Congratulations to KITS

Interaction Driven Topological Phases in Two-Dimensional Systems

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Research is supported by NSF (magnetic systems) and DOE (topological matter)

Research collaborations

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Tiansheng Zeng (CSUN), Wenjun Hu (RU)

Y. C. He and Yan Chen, Balents on CSL;
Liang Fu on Spontaneous QHE

Duncan Haldane, F. Becca
Kun Yang, O. Starykh

Outline

I. Introduction: Quantum spin liquid (SL) and topological order;

Time reversal symmetry (TRS) spontaneously breaking Chiral spin liquid (CSL) proposed 30 years ago ---no known examples until our recent work

II. Establish robust CSL in Kagome SU2 model (J1-J2-J3) small perturbation J2-J3, ~0.07)

comparing to other spin models (triangular etc)

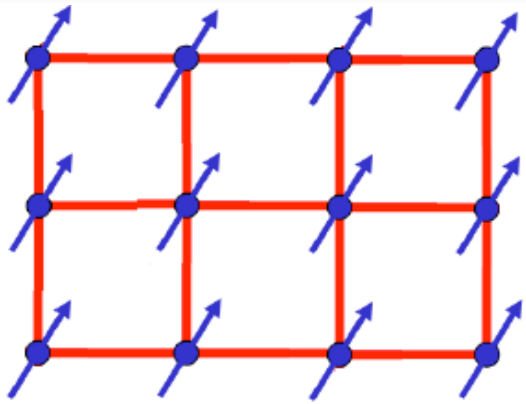
VMC simulations, indication of Dirac to CSL transition

III. Frustration in Electron Systems:

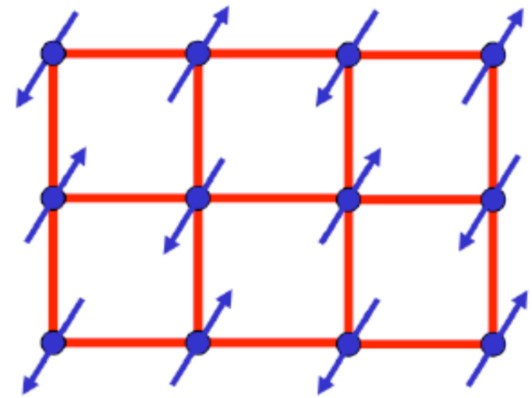
Spontaneous Quantum Hall Effect

Magnetic systems can develop orders by breaking symmetry

$J<0$

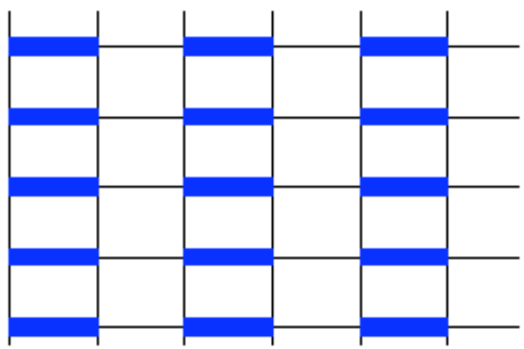


Ferromagnetic order

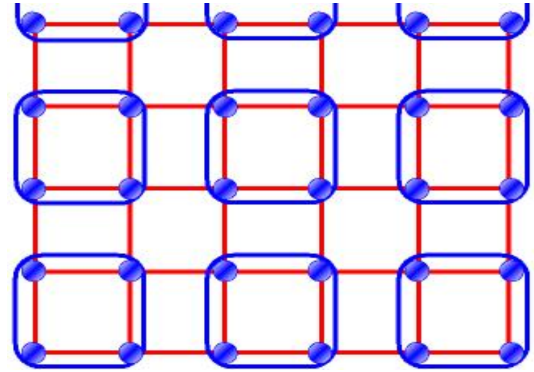


$J>0$

Antiferromagnetic order with NN J1

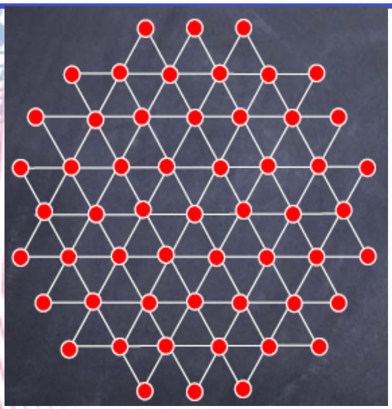


Valence Bond Solid (VBS) order

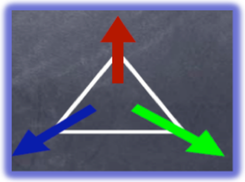


Plaquette VBS (possibly identified in J1-J2 square model)

Spin disordered phase in Heisenberg models with increasing frustration



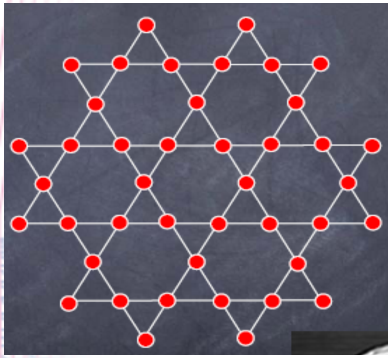
Triangular lattice



Three-sublattice AF order

Resonating valence bond (P. W. Anderson), RVB is a quantum spin liquid (SL)1973, 1989

NN Kagome lattice: A primary candidate for Z2 SL

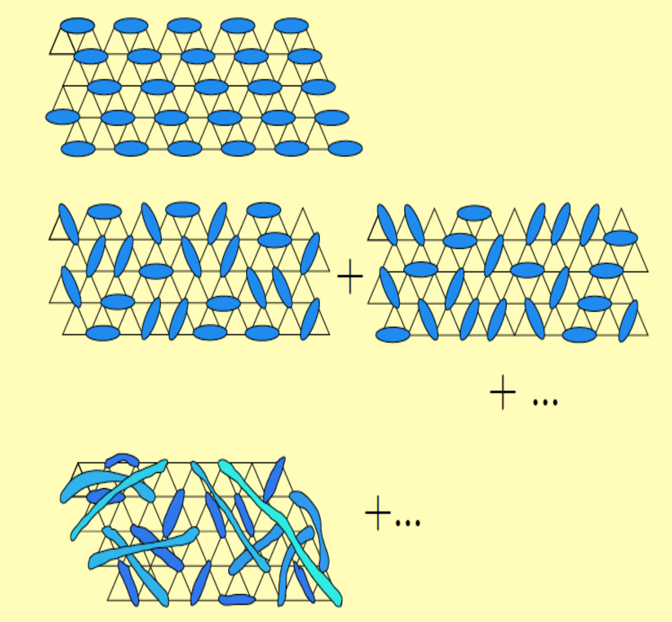


Spin Liquid
H.C. Jiang, Z.Y. Weng, DNS (2008), not converged DMRG for wider systems
Yan, Huse, White, Science (2010)---new milestone: A possible gapped spin liquid

Jiang, Wang, Balents (2012)
Depenbrock et al. (2012).

Spin Liquid (SL) State

A new state of matter with no symmetry broken, with topological order /entanglement, and fractionalized excitations X. G. Wen (1990, 1991)



Z2 SL in contrived theoretical models
Wen (1990,1991), Kivelson, Rokhsar, Sethna (1987), Senthil, Fisher (2000), Balents, Fisher, Girvin (2002)
Moessner and Sondhi, (2001), Senthil, Motrunich (2002) Senthil, Vishwanath, Balents, Sachdev, Fisher (2004)
PA Lee (2008), Balents (2010) ………

Gapped SL State

Gaps to all spin excitations

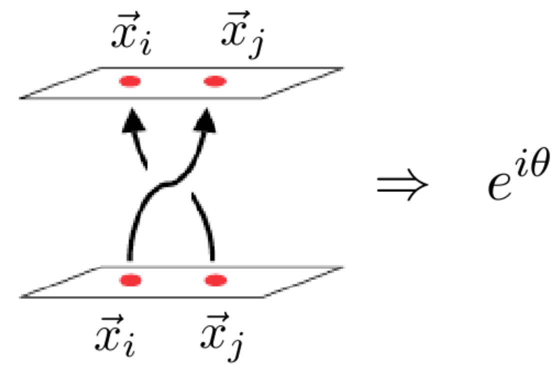
exponential decay correlations

J1-J2-J3 (Ising) kagome, Balents et al and J1 kagome model

Time reversal symmetry (TRS) broken chiral Spin liquid (CSL): a bosonic FQHE state

Kalmeyer and Laughlin 1987
Wen, Wilczek, Zee 1989

anyon quasiparticles obey fractional statistics



(bosonic)FQHE without B field

Induce CSL using (S1xS2)*S3 interaction

Haldane and Arovas 1995

Yang, Warman and Girvin1993.

Yao and Kivelson: a contrived CSL state (for Kitaev model) 2007

Schroeter et al 2007, Thomale et a. 2009, Nielsen et al. 2012, Bauer et al. 2013-2014 (Mott materials)

Approx. Methods find CSL
Hermele et al (2009), Messio et al. For kagome J1-J2-J3 (2012) Cuboc

Experimental Candidates

Kagome magnets

Bilayer magnets

Triangular organic compounds

Rare-earth spin-orbit coupling inorganic compounds

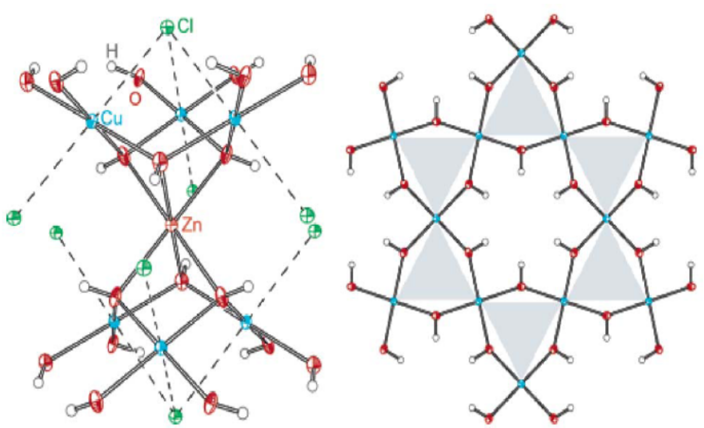
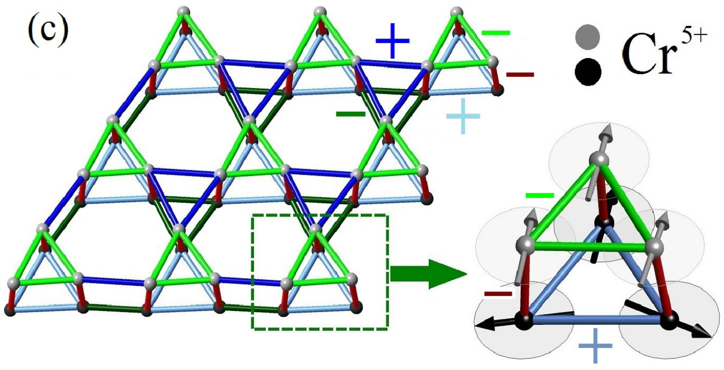
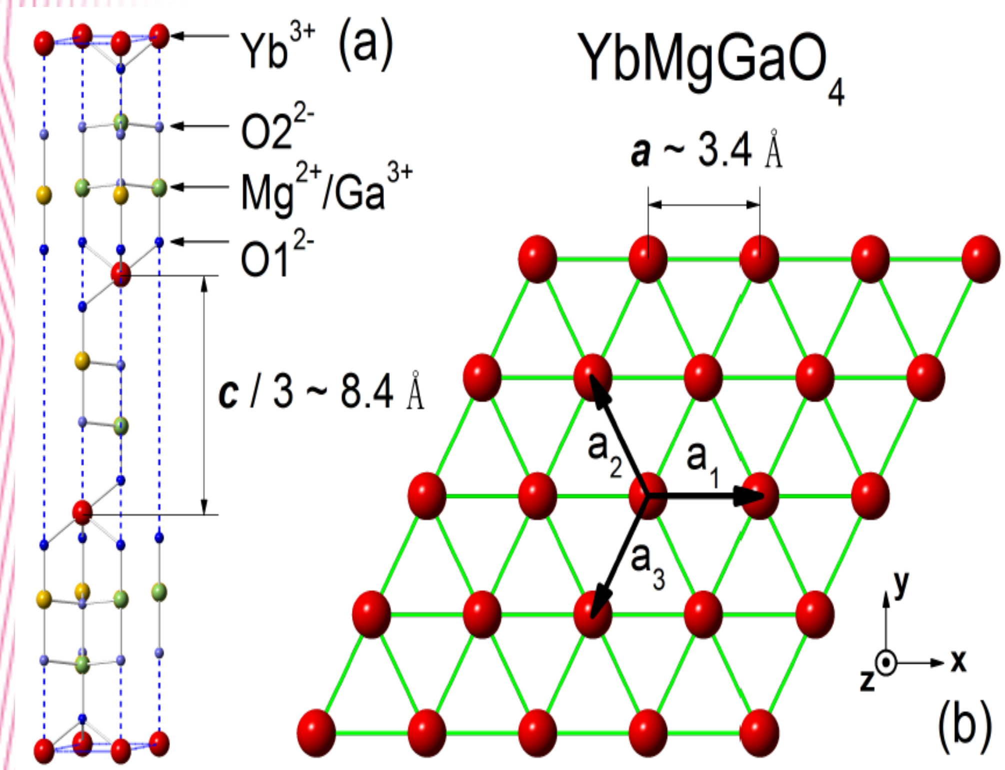


Figure 1. Crystal structure of Zn−paratacamite (**1**), $Zn_{0.33}Cu_{3.67}(OH)_6Cl_2$.

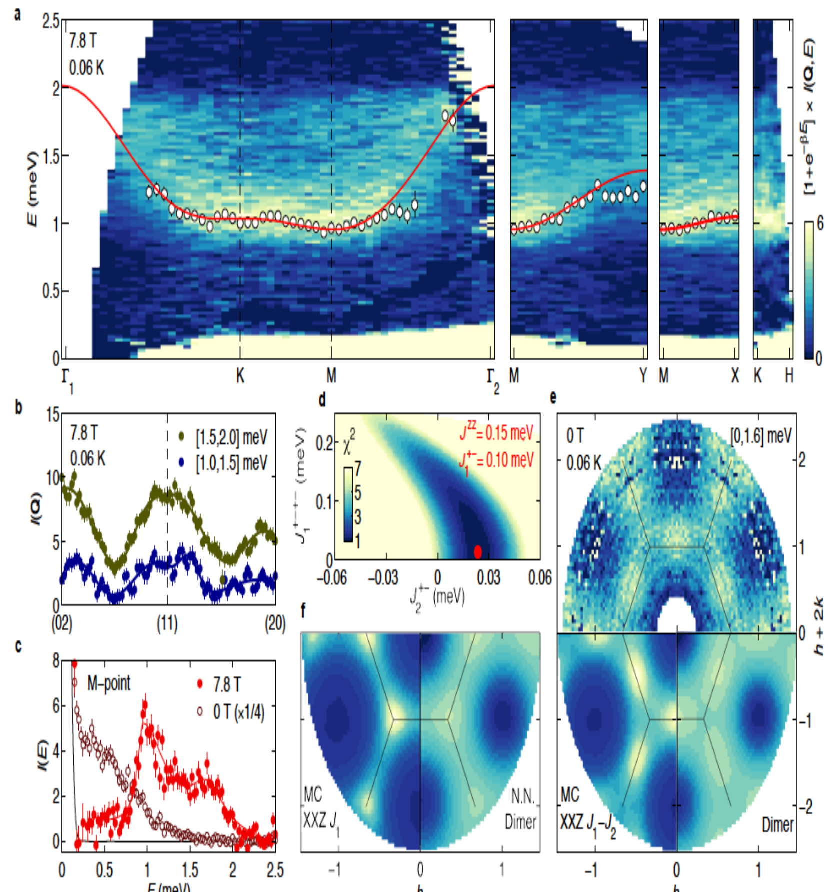


Rare-earth spin-orbit coupling triangular inorganic compounds

Y. Li et al 2015, PRL



Paddison et al 2016



J1-J2 Heisenberg models with spin nonconserved terms (S+S+)

Numerical Studies

What phases are realized in realistic simple spin models with strong frustration ?

--- density matrix renormalization group (DMRG) numerical studies

DMRG convergence is controlled by kept states (m)
m=6000--30000 (U1 equivalent) states
---utilizing SU2 symmetry (Gong et al.)

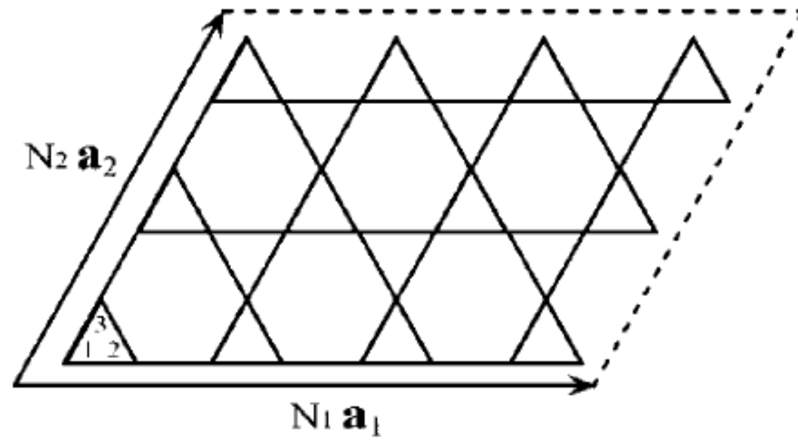
S. R. White 1992

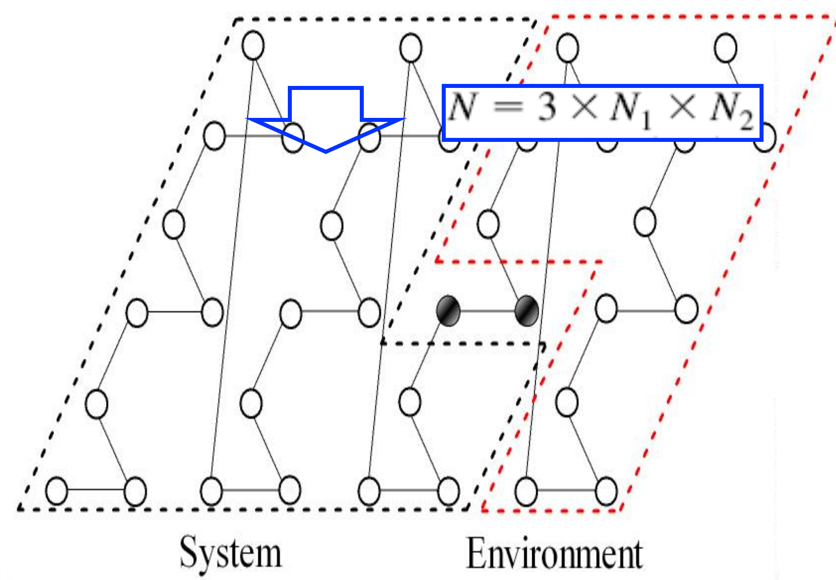
1D DMRG algorithm

Mapping a 2D lattice onto a 1D

Jiang, ZY Weng, DNS(2008)

DMRG with Su2 invariant
Depenbrock et al. 2012
Shoushu Gong 2012—2014
~30,000 states

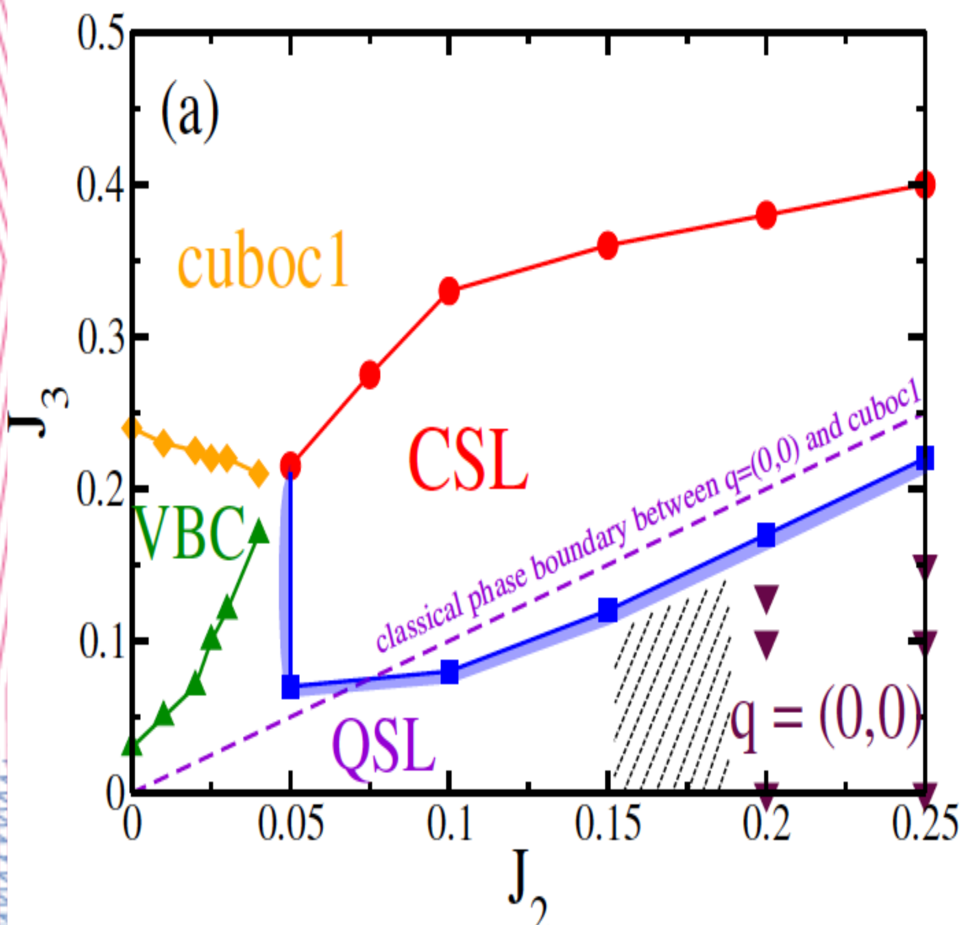


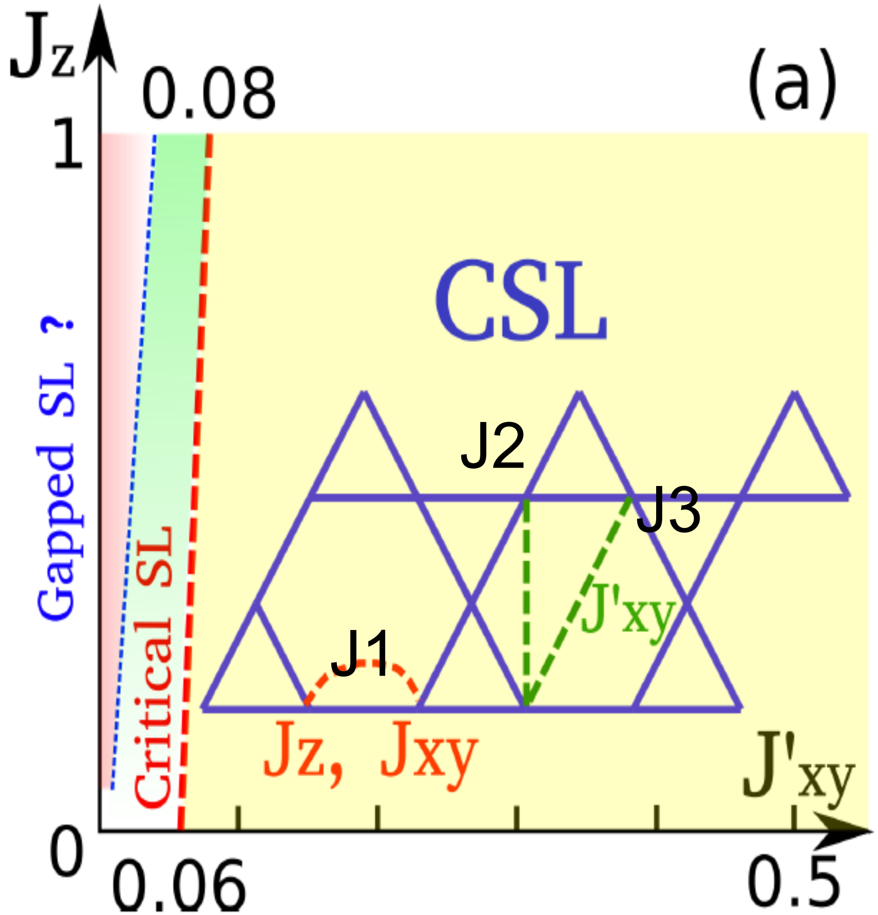


Discovering of Chiral Spin Liquid (CSL) in kagome spin ½ models

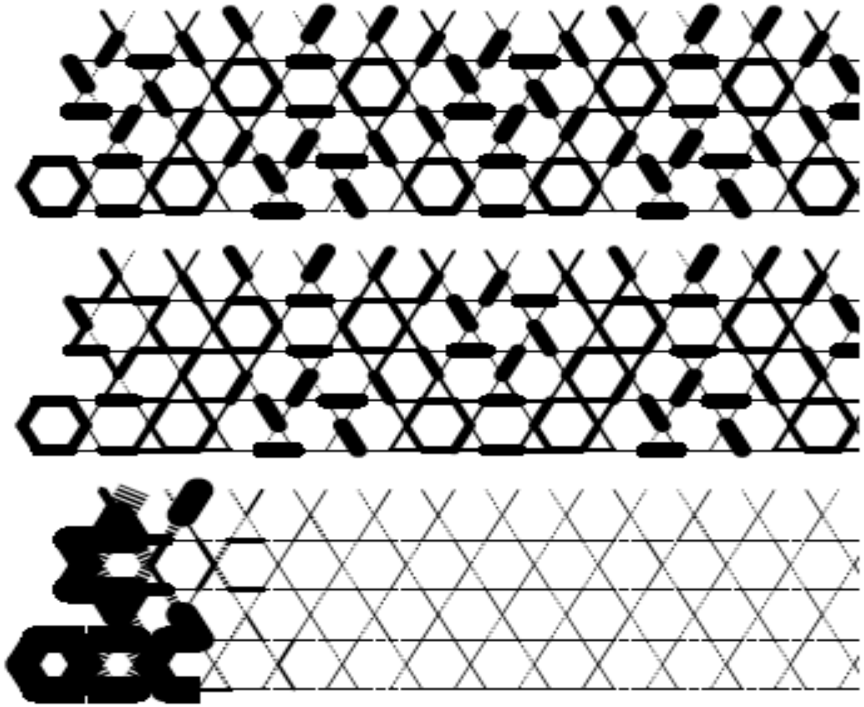
J1-J2-J3 Heisenberg model

$$H = J_1 \sum_{\langle i,j \rangle} S_i \cdot S_j + J_2 \sum_{\langle\langle i,j \rangle\rangle} S_i \cdot S_j + J_3 \sum_{\langle\langle\langle i,j \rangle\rangle\rangle} S_i \cdot S_j$$

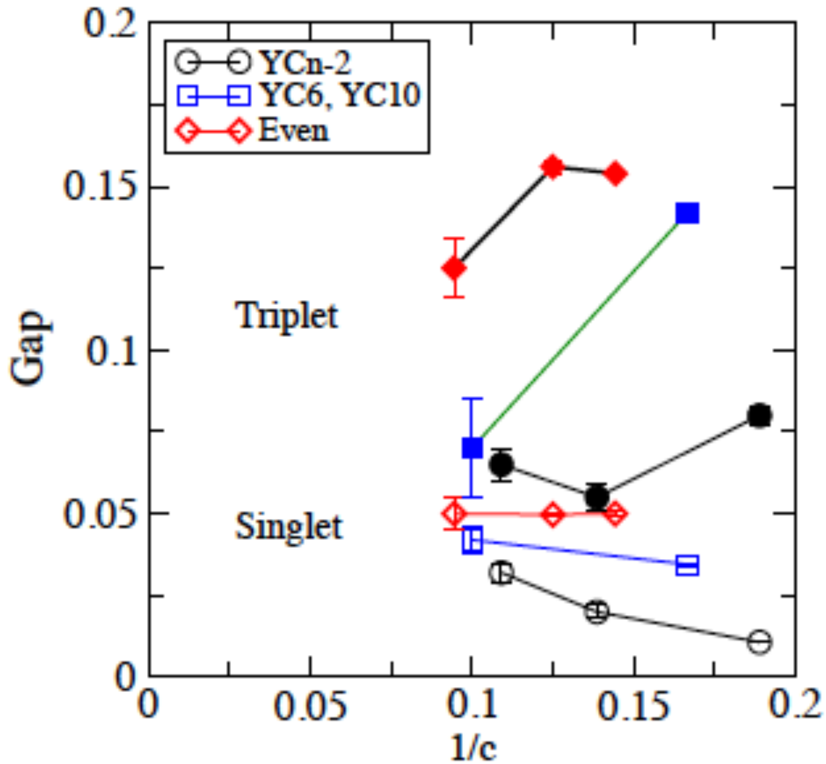




Main evidence for possible SL for J1 kagome spin1/2 (DMRG, Yan et al)

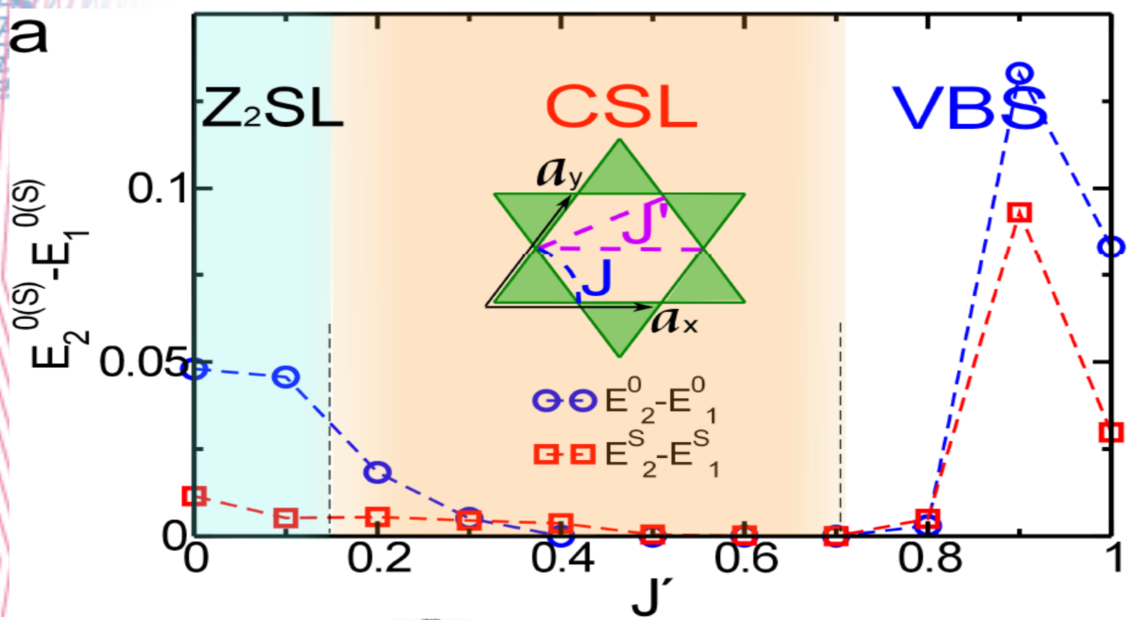


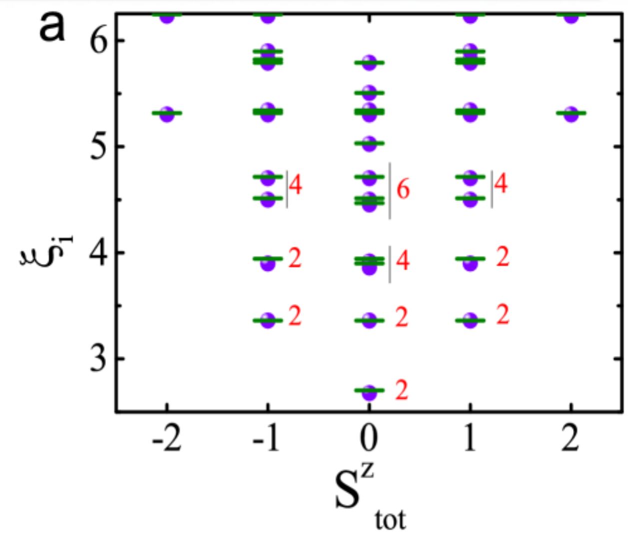
Evolving to uniform SL from VBS on kagome

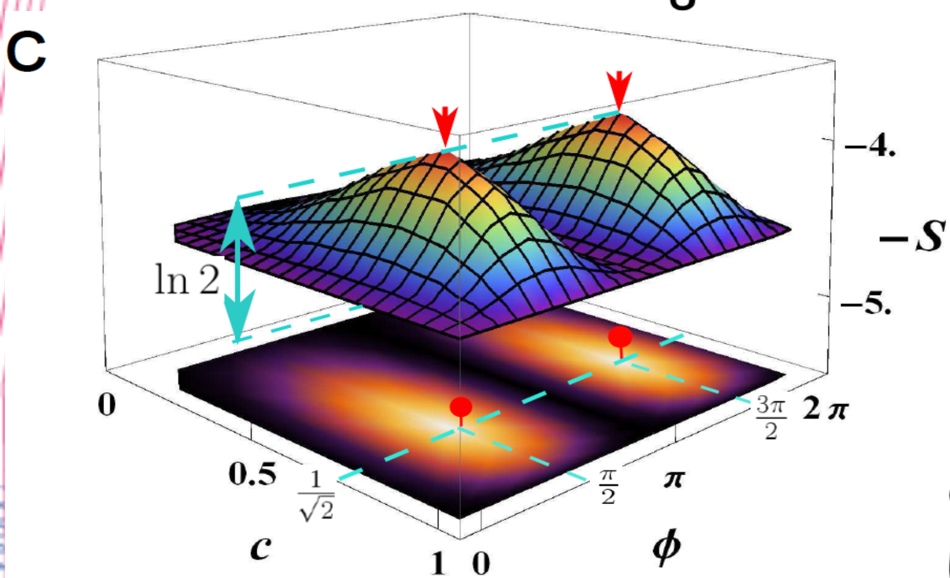


Yan, Huse, White, 2010 found two topological sectors in DMRG; Challenge to find four sectors to fully establish topological nature of the phase

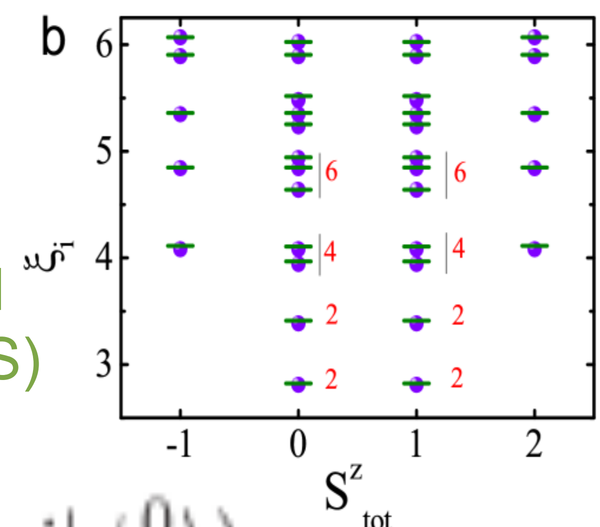
Four fold ground state degeneracy and entanglement spectra





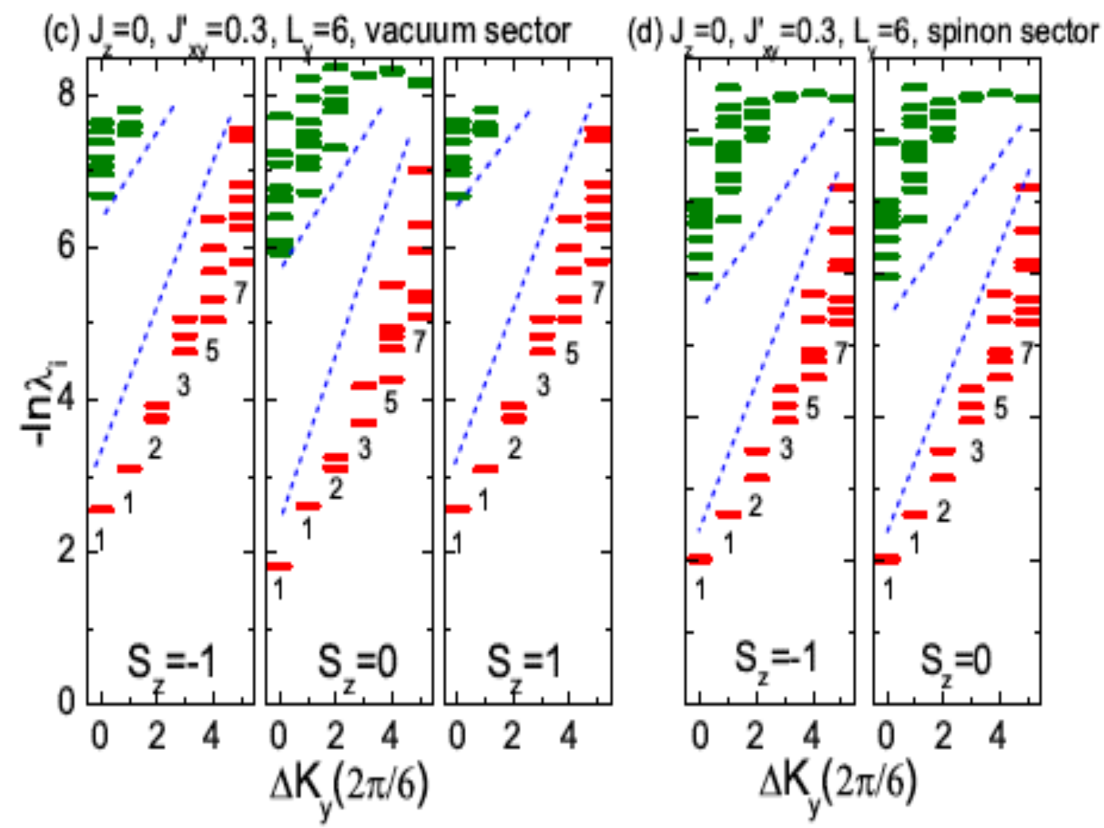


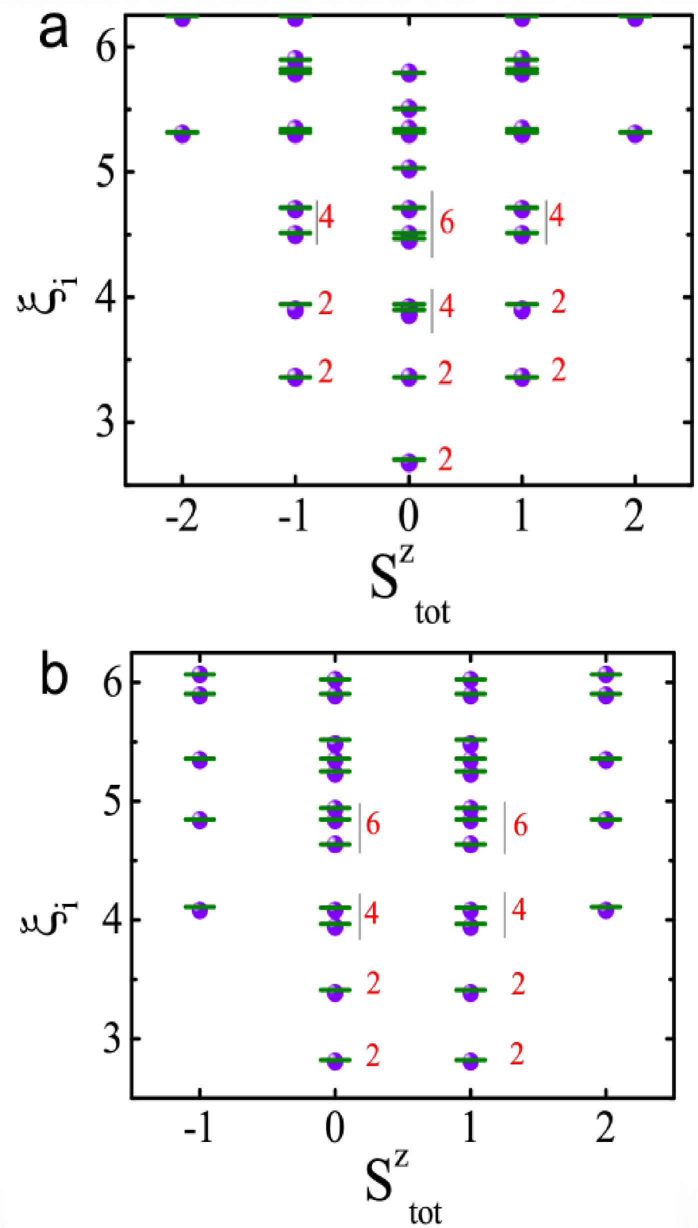
Minimum Entangled State (MES)



$$\frac{1}{\sqrt{2}}(|\psi_1^0\rangle \pm i|\psi_2^0\rangle).$$

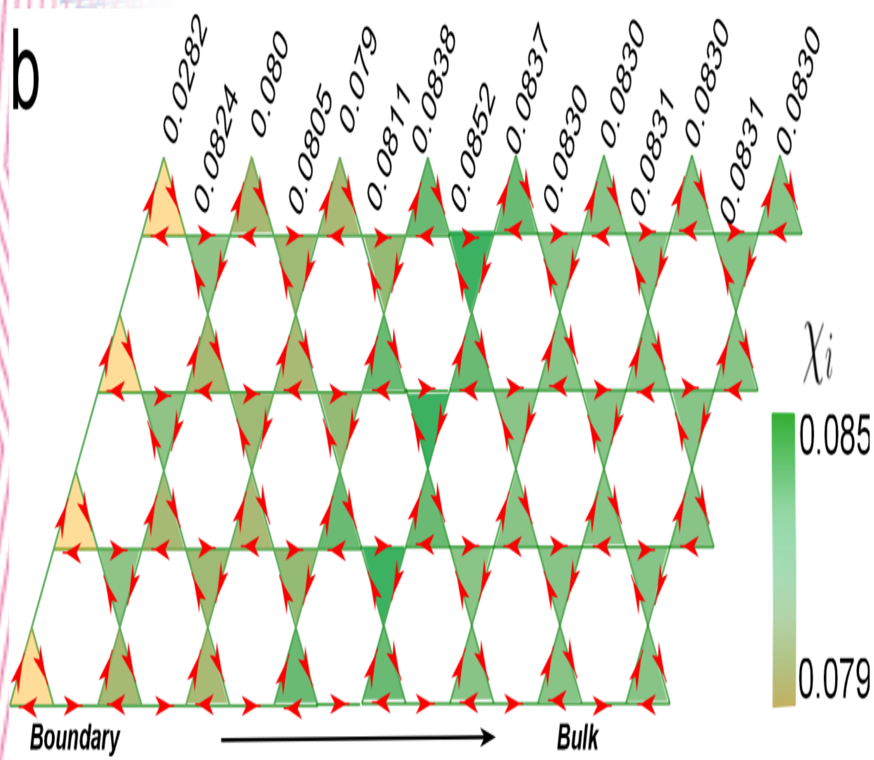
Entanglement spectra of CSL: emerging Laughlin ν=1/2 FQHE

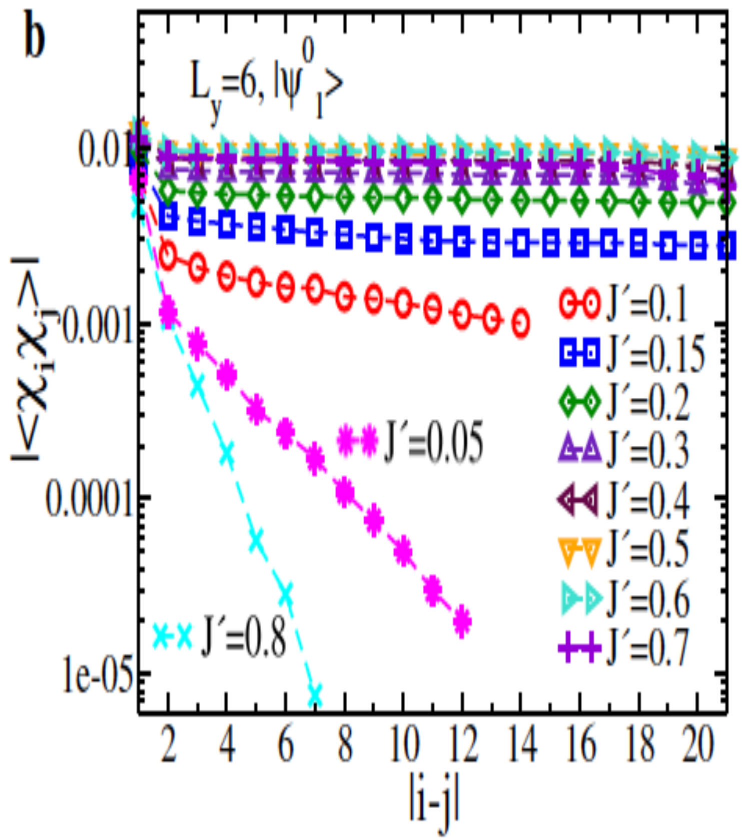




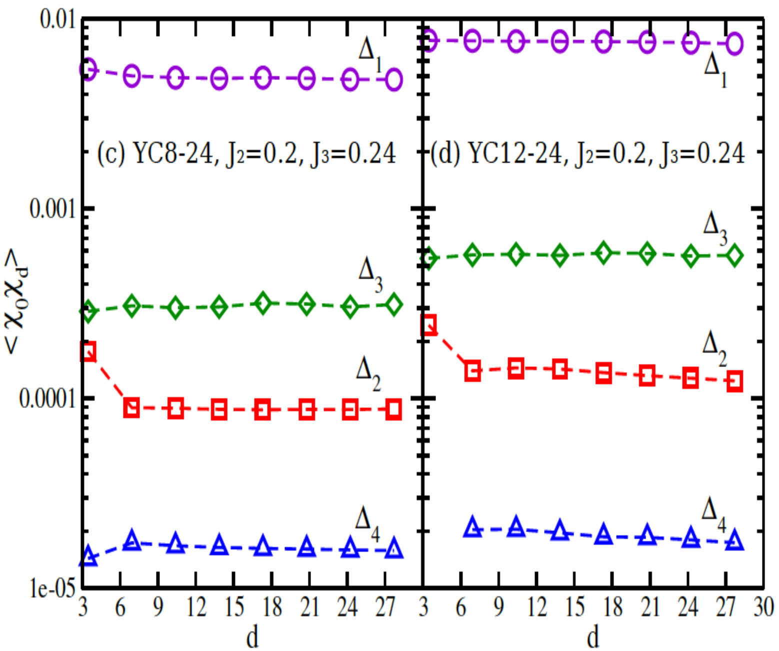
Inserting flux turns the ground state from vacuum to spinon sector

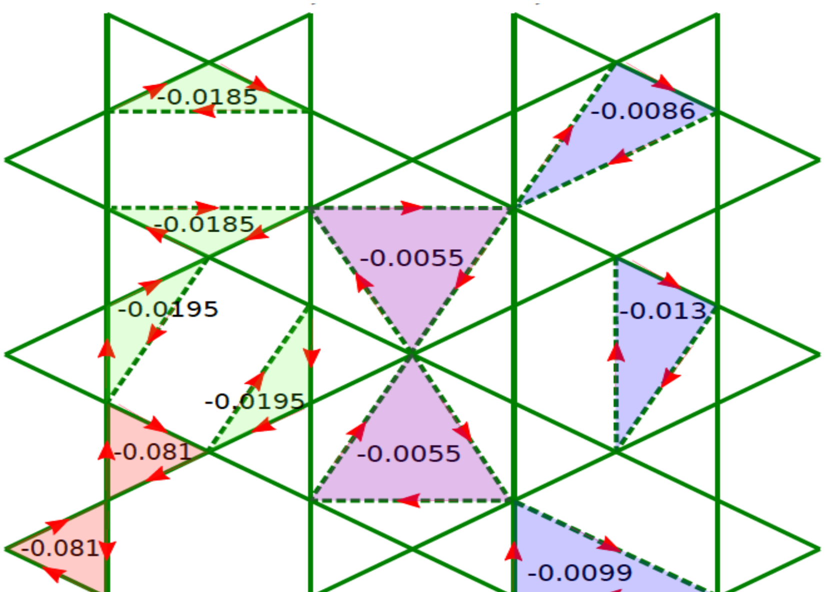
Chiral SL with nonzero spin chirality Xi=(S1xS2)*S3

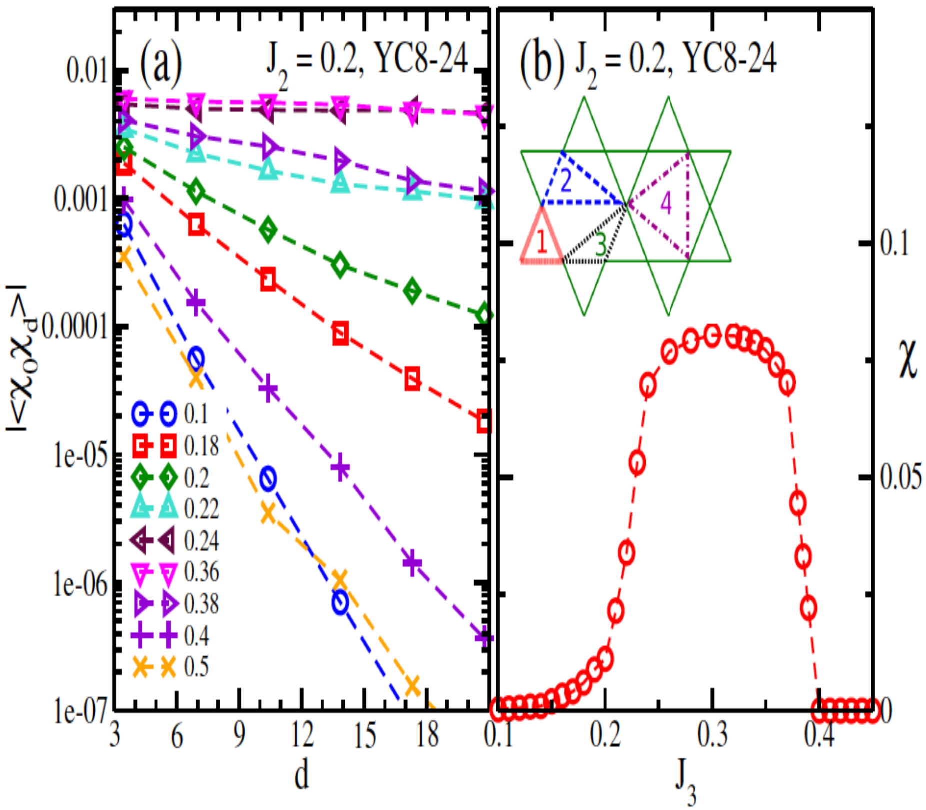




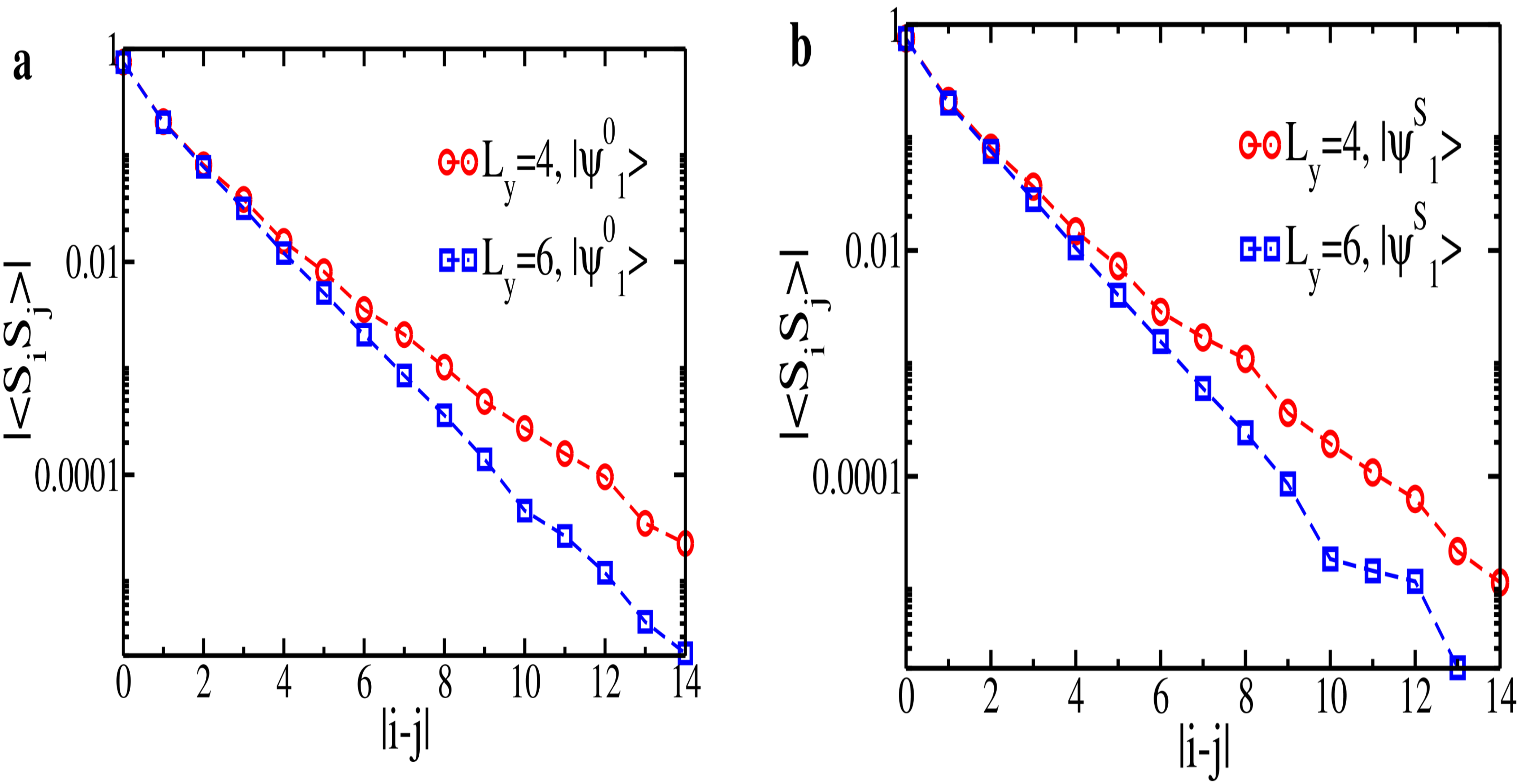
Chiral SL with nonzero spin chirality Xi=(S1xS2)*S3





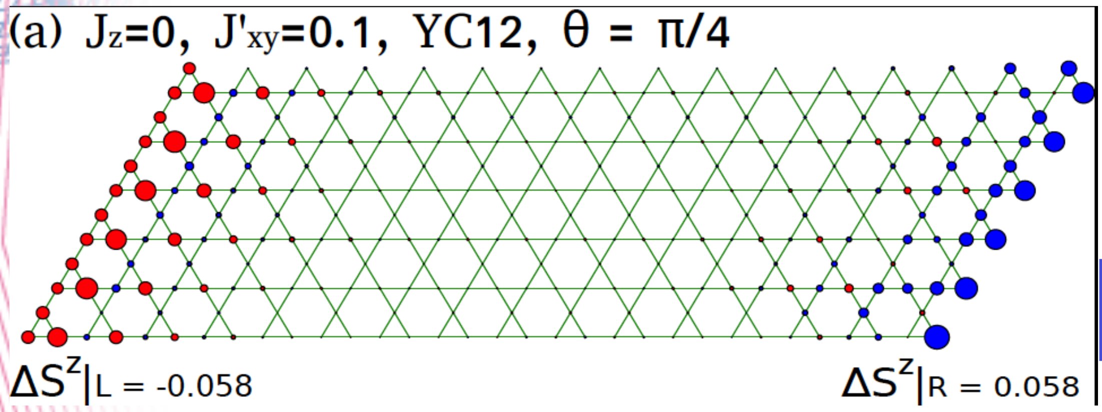


Exponential decay of the spin correlations—J'=0.5



Inserting flux adiabatically, spin pump, and C=1/2 quantization

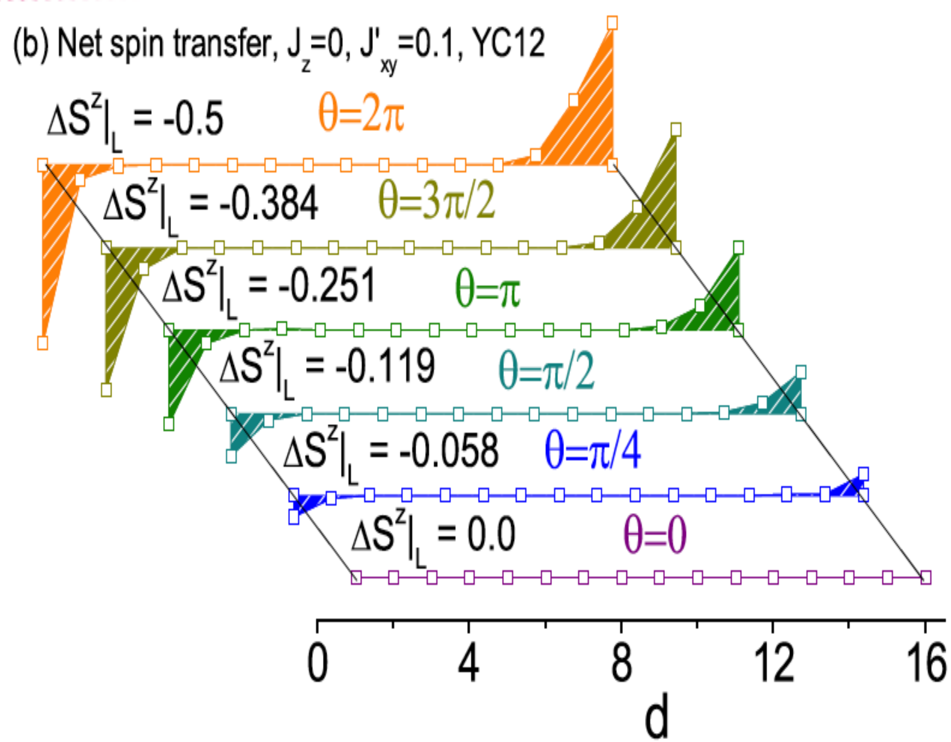
Signature of nu=1/2 Laughlin state and CSL



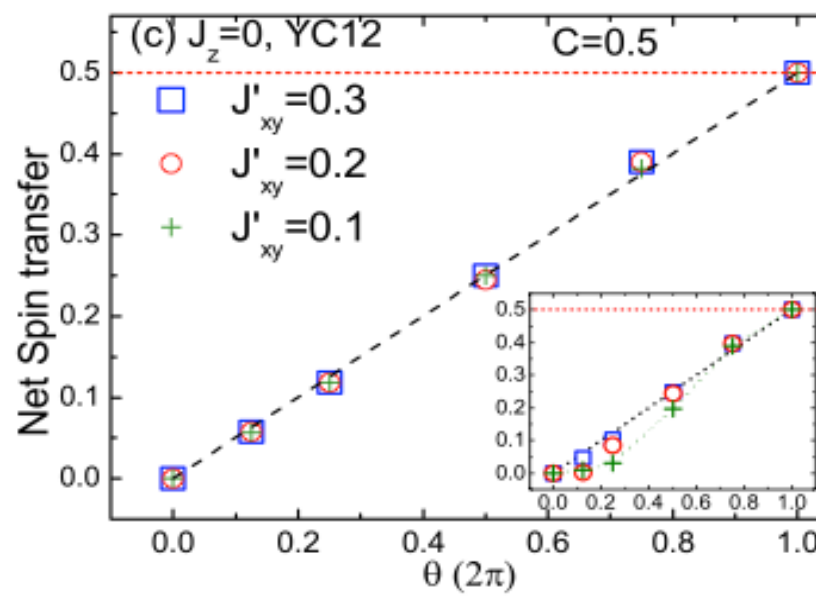
Flux=twist boundary

Phase $S^+_l S^-_j \exp(i\theta)$

Haldane and Arovas 1995



Ly=12



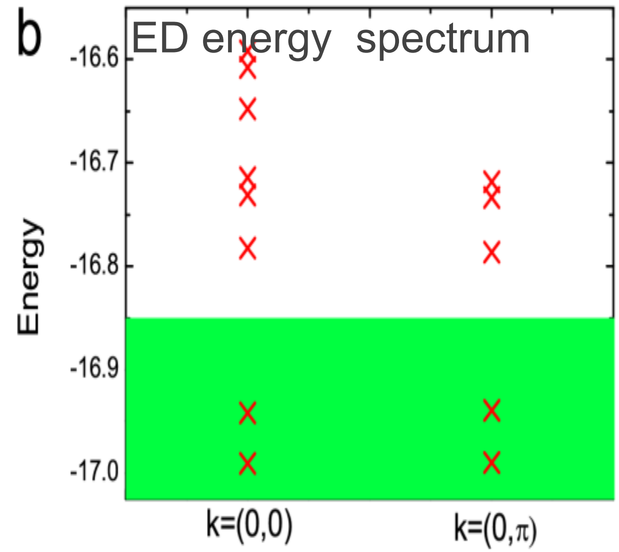
Entanglement spectrum and modular matrix: matching to 1/2 FQHE (model independent)

DMRG "torus" modular matrix

$$S = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} + \frac{10^{-2}}{\sqrt{2}}\begin{pmatrix} -0.42 & -2.2 \\ -1.26 & 0.76-0.15i \end{pmatrix}$$

and

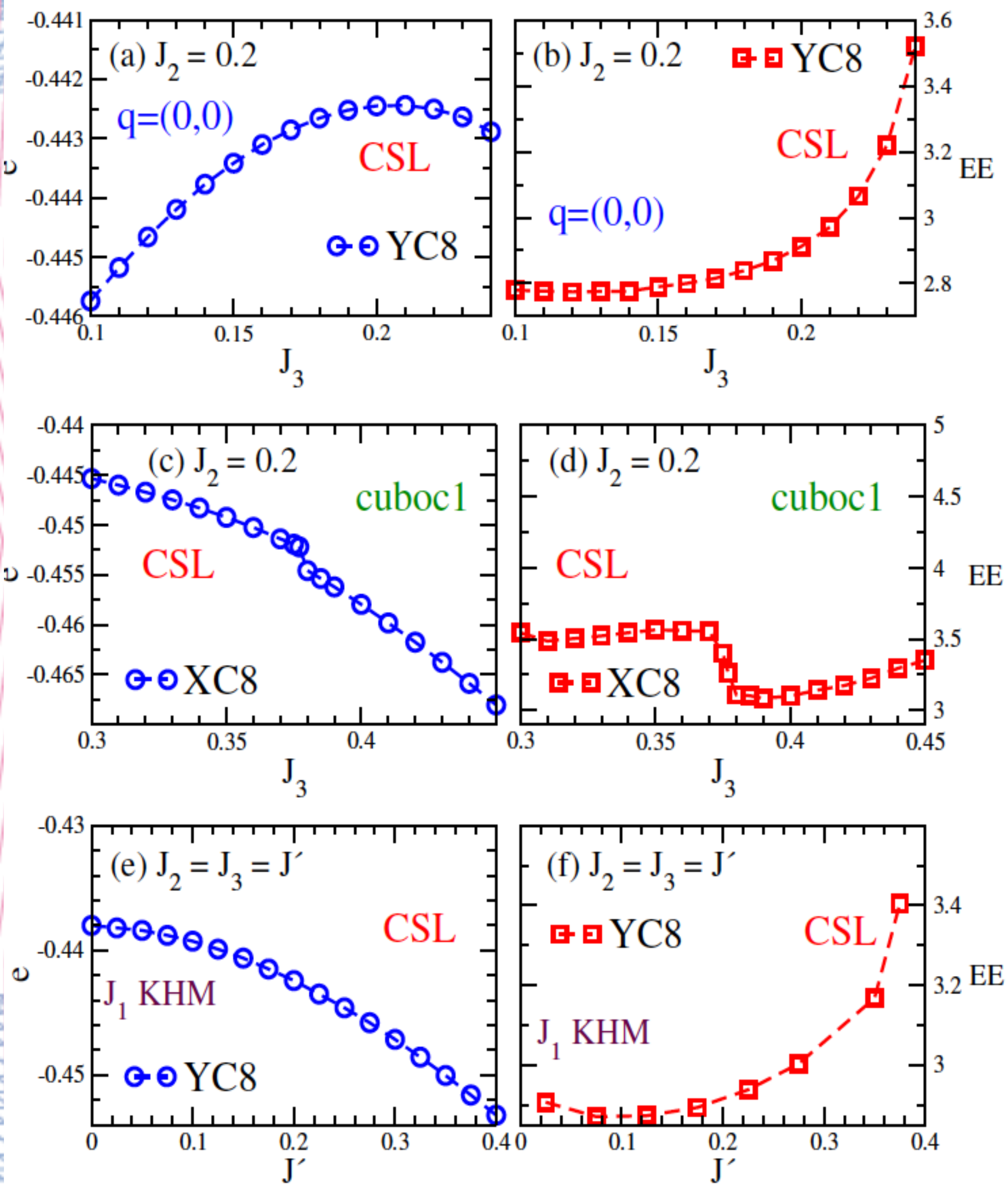
$$U = e^{-i(2\pi/24)}\begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix} \times \begin{pmatrix} e^{0.011i} & 0 \\ 0 & e^{-0.006i} \end{pmatrix}$$

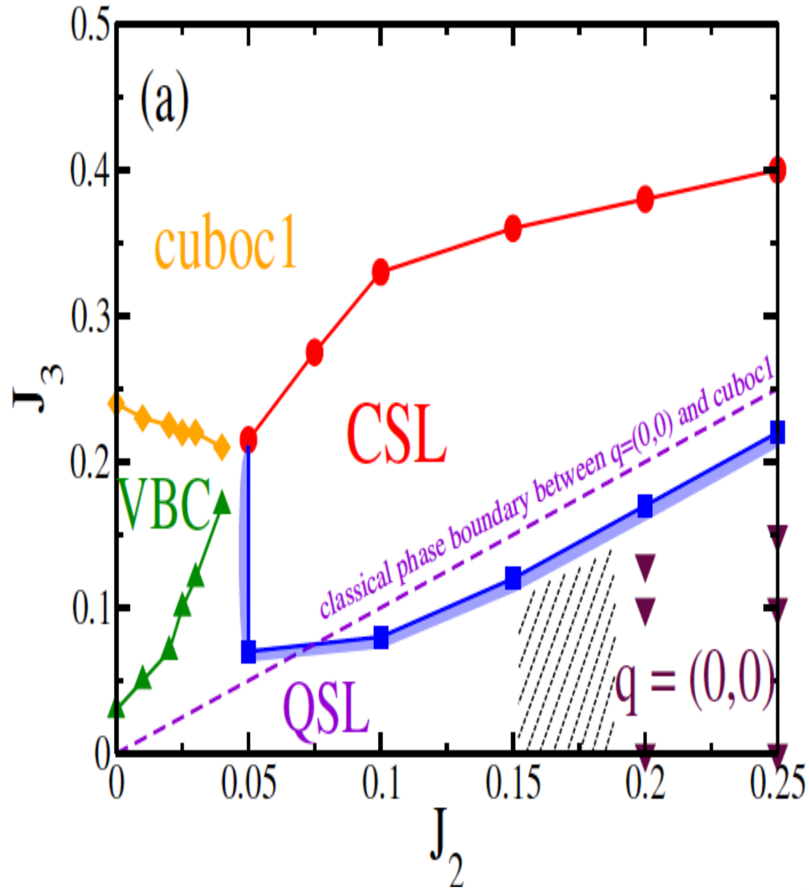


Semion with topological spin h=1/4

Wen 1990; Li, Haldane 2008
Y. Zhang et al 2012
Cincio &Vidal2013
Zhu, Gong,Haldane, DNS 2013-2014

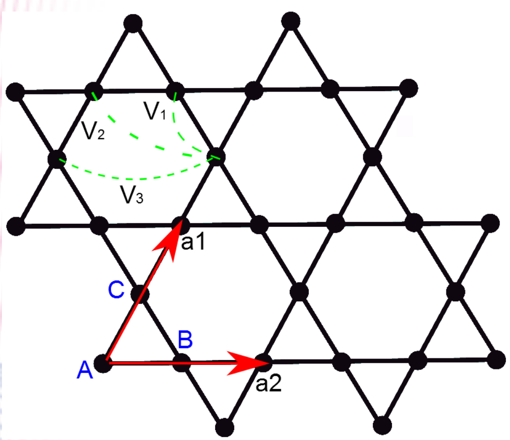
Quantum phase transitions

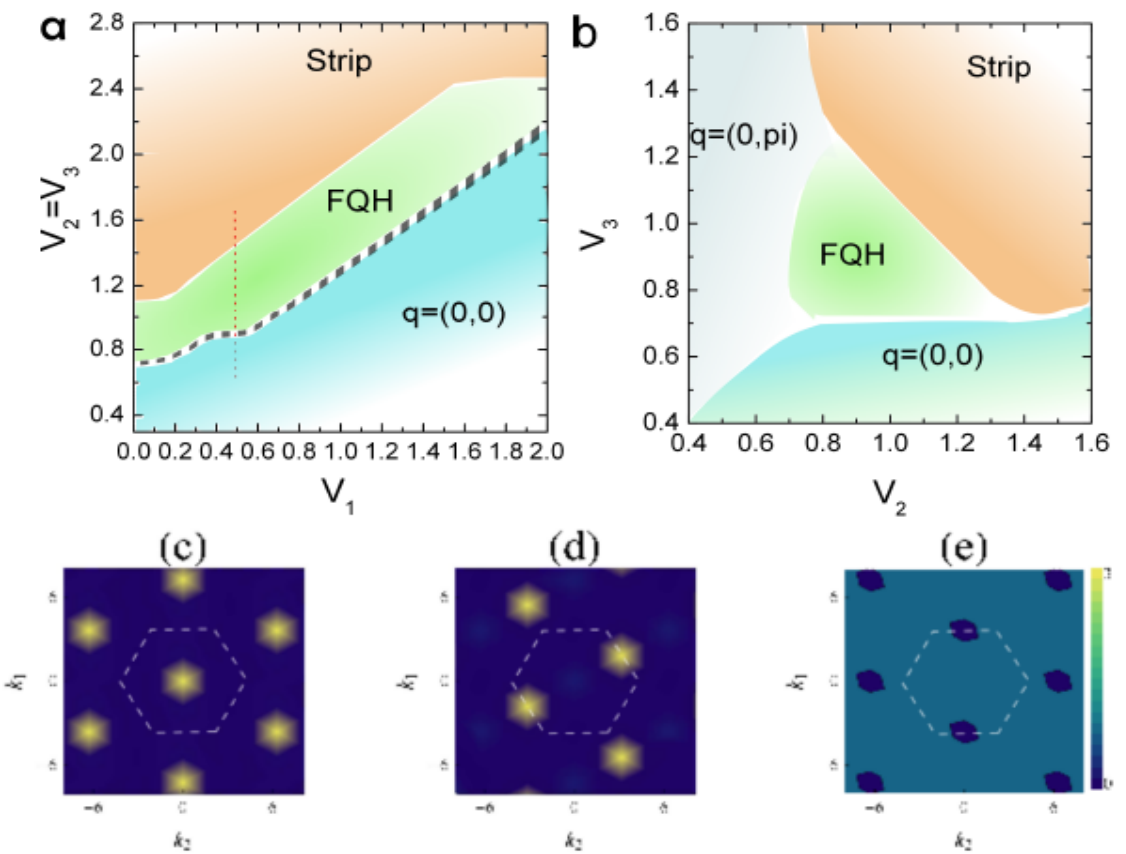




Boson at filling 1/3 ---- CSL (FQH)

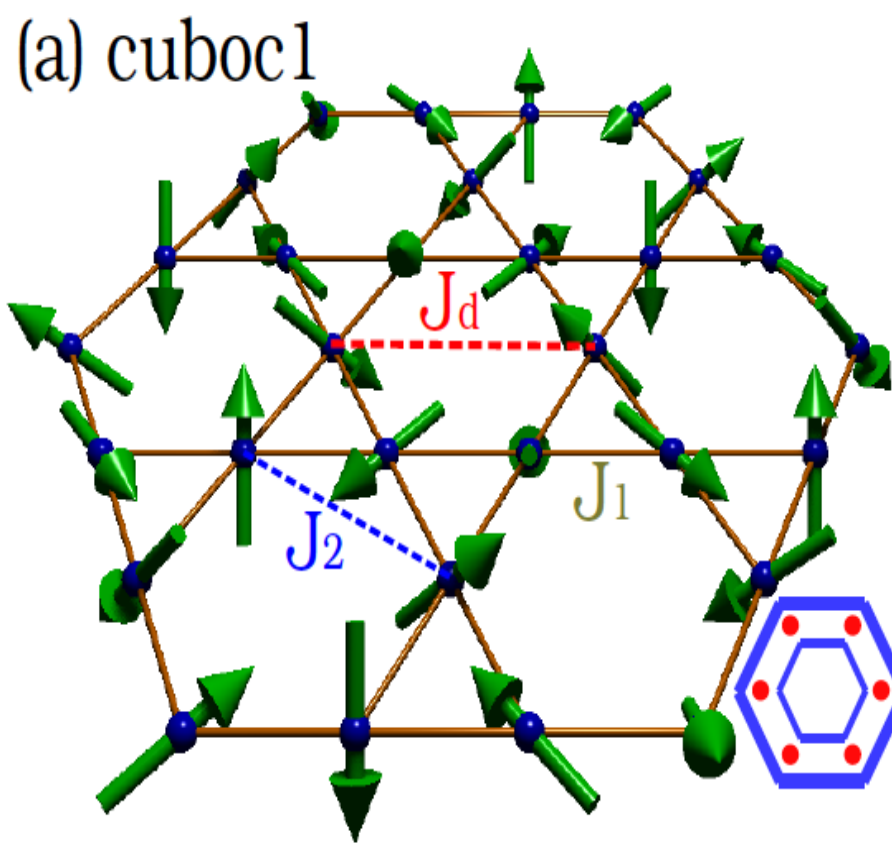
$$H = t\sum_{\langle \mathbf{r}\mathbf{r}'\rangle}\left[b^\dagger_{\mathbf{r}'}b_{\mathbf{r}} + \mathrm{H.c.}\right] + V_1\sum_{\langle \mathbf{r}\mathbf{r}'\rangle} n_{\mathbf{r}}n_{\mathbf{r}'} + V_2\sum_{\langle\langle \mathbf{r}\mathbf{r}'\rangle\rangle} n_{\mathbf{r}}n_{\mathbf{r}'} + V_3\sum_{\langle\langle\langle \mathbf{r}\mathbf{r}'\rangle\rangle\rangle} n_{\mathbf{r}}n_{\mathbf{r}'}$$

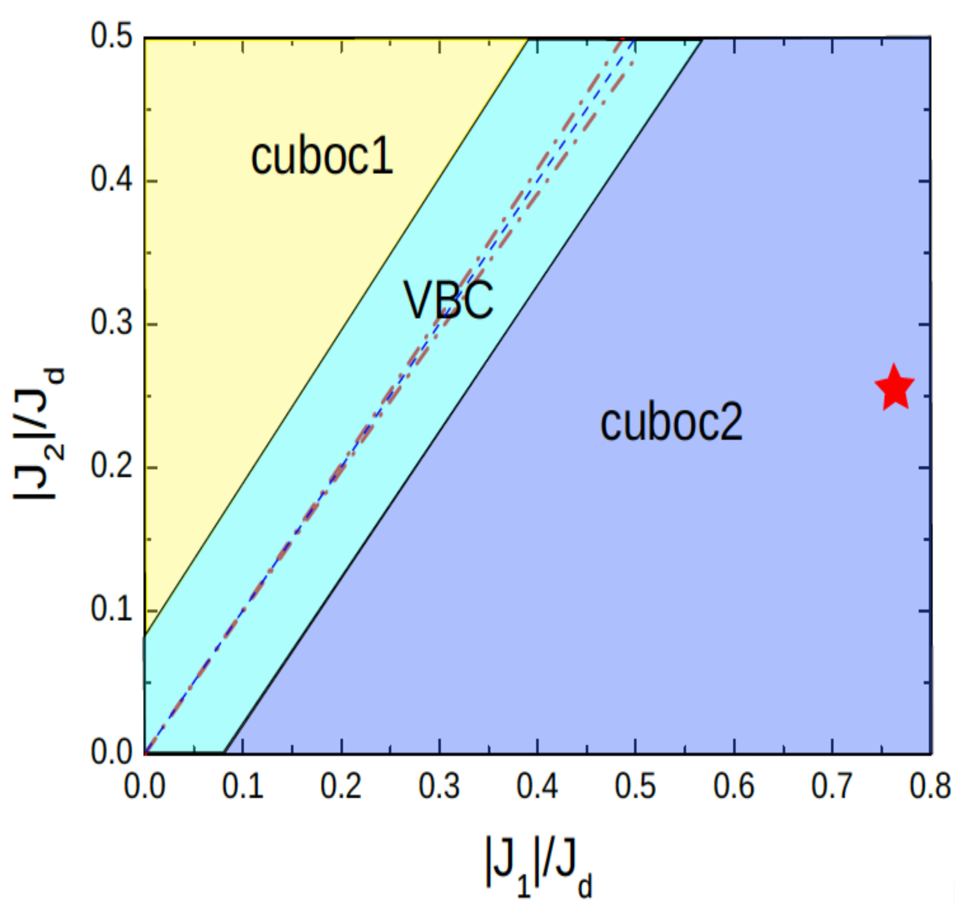




Boson at filling 1/3: W Zhu, et. al, PRB 2016

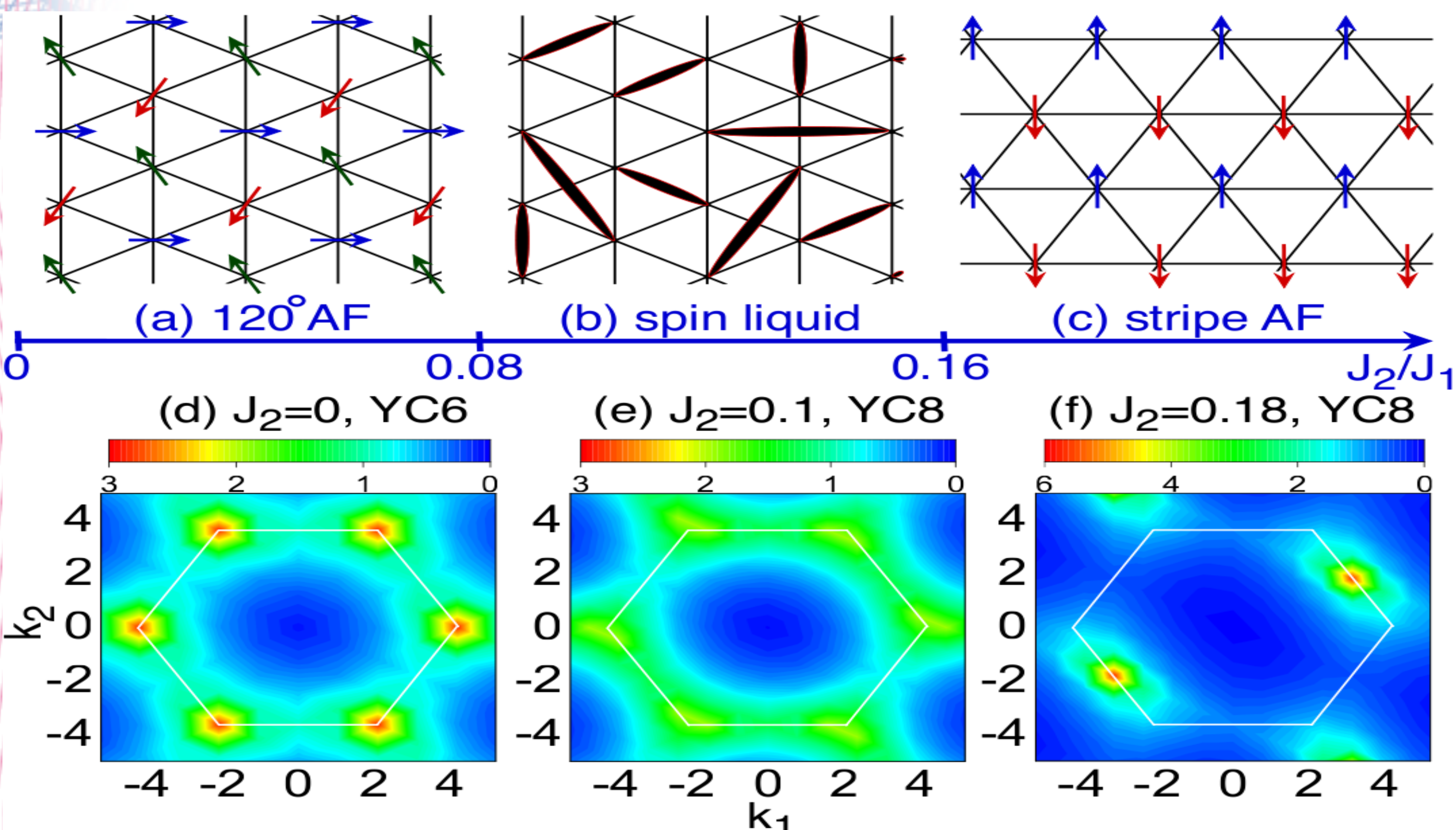
No SL nearby two cuboc phases (related to Kapellasite), disagree with slave-particle





Gong et al 2016

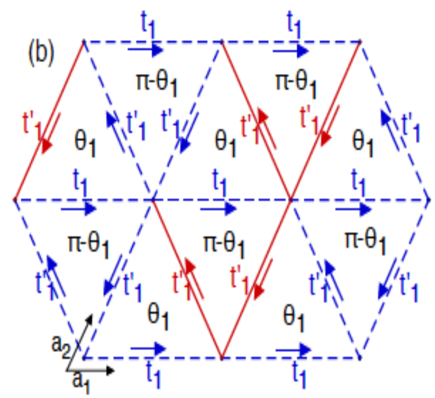
Frustrated triangular lattice spin ½ J1-J2 models

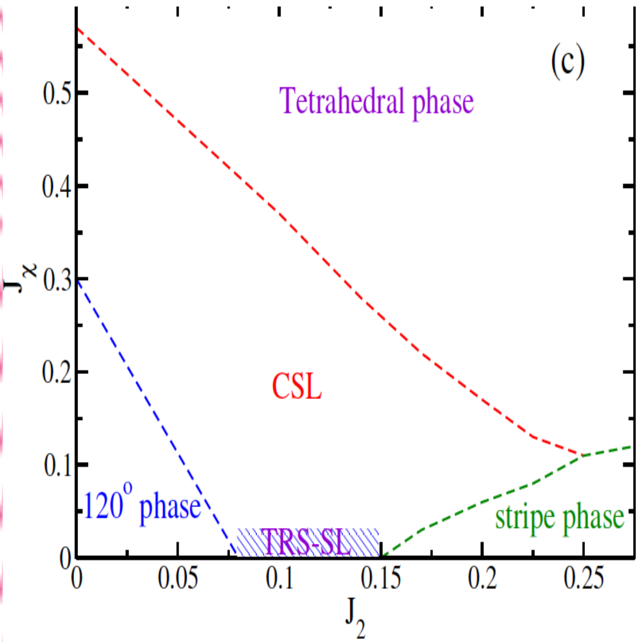


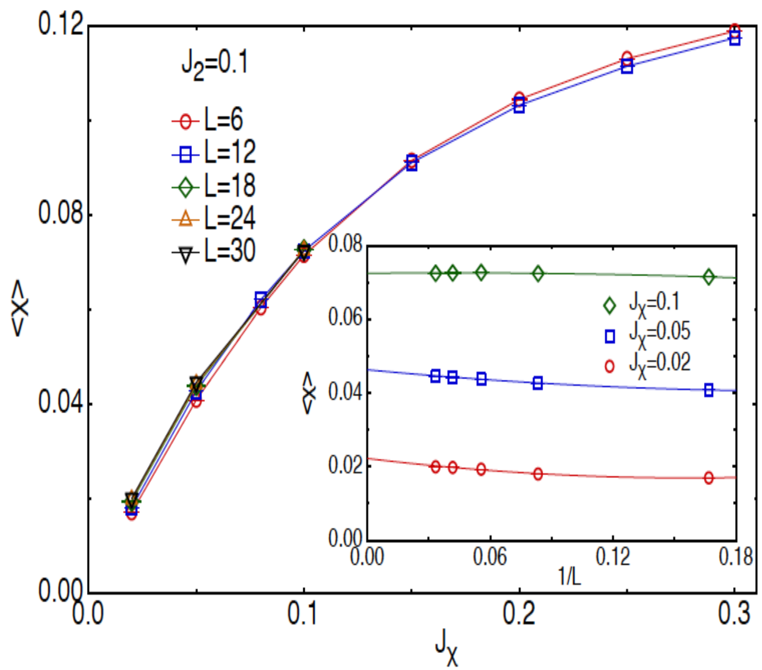
Z. Zhu and S. R. White, PRB; W . J. Hu et al., PRB

DMRG and VMC pictures: Dirac to CSL

$$H = J_1 \sum_{\langle i,j \rangle} \vec{S}_i \cdot \vec{S}_j + J_2 \sum_{\langle\langle i,j \rangle\rangle} \vec{S}_i \cdot \vec{S}_j + J_\chi \sum_{\Delta/\nabla} (\vec{S}_i \times \vec{S}_j) \cdot \vec{S}_k$$





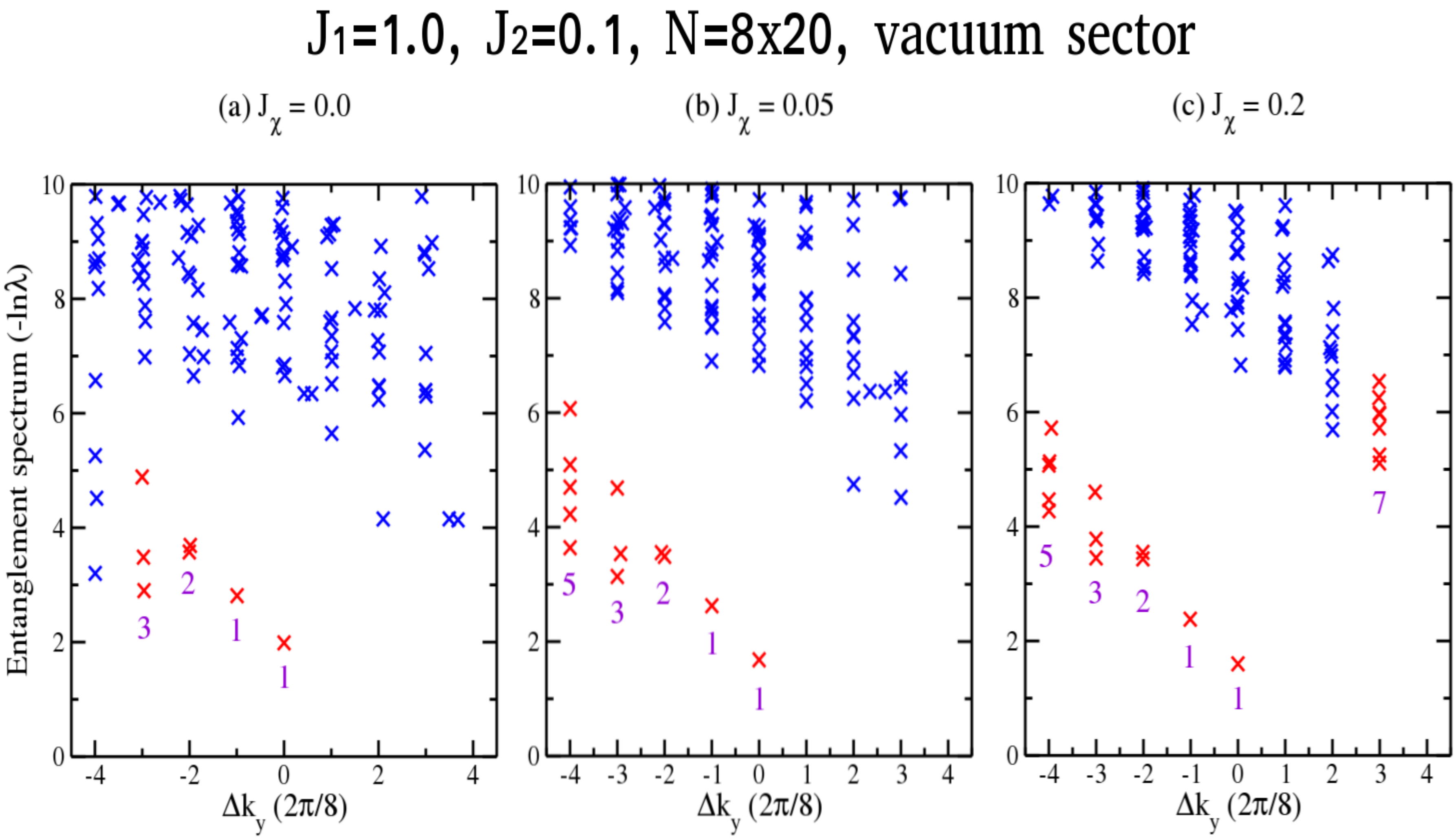


Bauer et al. (2014)
He et al.(2016)

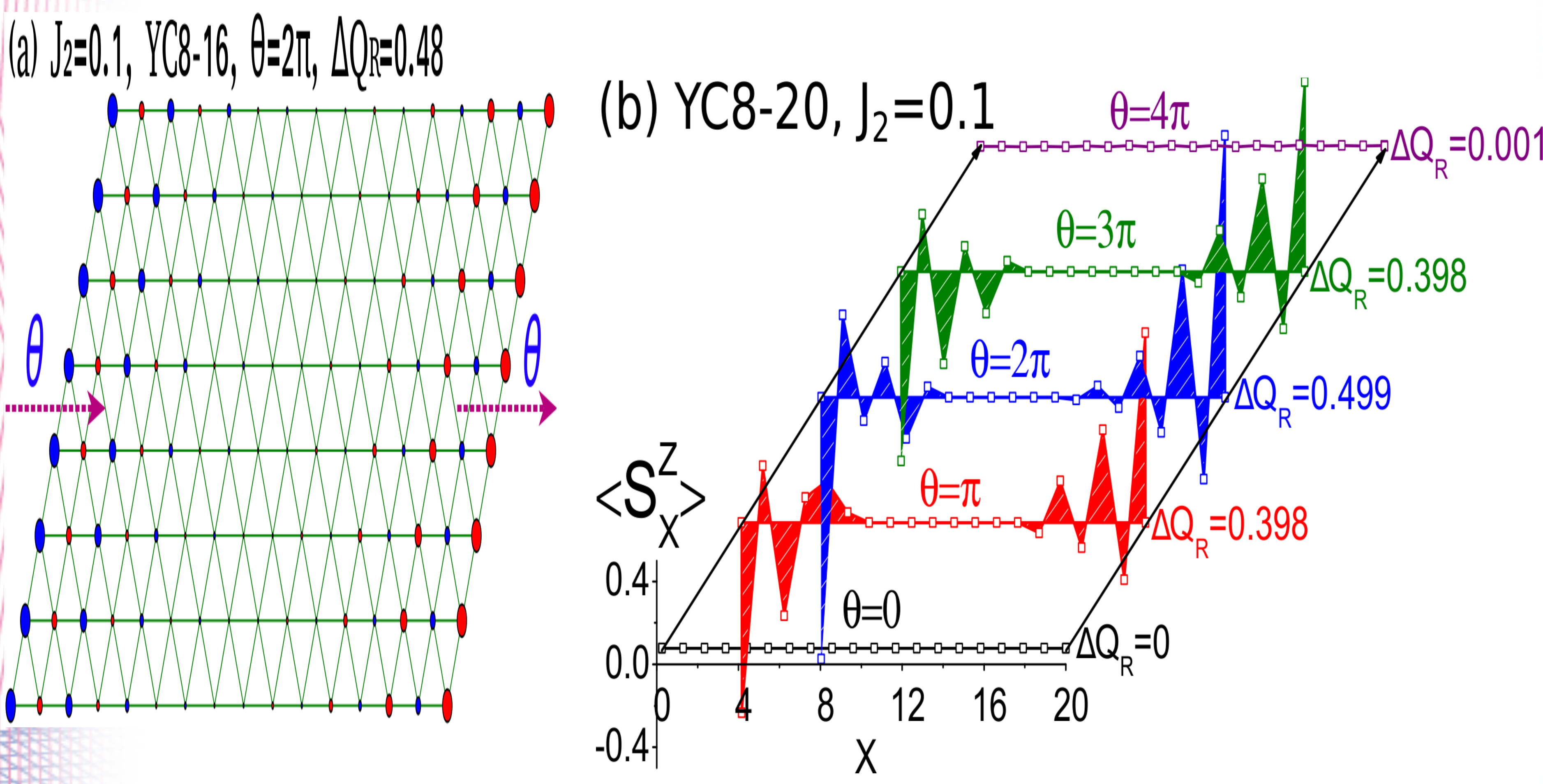
Zhu et al. 2015
Hu et al. 2015
Similar critical phase
For kagome

Agree with ED phase diagram, Lauchli et al.

Entanglement spectrum of the vacuum sector and additional Jx (S1xS2)*S3

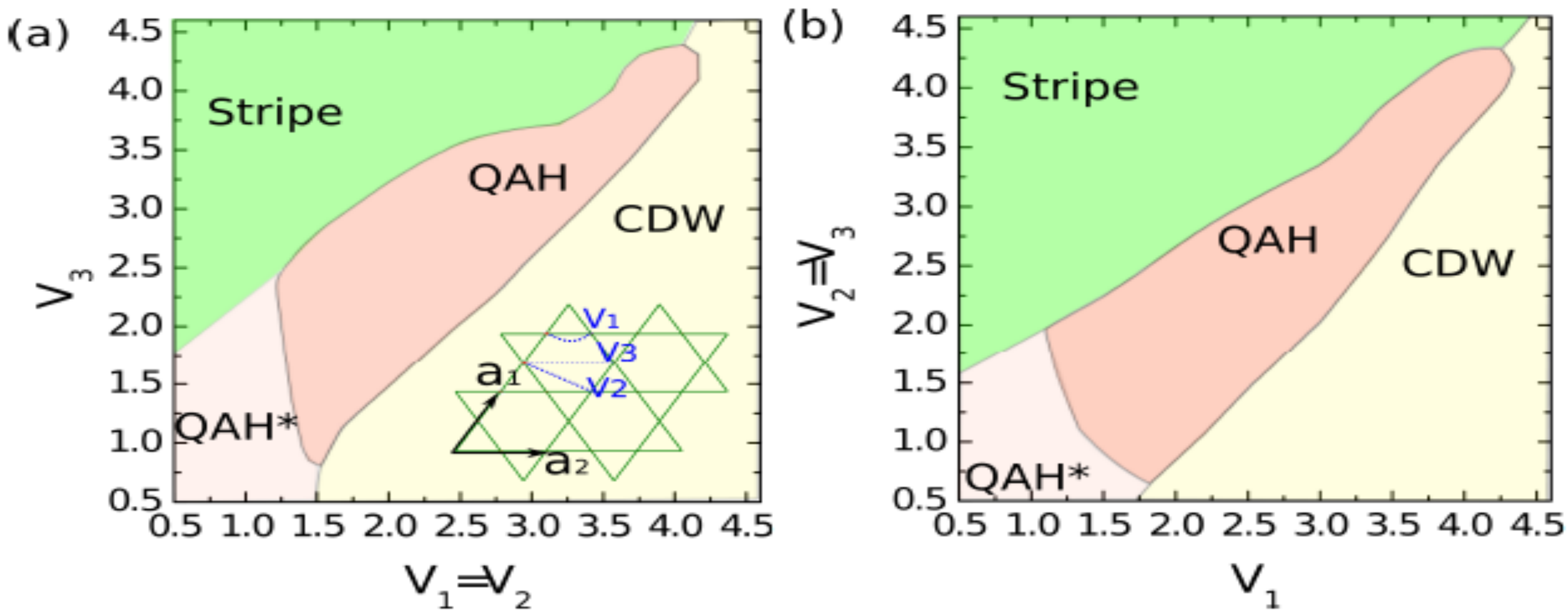


SL phase around J2=0.1, inserting flux 2pi gives fermionic spinon sector



Interaction-driven QAH ---- Fermion-Hubbard on kagome at 1/3

$$H = t \sum_{\langle \mathbf{r}\mathbf{r}' \rangle} \left[c^\dagger_{\mathbf{r}'} c_{\mathbf{r}} + \mathrm{H.c.} \right] + V_1 \sum_{\langle \mathbf{r}\mathbf{r}' \rangle} n_{\mathbf{r}} n_{\mathbf{r}'} + V_2 \sum_{\langle\langle \mathbf{r}\mathbf{r}' \rangle\rangle} n_{\mathbf{r}} n_{\mathbf{r}'} + V_3 \sum_{\langle\langle\langle \mathbf{r}\mathbf{r}' \rangle\rangle\rangle} n_{\mathbf{r}} n_{\mathbf{r}'}$$

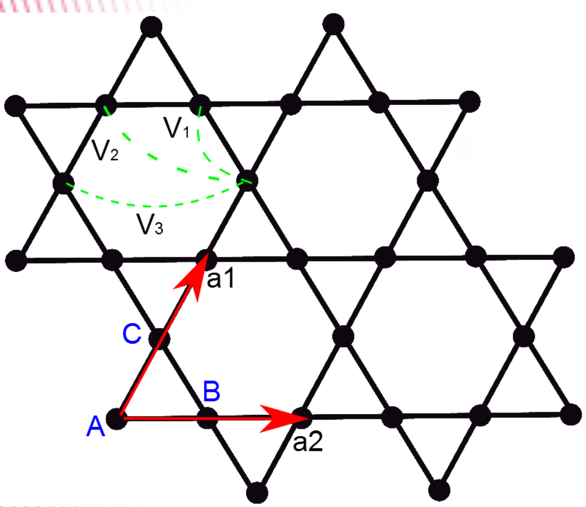


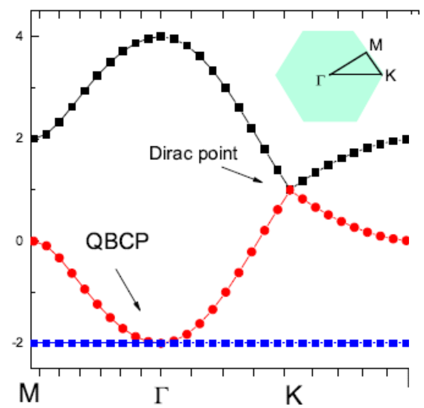
How to identify QAH? Using ED and DMRG

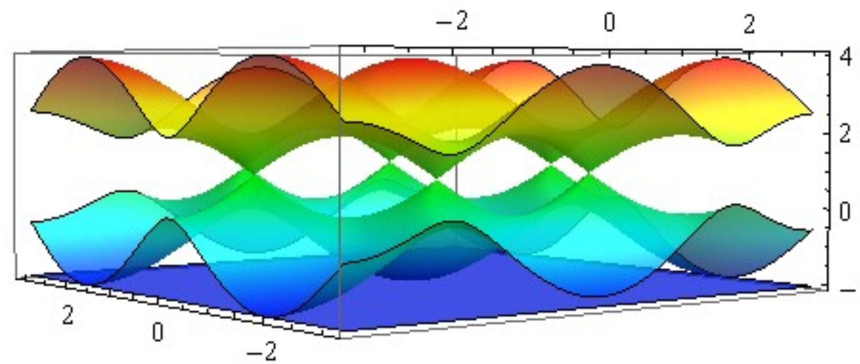
- Time-Reversal Symmetry Spontaneously Breaking
  Emergent Staggered Magnetic Flux
- Topological Chern Number (Hall Conductance)

Interaction-driven QAH ---- Fermion-Hubbard on kagome at 1/3

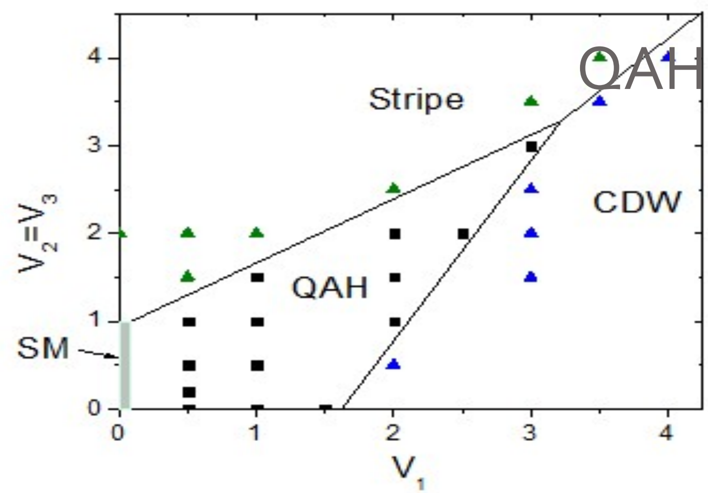
$$H = t \sum_{\langle \mathbf{r}\mathbf{r}' \rangle} \left[c^{\dagger}_{\mathbf{r}'} c_{\mathbf{r}} + \mathrm{H.c.} \right] + V_1 \sum_{\langle \mathbf{r}\mathbf{r}' \rangle} n_{\mathbf{r}} n_{\mathbf{r}'} + V_2 \sum_{\langle\langle \mathbf{r}\mathbf{r}' \rangle\rangle} n_{\mathbf{r}} n_{\mathbf{r}'} + V_3 \sum_{\langle\langle\langle \mathbf{r}\mathbf{r}' \rangle\rangle\rangle} n_{\mathbf{r}} n_{\mathbf{r}'}$$







Mean-Field Phase Diagram



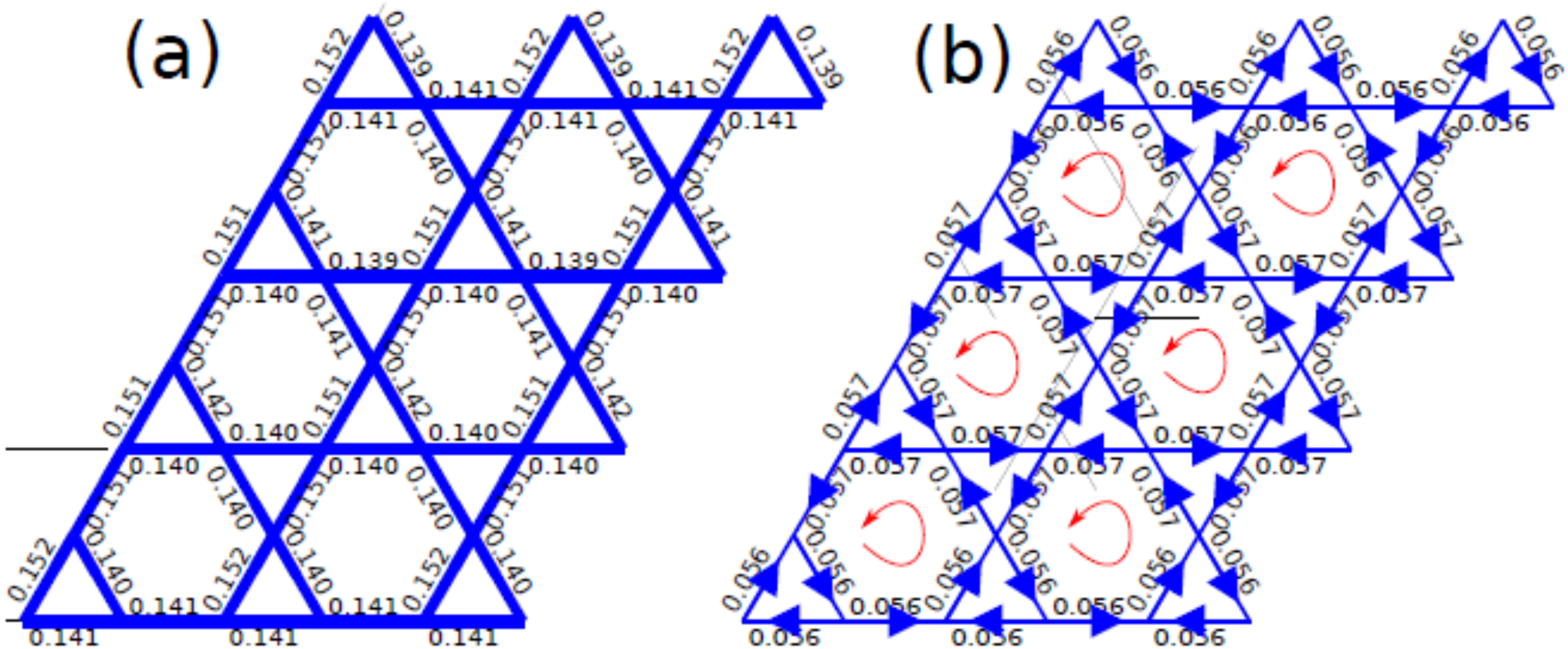
Wu et al. (2007),Bermann et al (08)
Sun et al (09)
Zhu, Gong, Zeng, Fu, Sheng (16)

Interaction-driven QAH ---- Emergent Magnetic Flux

$$H = t\sum_{\langle \mathbf{r}\mathbf{r}'\rangle}\left[c^{\dagger}_{\mathbf{r}'}c_{\mathbf{r}} + \mathrm{H.c.}\right] + V_1\sum_{\langle \mathbf{r}\mathbf{r}'\rangle} n_{\mathbf{r}}n_{\mathbf{r}'} + V_2\sum_{\langle\langle \mathbf{r}\mathbf{r}'\rangle\rangle} n_{\mathbf{r}}n_{\mathbf{r}'} + V_3\sum_{\langle\langle\langle \mathbf{r}\mathbf{r}'\rangle\rangle\rangle} n_{\mathbf{r}}n_{\mathbf{r}'}$$

$$\mathcal{J}_{ij} = \langle \Psi^L | c^{\dagger}_i c_j | \Psi^L \rangle$$

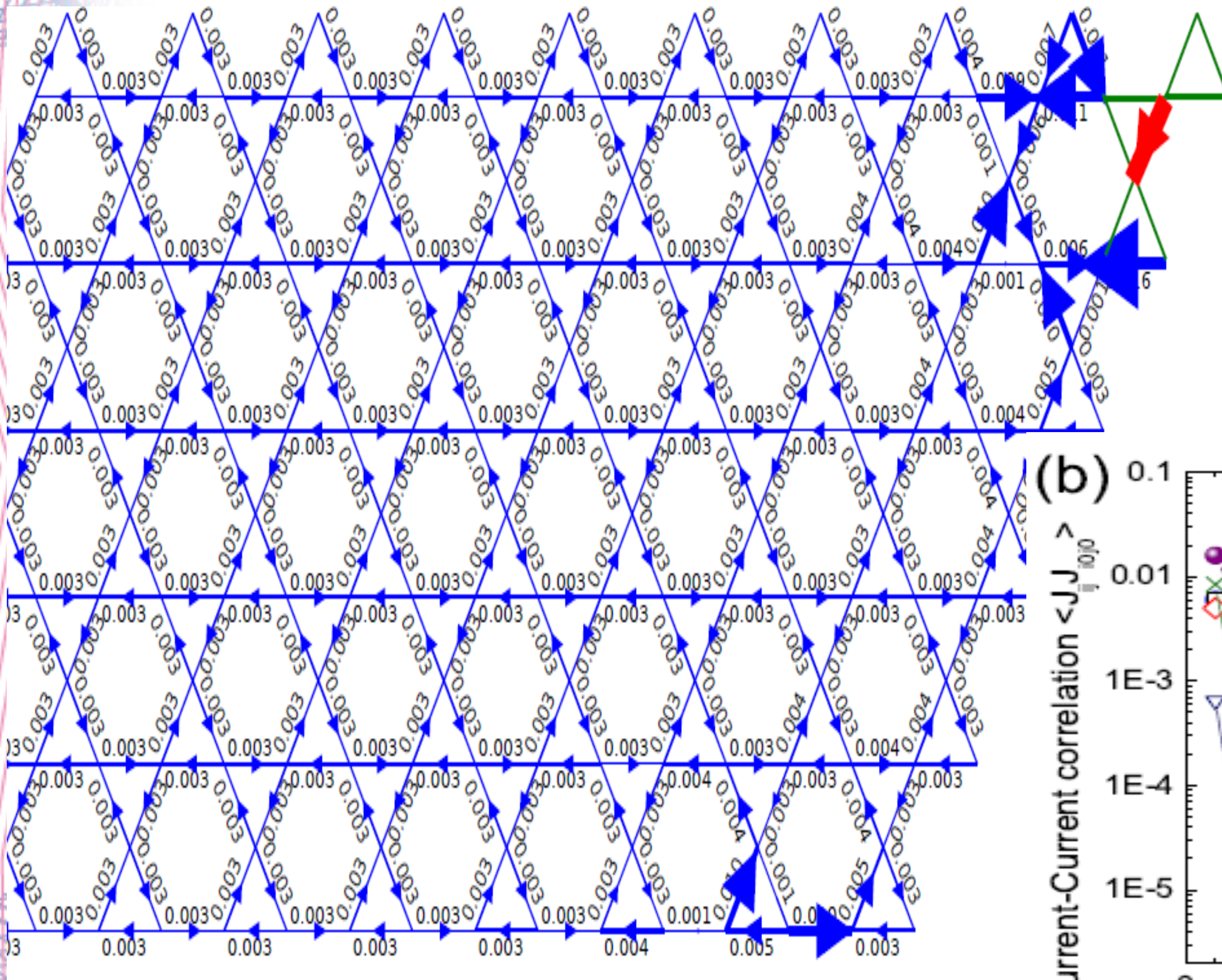
$Re\mathcal{J}_{ij}$ $Im\mathcal{J}_{ij}$

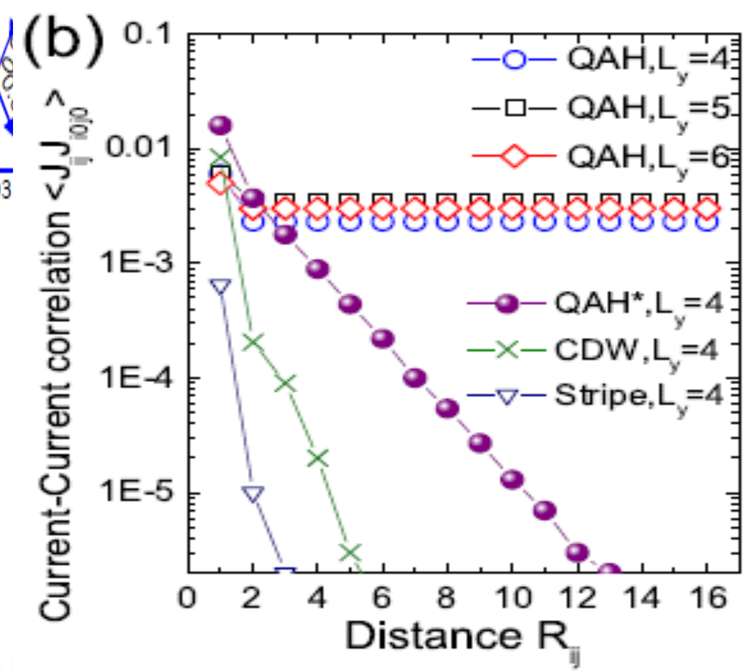


Staggered Magnetic Flux!!

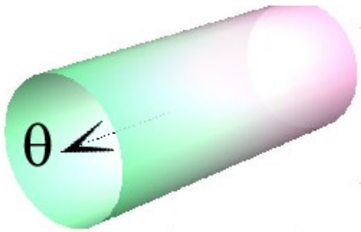
Current-Current correlation

$$\mathcal{J}_{ij} = i\langle\Psi^{L(R)}|c_i^\dagger c_j - c_j^\dagger c_i|\Psi^{L(R)}\rangle$$

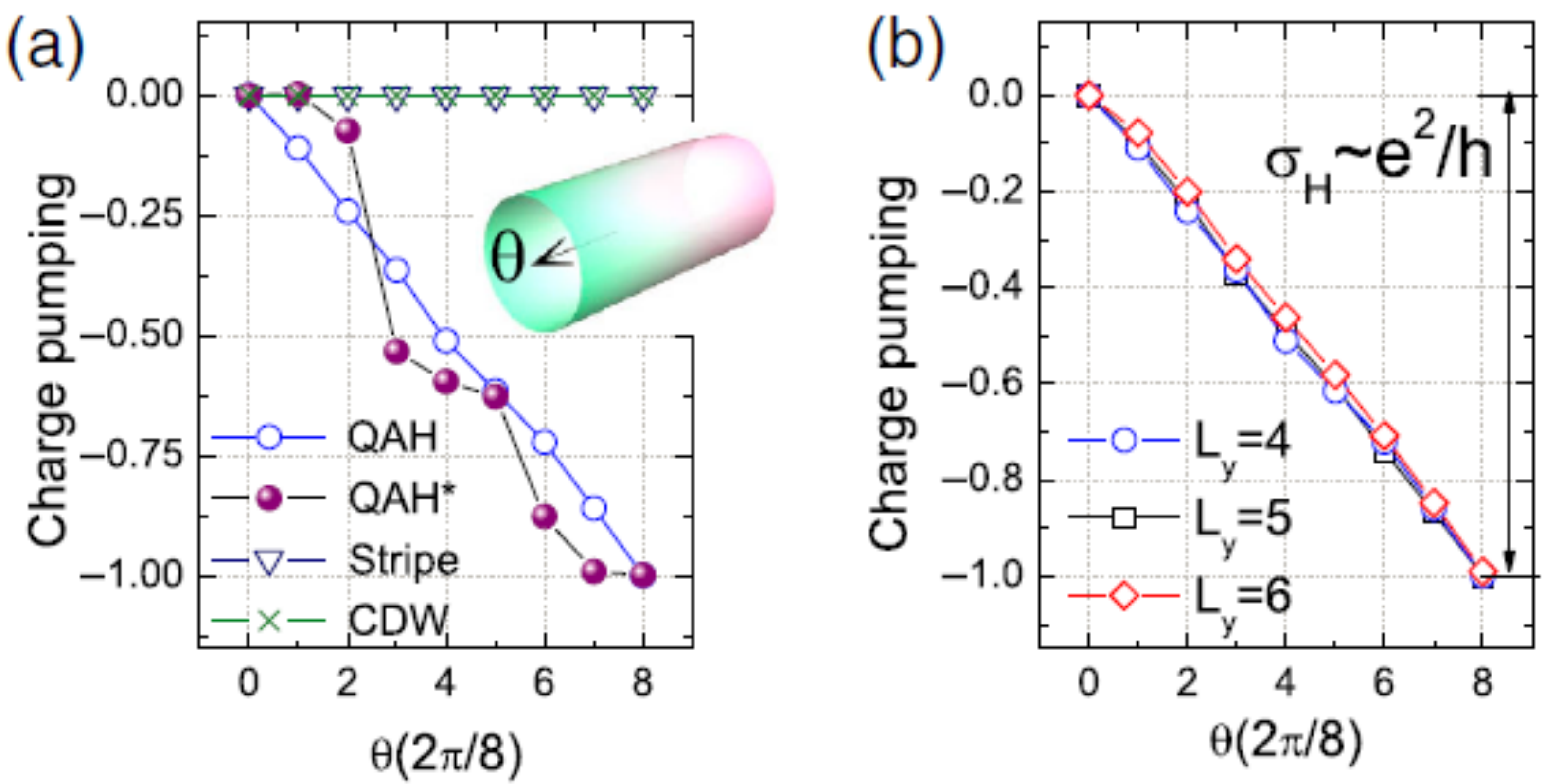




Interaction-driven QAH ---- Chern Number



Chern Number (Hall conductance):
C = 1 (Left chirality)
C= -1 (Right chirality)



Summary and discussions

Most experimental SL candidates have triangular or kagome lattices.

Numerically we also found frustrated triangles in these lattices play essential role in stabilizing SLs.

We discovered long-sought chiral spin liquid (CSL) in kagome lattice model and establish the instability toward CSL for triangular J1-J2 model.

VMC/DMRG studies indicate a continuous Dirac SL to CSL transition

Fermionic systems with such frustrations can realize Spontaneous Quantum Hall Effect and other exotic phases (ongoing work).

Research collaborations

Shoushu Gong (FS), Wei Zhu (LANL)
Tiansheng Zeng (CSUN), Wenjun Hu (RU)

Y. C. He and Yan Chen, Balents on CSL;
Liang Fu on Spontaneous QHE

Duncan Haldane, F. Becca,
Kun Yang, O. Starykh

Thank you !