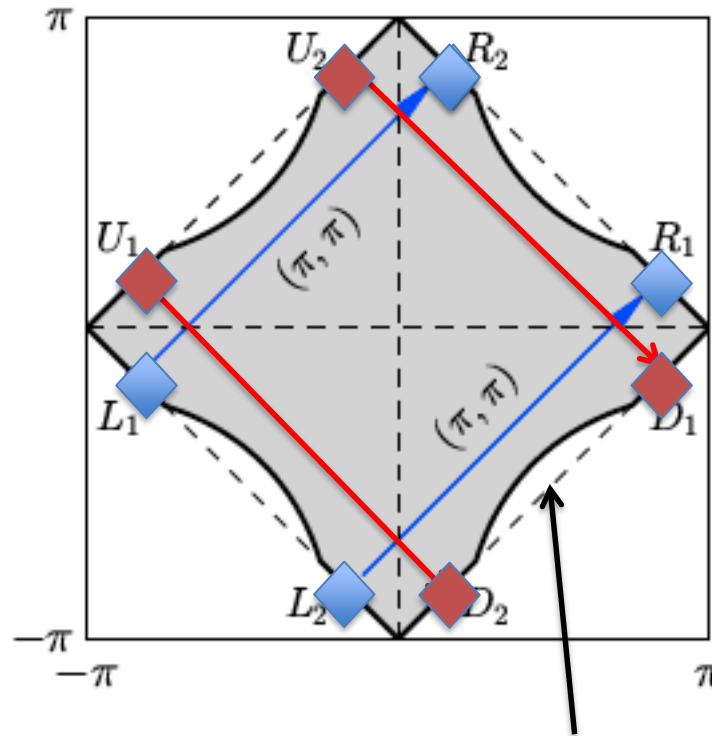
U-Scattering Processes on the U-Surface

(π, π) U Scattering Processes connect the perpendicular $(1,1)$ & $(1,-1)$ directions separately
 i.e. not between $\{U, D\}$ & $\{R, L\}$ patches \Rightarrow Treat each separately as 2-Leg Ladders

Map each set of $\{R, L\}$ and $\{U, D\}$ patches to 2-Leg Ladders with k - dependent parameters

\Rightarrow Pseudogap Energy Gaps to add both 1-el (E_1) & 2-els (E_2) leading to Insulating behavior

\Rightarrow also SRO AF (π, π) in dSC ($T < T_c$) & normal states ($T > T_c$)



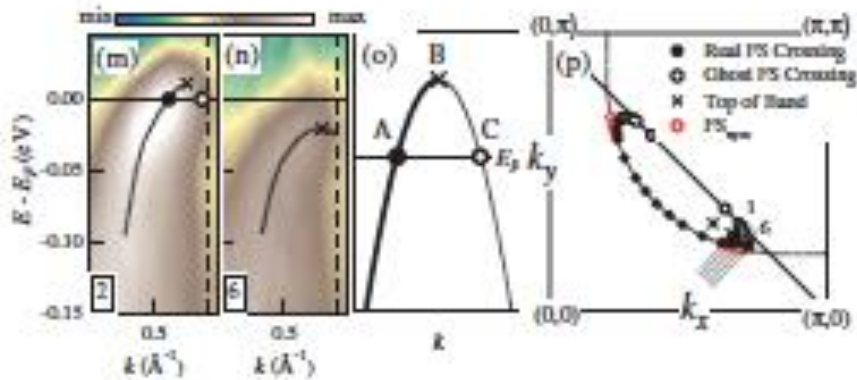
Pair Scattering Processes on the 4 Fermi Arcs \Rightarrow d-wave SC pairing $\Rightarrow E_2 = 0$

YRZ Ansatz for propagator with Energy Gap fixed on US

$$G^{RVB}(k, \omega) = \frac{g_t}{\omega - \xi(k) - \frac{\Delta_R^2}{[\omega + \xi_0(k)]}} + G_{inc}$$

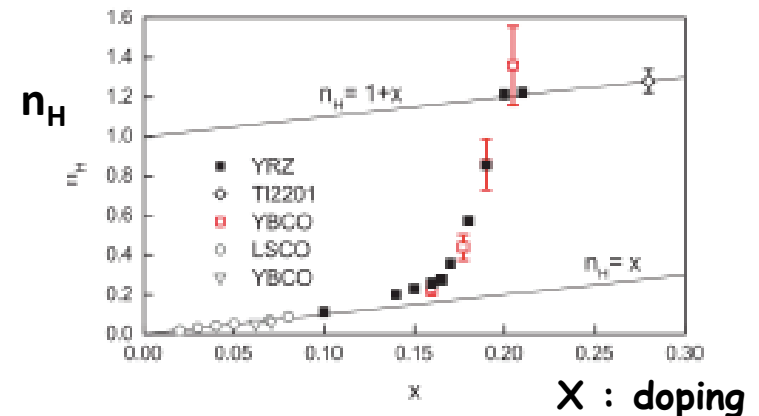
original FS open gap at US
 YES nnn hopping NO

Yang, Rice, Zhang, PRB '06



ARPES H. B. Yang et al, PRL 2011 BNL

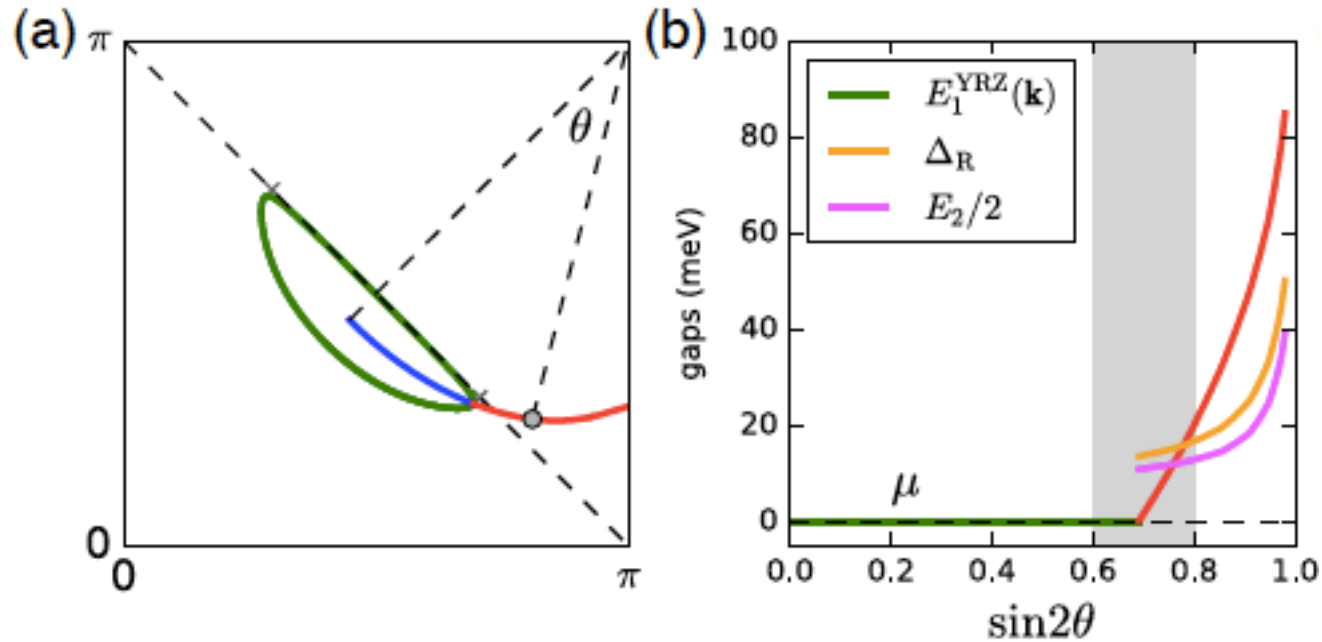
Hall Effect at onset of Pseudogap



J. G. Storey, EPL 2016

Expts : Taillefer group 2016

Normal State Energies



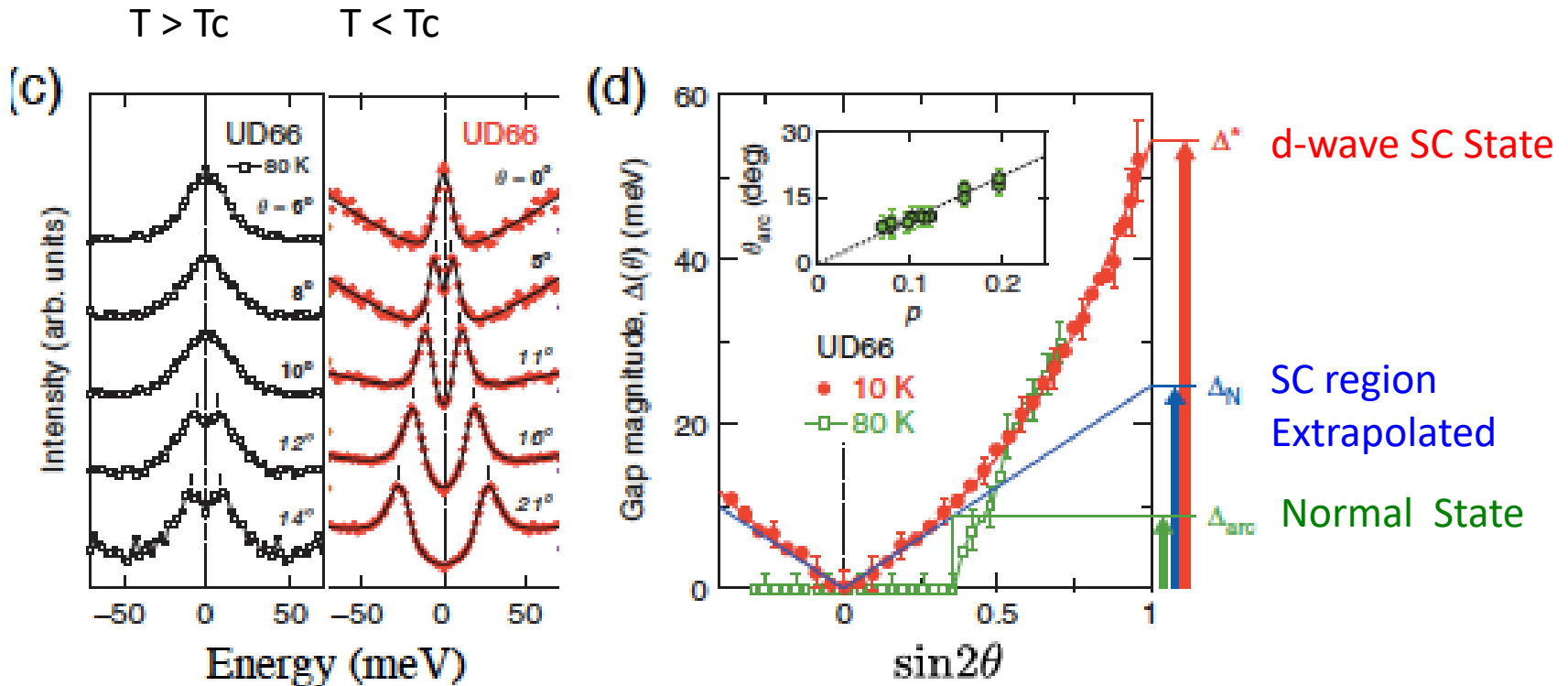
Transition Region

Single Particle Energy : along Nodal Pocket (YRZ form) —

Minimum hole energy not along Umklapp Surface —

Cooper Pair Energy (per Hole) along Umklapp Surface —
 Indicating pairing short range correlations $E_1 > E_2 > 0$

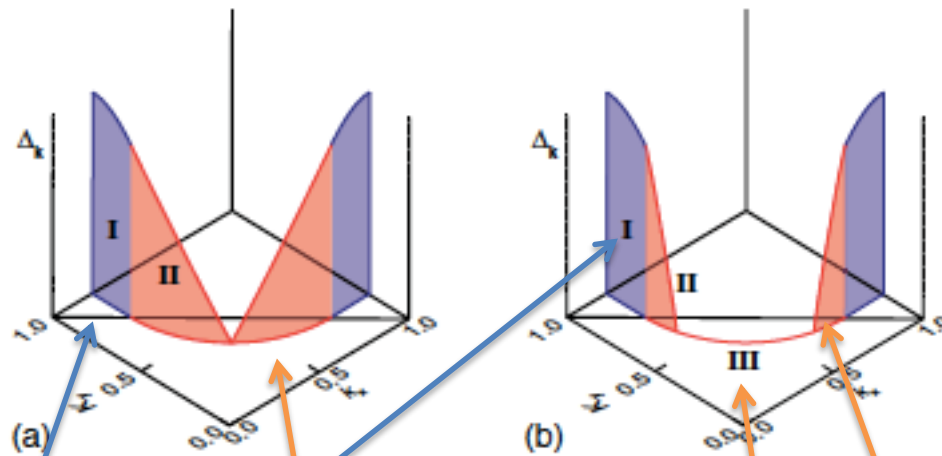
ARPES{ single hole} Spectra{Symmetrized} Anzai et al(2013) on UD BSCCO sample $T_c = 66\text{K}$



Energy Symmetrized ARPES Spectra
 Note: Pairing Fluctuations at crossover
 $T \sim T_c$ broadens peaks

Single Particle Energy Gaps
 Note: Pseudogap constant at $T > T_c$

Scanning Tunneling Microscopy Results for $E_1(\mathbf{k})$ - Davis Group
Fujita et al JPSJ (2012)



SC state; I : Pseudogap

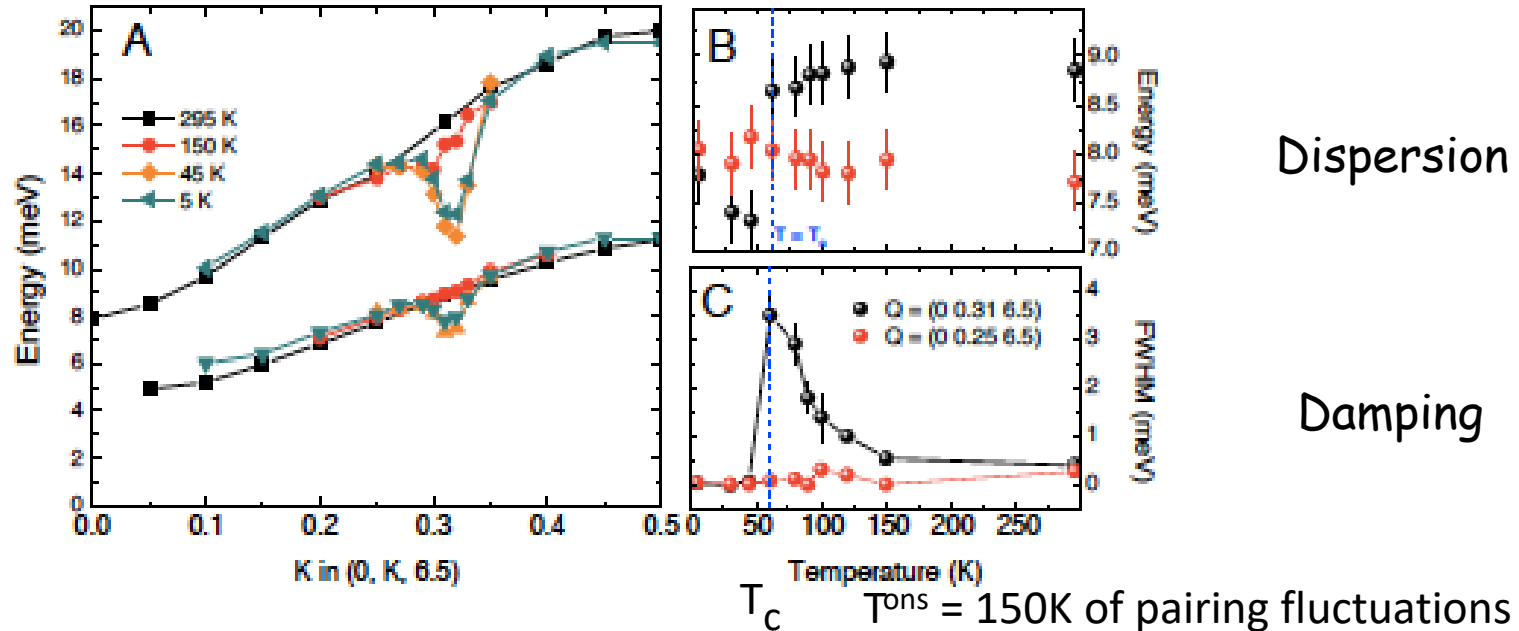
II : d-wave SC gap

Normal state; II : Transition Region

III : Nodal Fermi Arc

Unusual Giant Phonon Anomaly accompanies pairing Fluctuations

IXS Scattering Expts. LeTacon et al 2014



$T^{\text{ons}} > T > T_c$: Strong Damping of Phonons increasing as $T \rightarrow T_c$
 Not a softening as a precursor to a CDW instability
 $T < T_c$ Damping disappears and Dispersion dip appears

Conclusion : GPA is associated with the anomalous pairing fluctuations

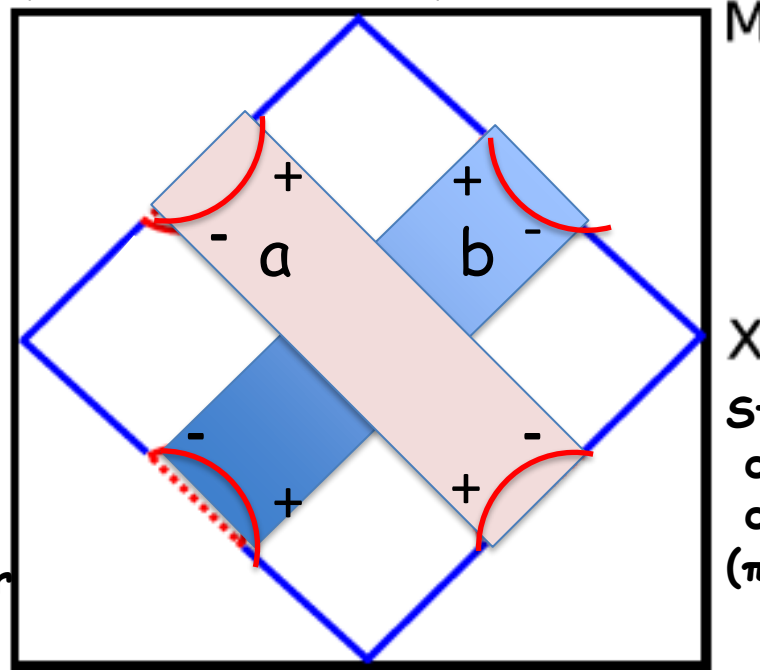
Puzzle : How are Finite Energy & Finite Wavevector Phonons
 coupled to Superconductivity Fluctuations at $(\mathbf{k}, \omega) \approx 0$?

Answer \Rightarrow Y. H. Liu, R. Konik, T.M.Rice & F.C.Zhang

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Fermi Surface Breakup => SC Breakup into 2 sets of Cooper "subbands"?

d-wave symmetry Driver
in overdoped region:
Large Antinodal SC gaps
connected by Strong
Repulsive (π, π) Cooper pair
scattering are absent in
pseudogap region



Strong Intrasubband {a&b}
d-wave Pairing possible
due to Strong Repulsive
 (π, π) Cooper pair scattering

Intersubband Cooper Pair Scattering should be weak due to
cancellation of approximately equal and opposite sign regions

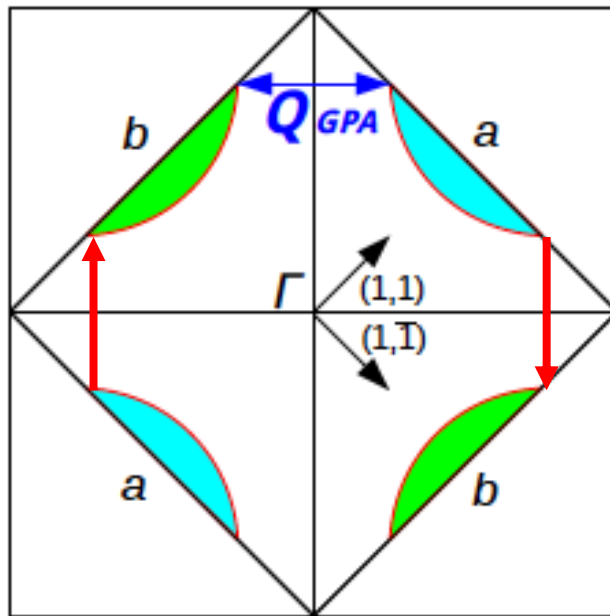
$$\Delta_a^* \Delta_b \sim \sum_{k, k'} V(k, k') \{ \langle c_{k, \sigma}^+ c_{-k, -\sigma}^+ \rangle_{a+} \langle c_{k', \sigma} c_{-k', -\sigma} \rangle_{b+} + (a-, b-) \\ + \langle c_{k, \sigma}^+ c_{-k, -\sigma}^+ \rangle_{a+} \langle c_{k', \sigma} c_{-k', -\sigma} \rangle_{b-} + (a-, b+) \}$$

Intrasubband Pairing Scale \gg Intersubband Pairing Scale
=> Low Energy Leggett Mode due to Intersubband Phase Fluctuations; Leggett ('66)

How do Intersubband Phase Fluctuations couple to Phonons ?

An intersubband **Q-phonon** moves a single QP between subbands.

Cooper Pairs require Absorption & Emission of **Q-phonons**.



GPA : Giant Phonon Anomaly

$T > T_c$ Leggett Mode Propagator

$$L_{\mathbf{q},\omega}^R = \frac{T}{cN_0} \frac{1}{i\omega - \Gamma_{\mathbf{q}}^{(LM)}}$$

$$\Gamma_{\mathbf{q}}^{(LM)} = \tau^{-1} + Dq^2$$

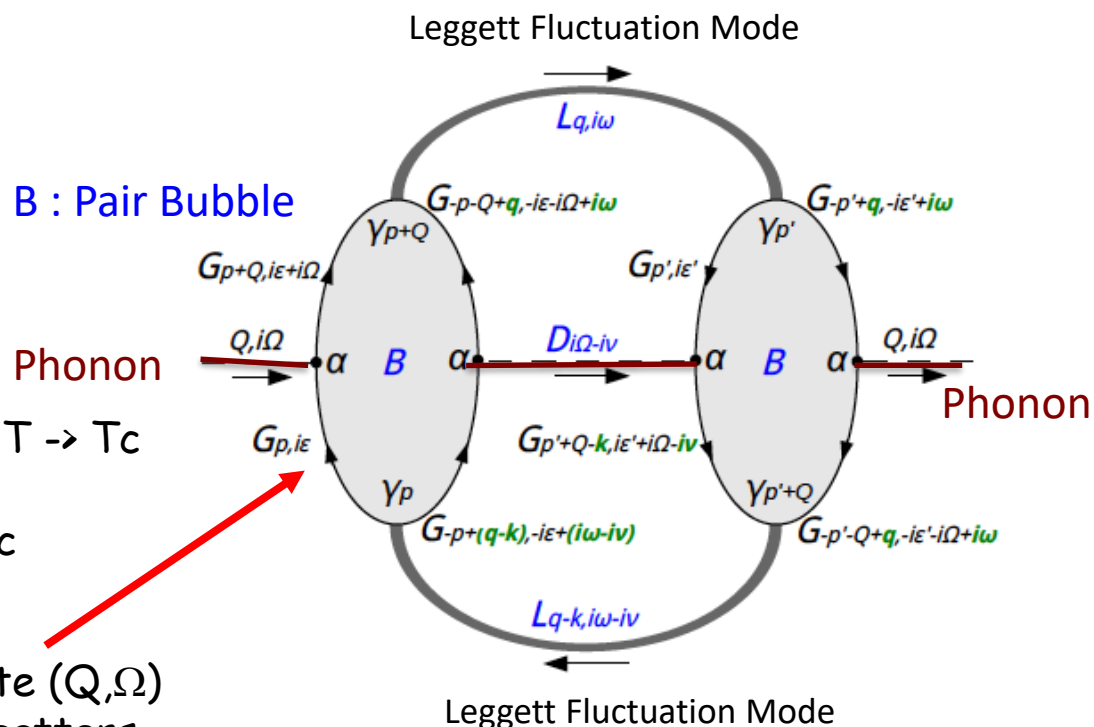
Overdamped Leggett Fluctuations as $T \rightarrow T_c$

Finite Energy Leggett Mode at $T < T_c$

Absorbing a Phonon with finite (Q, Ω) & Emitting a nearby Phonon scatters a Cooper pair from subband $a \rightarrow b$

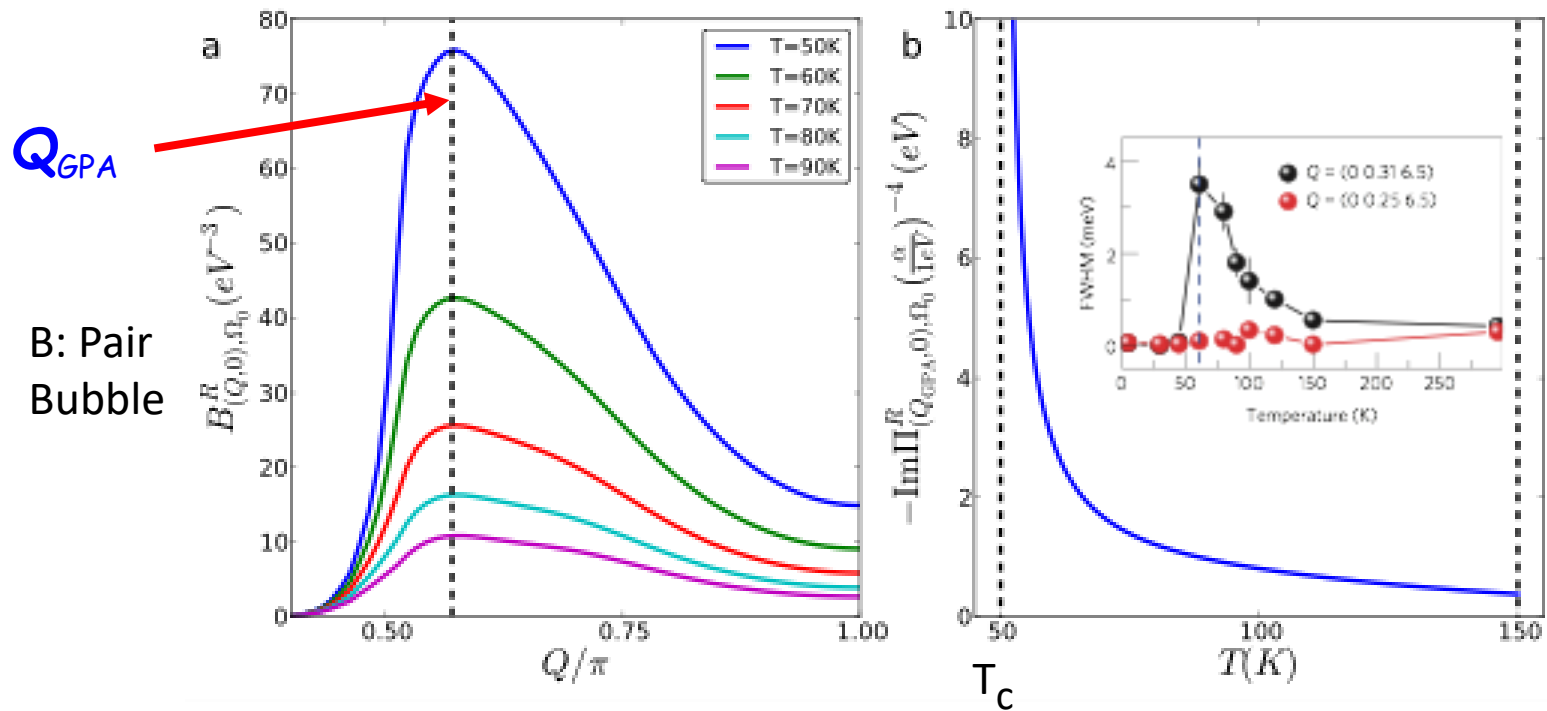
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Phonon Self Energy



Almost Resonant Scattering of
Phonon \rightarrow
Phonon + 2 Overdamped Leggett Modes

$T > T_c$ Strong Phonon Damping as $T \rightarrow T_c$ at $Q \approx Q_{GPA}$



$T < T_c$ GPA Very Weak Phonon Damping at $Q \approx Q_{GPA}$

Finite Energy Leggett Mode \Rightarrow Damping $\rightarrow 0$
 Dip in Phonon Dispersion remains at $T = 0$

$$\begin{aligned} \Pi &\propto \text{Tr} I_{\mathbf{q}, \mathbf{k}, \Omega} \\ &= \text{Tr} \int \frac{d\omega}{2\pi i} \frac{d\nu}{2\pi i} L_{\mathbf{q}, \omega} L_{\mathbf{q}-\mathbf{k}, \omega-\nu} D_{\Omega-\nu} \\ &= \frac{4\Delta^4}{N_0^2} \frac{1}{\omega_{\mathbf{q}} \omega_{\mathbf{q}-\mathbf{k}}} \frac{2(\Omega_0 + \omega_{\mathbf{q}} + \omega_{\mathbf{q}-\mathbf{k}})}{\Omega^2 - (\Omega_0 + \omega_{\mathbf{q}} + \omega_{\mathbf{q}-\mathbf{k}})^2 + i\delta} \end{aligned}$$

Conclusions

- PS gap : Maximising U-scattering leads to Short Range Order due to conflicting AF & d-wave Pairing Correlations => YRZ Scenario
- Breakup of the Fermi Surface leads to 2-band Superconductivity with a soft Leggett mode and SC fluctuations over a large T interval $> T_c$ in the Pseudogap phase
- Low Energy Intersubband Phase Fluctuations couple to Phonons and lead to Strong Phonon Damping at $T > T_c$ but damping vanishes at $T < T_c$ replaced by Dip in Phonon Dispersion
- Intrinsic Connection between Anomalous Pseudogap Phase, Enhanced Superconducting Fluctuations and Giant Phonon Anomalies
- No Static Long Range Charge Order - also not in NMR Wu et al Nature Commun. 2015

Y. H. Liu, R. Konik. T.M.Rice & F.C.Zhang Nature Commun. **7** 10378 (2016)