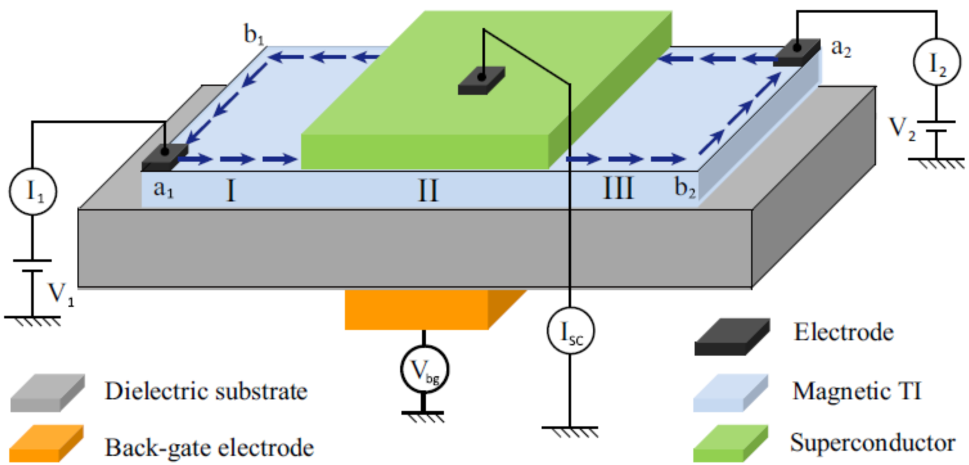
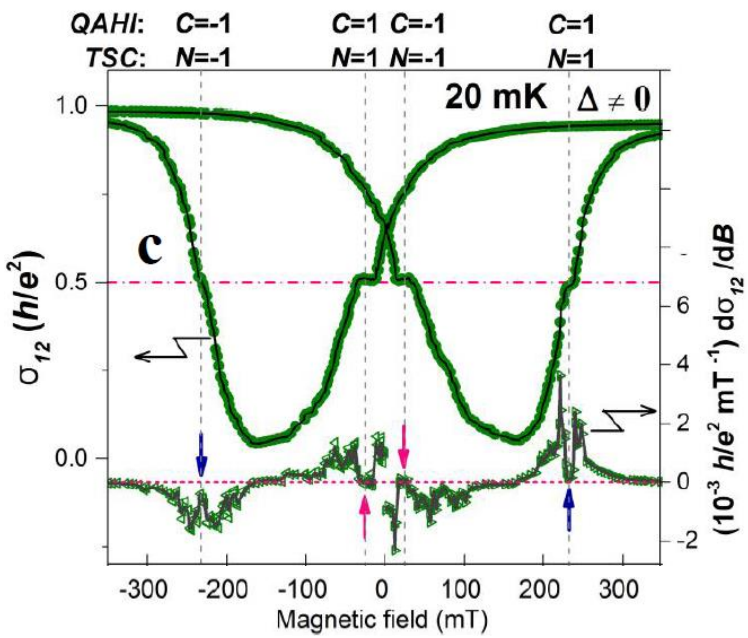
Discovery of the chiral Majorana fermion and its application to quantum computing





KITPC 2018/5

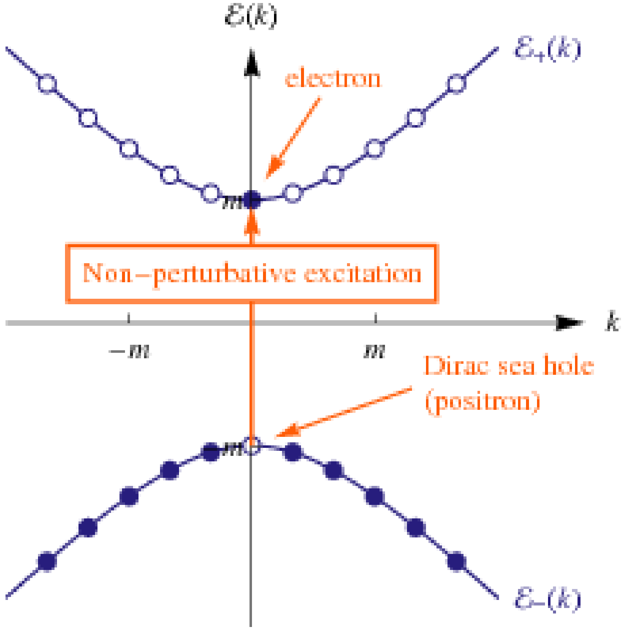
Shoucheng Zhang, Stanford University

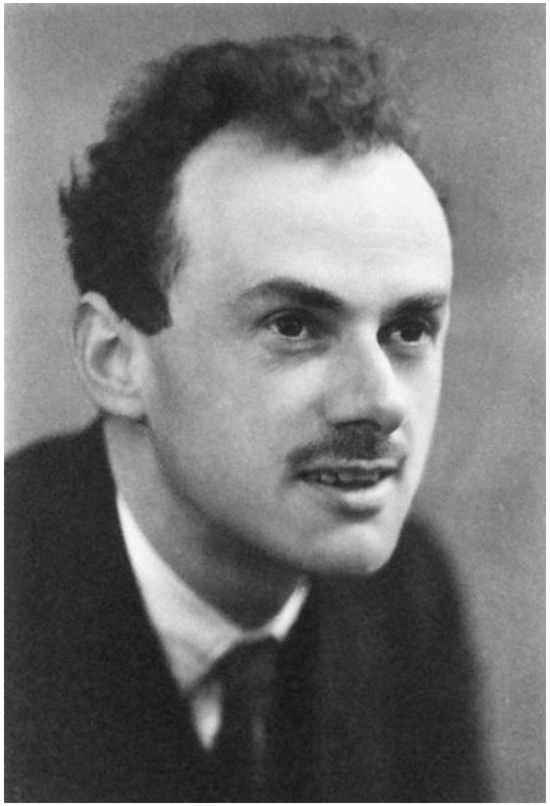
Dirac equation and the anti-particle

In 1928, Dirac unified Einstein's special theory of relativity with quantum mechanics, and introduced Dirac equation:

$$(i\gamma^{\mu}\partial_{\mu} - m)\psi = 0$$

Dirac equation gives negative energy solutions, which led Dirac to predict the existence of anti-particle.





In 1932, the positron, the anti-particle of the electron was discovered by CD Anderson in cosmic rays.

Majorana and his fermion

In 1937, Ettore Majorana asked the question: can fermions be their own antiparticles?

The ***Dirac equation*** is known to describe charged fermions:

$$(i\gamma^{\mu}\partial_{\mu} - m)\psi = 0$$

where γ^{μ} are Dirac's anticommuting *Gamma matrices*.

Ettore Majorana

Majorana claimed if all γ^{μ} are selected imaginary, one can make ψ real, describing a *charge neutral, spin ½ fermion being its own antiparticle*, i.e., ***Majorana fermion***, obeying ***Majorana equation***.

$$\tilde{\gamma}^0 = \begin{pmatrix} 0 & 0 & 0 & -i \\ 0 & 0 & -i & 0 \\ 0 & i & 0 & 0 \\ i & 0 & 0 & 0 \end{pmatrix} \quad \tilde{\gamma}^1 = \begin{pmatrix} 0 & 0 & i & 0 \\ 0 & 0 & 0 & i \\ i & 0 & 0 & 0 \\ 0 & i & 0 & 0 \end{pmatrix} \quad \tilde{\gamma}^2 = \begin{pmatrix} i & 0 & 0 & 0 \\ 0 & i & 0 & 0 \\ 0 & 0 & -i & 0 \\ 0 & 0 & 0 & -i \end{pmatrix} \quad \tilde{\gamma}^3 = \begin{pmatrix} 0 & 0 & 0 & -i \\ 0 & 0 & i & 0 \\ 0 & i & 0 & 0 \\ -i & 0 & 0 & 0 \end{pmatrix}$$

Gamma matrices in *Majorana equation*

Properties of the Majorana fermion

Neutrino could be a Majorana fermion, with Majorana mass term.

Majorana fermion is essential for supersymmetry.

Chiral Majorana fermion could exist in 1+1 and 9+1 dimensions, both essential for the superstring theory.

Majorana fermion could arise as quasi-particles of topological states of quantum matter.

Majorana fermion could be used for topological quantum computing.

Search for hypothetical particles/waves

Higgs boson, gravitational wave

Majorana fermion

Magnetic monopole

Axion

Dark matter particle

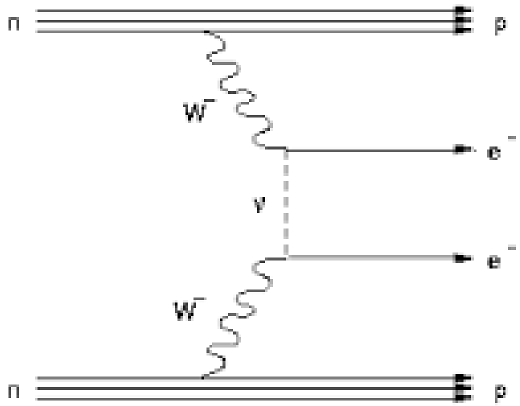
The race for Majorana fermion

Majorana returns

Frank Wilczek

In his short career, Ettore Majorana made several profound contributions. One of them, his concept of 'Majorana fermions' — particles that are their own antiparticle — is finding ever wider relevance in modern physics.

Neutrinoless double beta decay



Viewpoint

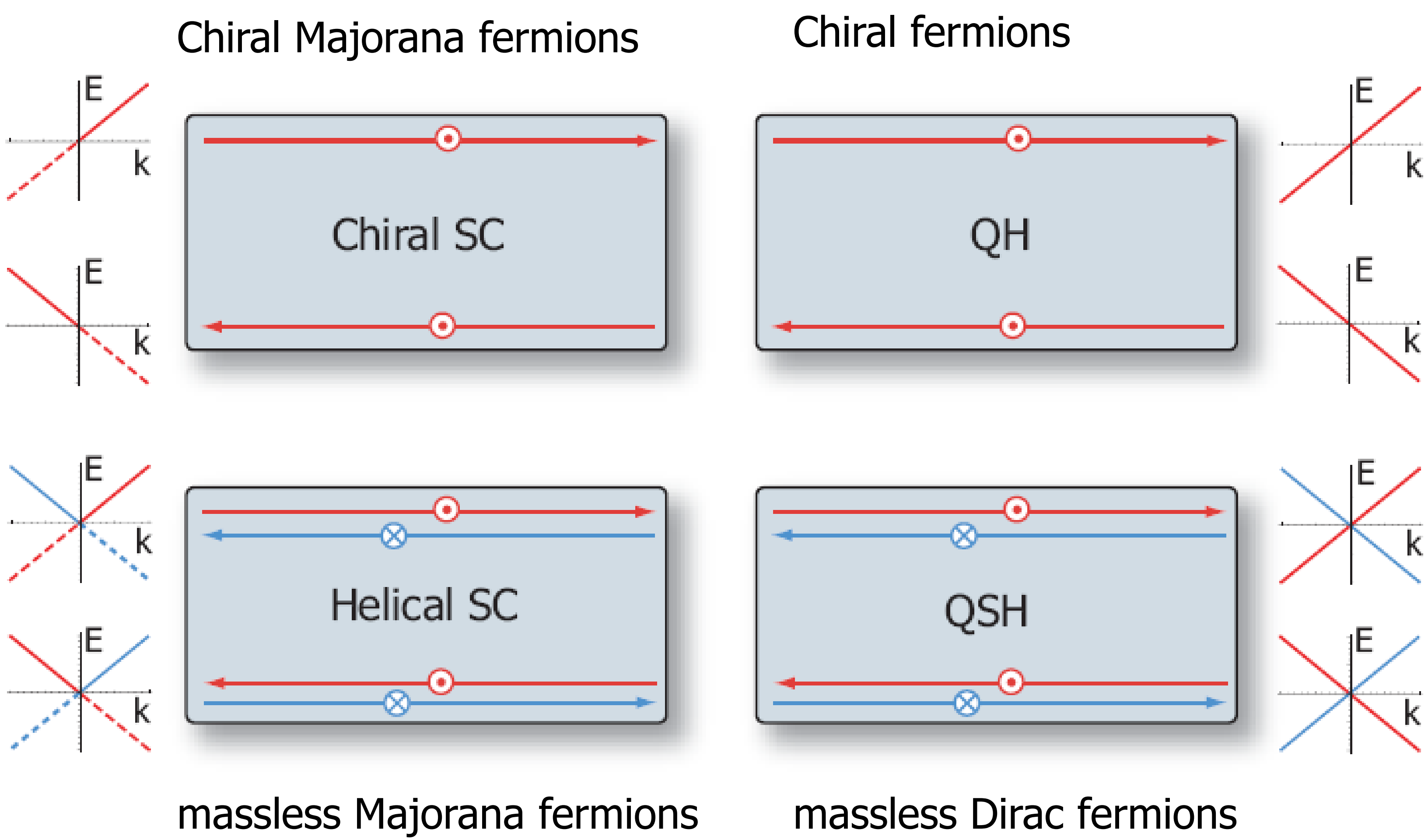
Race for Majorana fermions

Marcel Franz
Department of Physics and Astronomy, University of British Columbia, Vancouver, BC, Canada V6T 1Z1
Published March 15, 2010

The race for realizing Majorana fermions—elusive particles that act as their own antiparticles—heats up, but we still await ideal materials to work with.

Topological insulators and superconductors

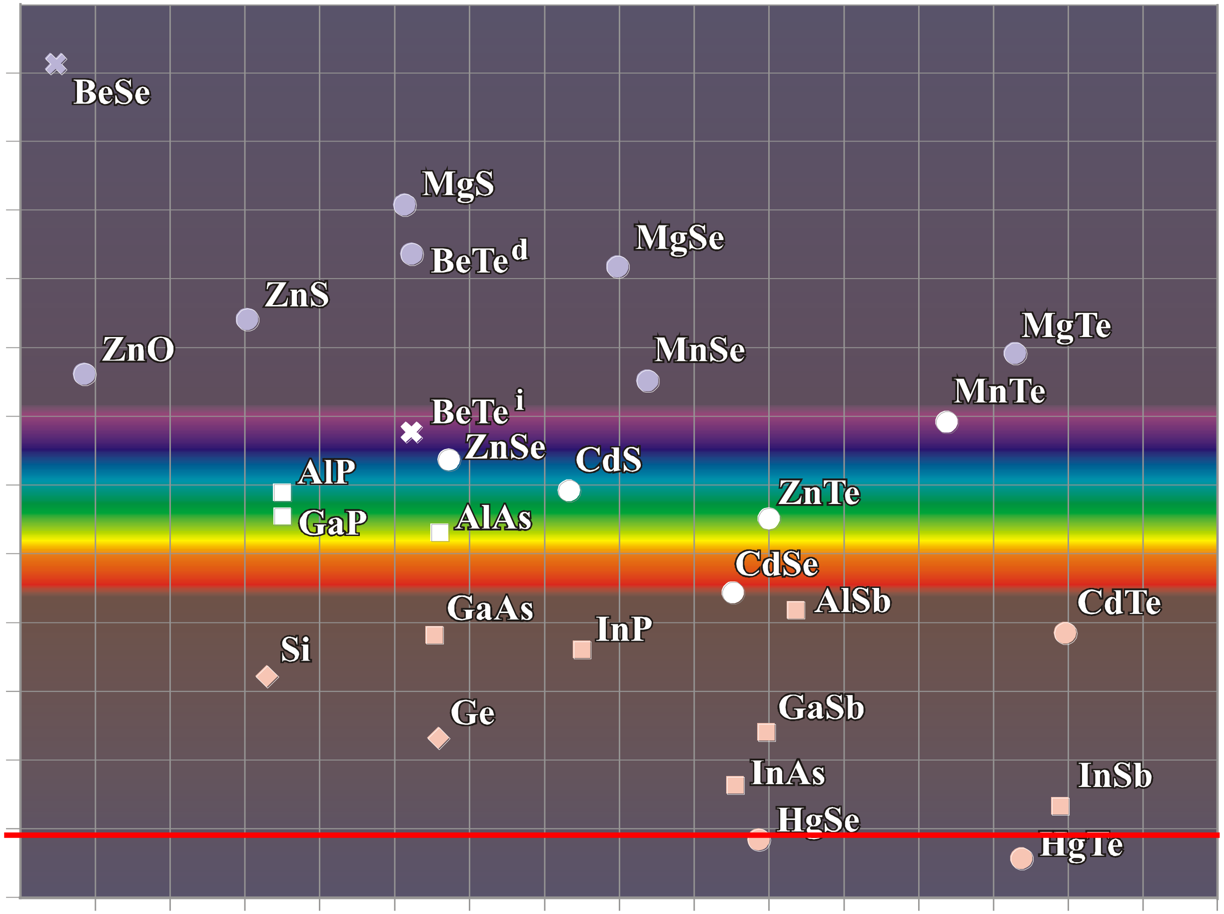
Full pairing gap in the bulk, gapless Majorana edge and surface states



(Qi, Hughes, Raghu and Zhang, PRL, 2009)

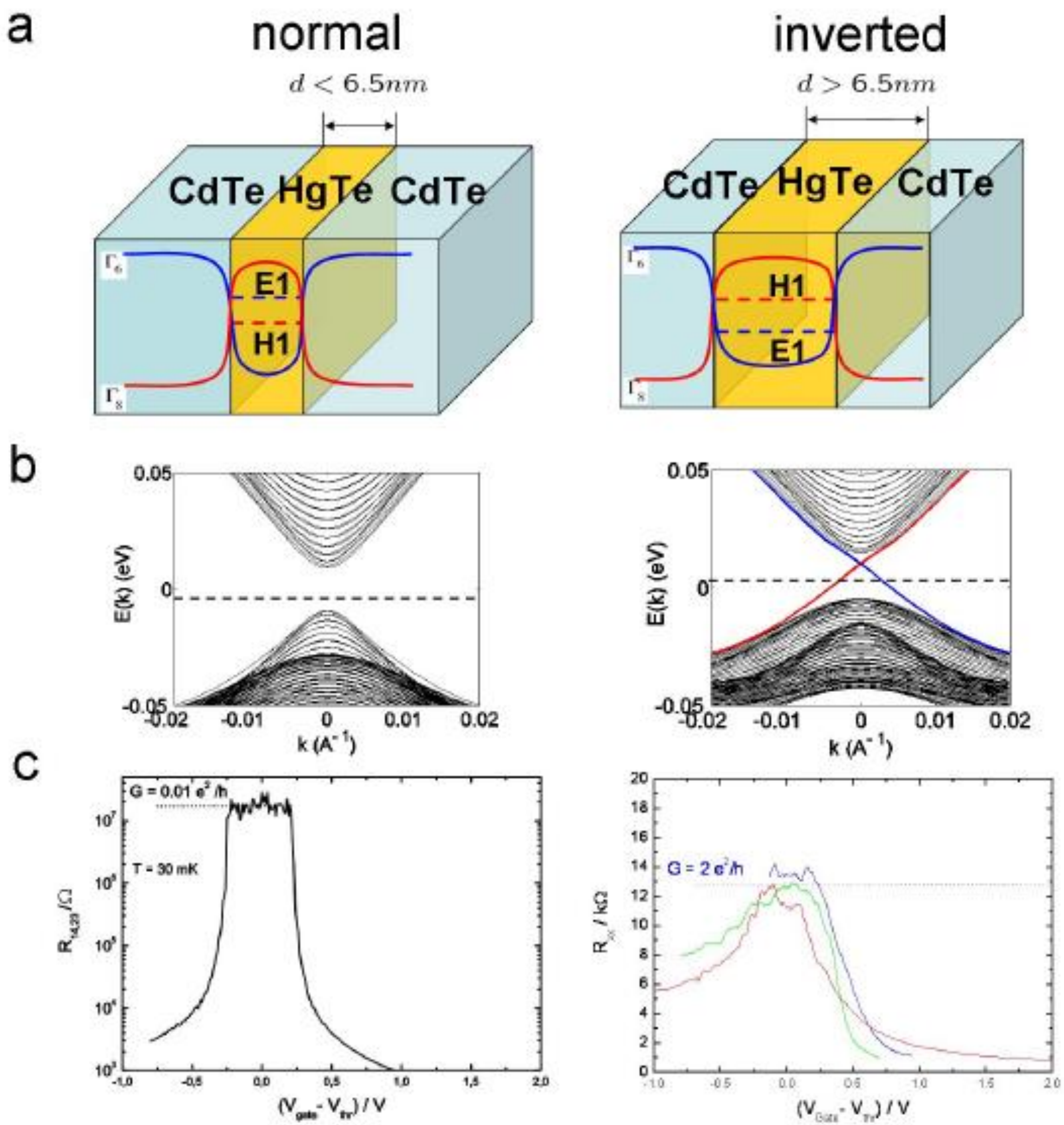
Quantum Spin Hall Effect and Topological Phase Transition in HgTe Quantum Wells

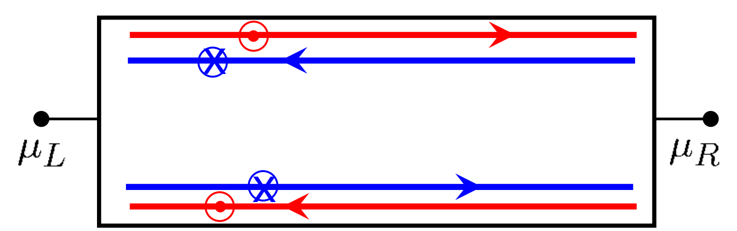
B. Andrei Bernevig,[1,2] Taylor L. Hughes,[1] Shou-Cheng Zhang[1]*



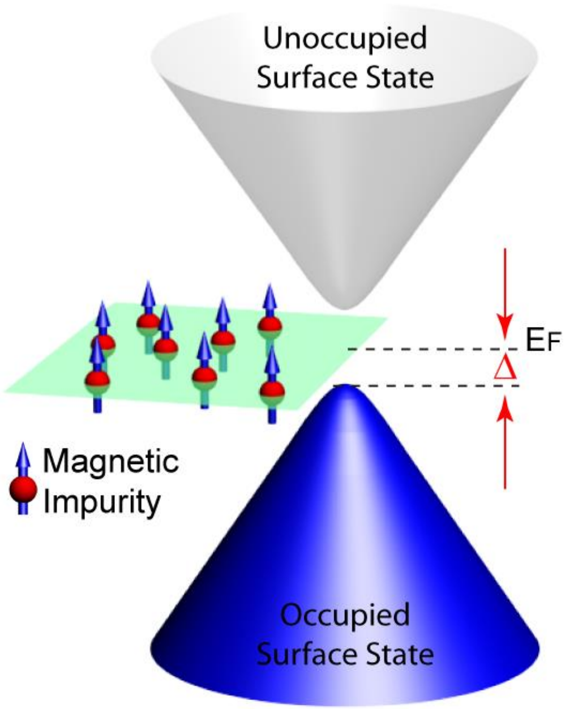
All topological insulators are predicted based on the principle of band inversion

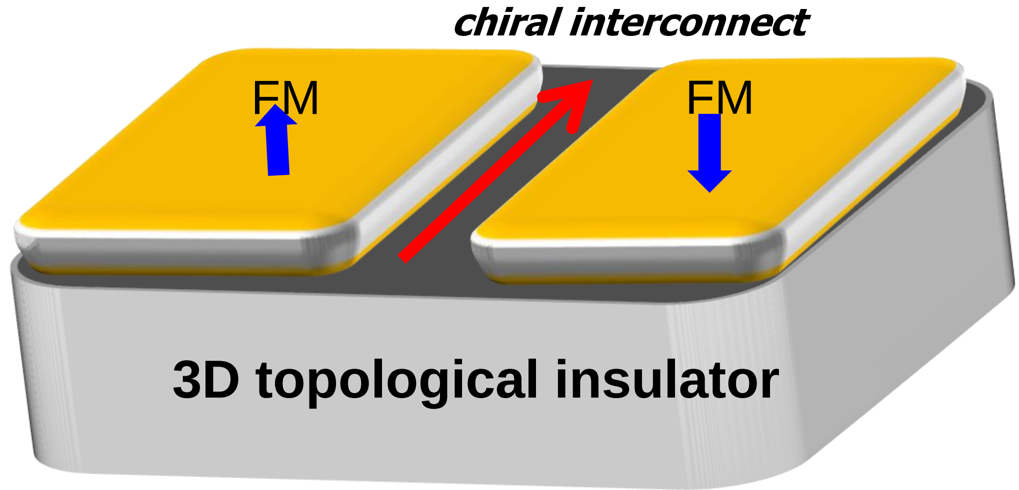
Experimental observation of the QSH edge state
(Konig et al, Science 2007)

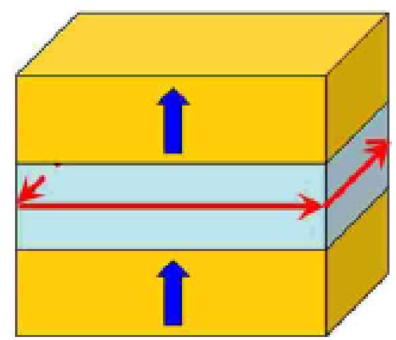




Gapped Dirac fermions on the surface, chiral fermions on the domain wall

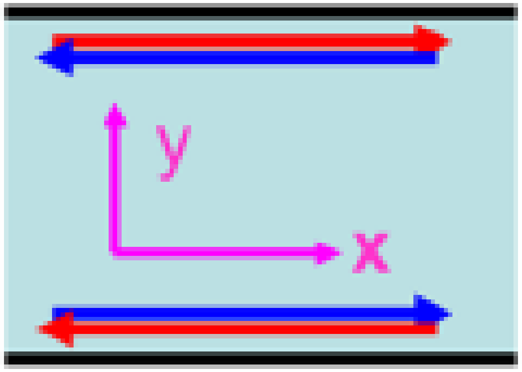




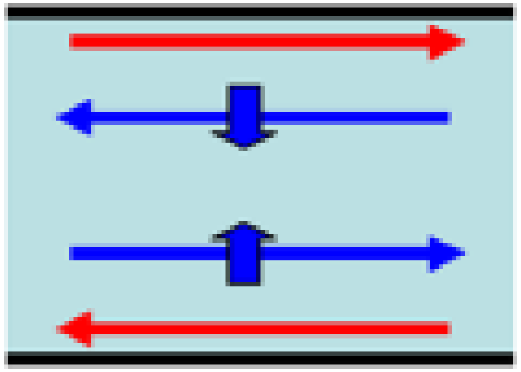


QAH can be realized in magnetic TI (Qi, Hughes, Zhang, PRB 2008)

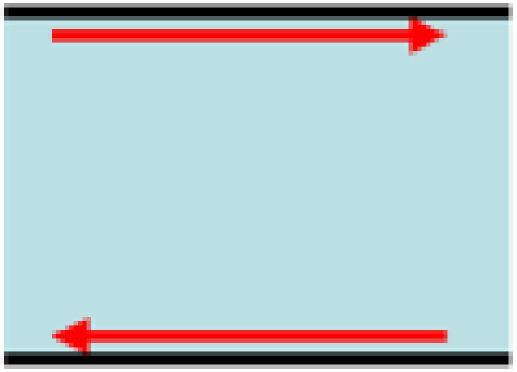
Evolution from QSH to QAH



Helical edge states of QSH protected by time reversal symmetry

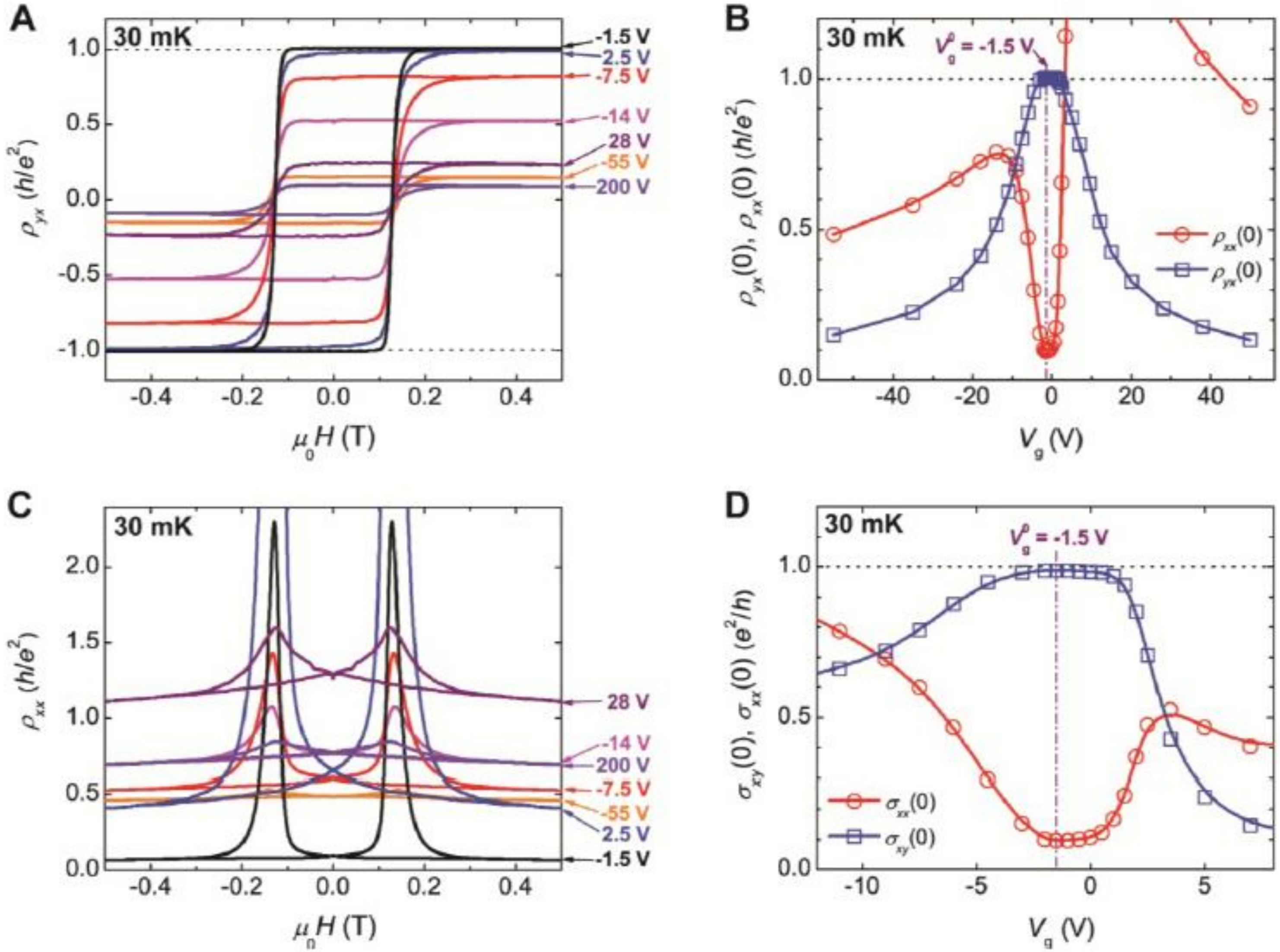


Magnetic order breaks time reversal symmetry, removes the degeneracy



In the extreme limit, only chiral edge states of the QAH state remains

Discovery of the QAH (Science 340, 167 (2013))



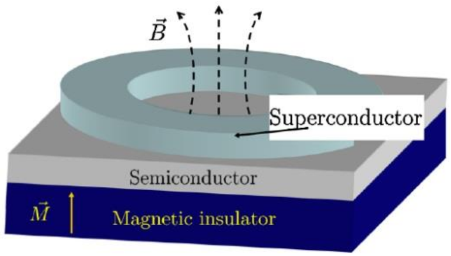
Majorana zero mode

Majorana zero mode (MZM) is predicted to appear

- on the 1D TSC boundary:

Kitaev 2000, Lutchyn, Sau, Das Sarma, 2010; Oreg, Refael, von Oppen, 2010

- or in the vortex of a 2D chiral TSC:



Moore, Read 2000, Sau et. al., 2010; Fu, Kane, 2010, Alicea 2010, Qi, Hughes, Zhang, 2010

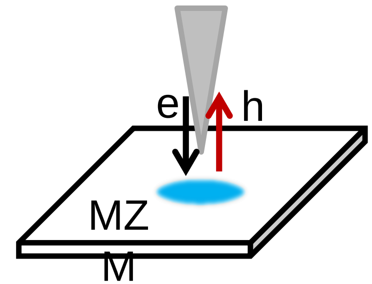
Detection method: STM tunneling

$$\frac{dI}{dV} \propto \text{Density of States}$$

which should exhibit a **zero bias peak**.

Quantized Zero Bias Peak in Theory

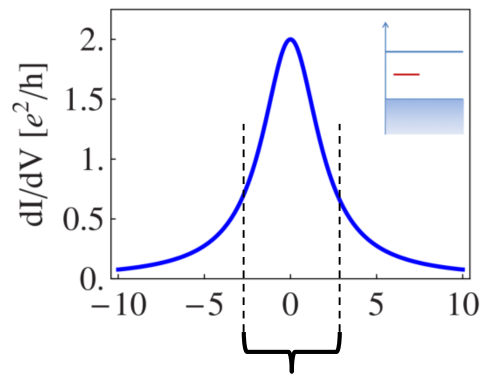
For STM tip solely coupled to a MZM (coupling strength Γ):



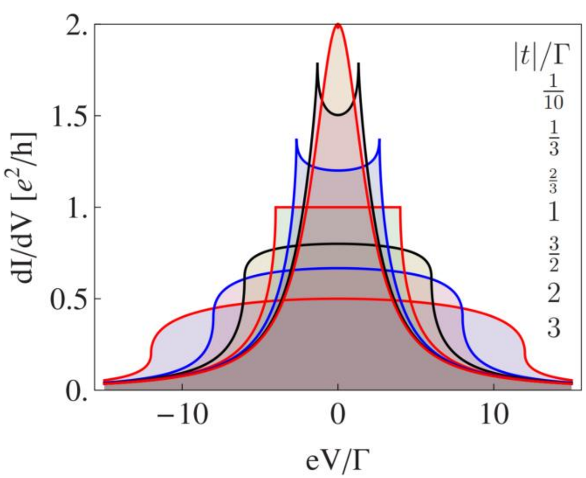
charge neutrality → resonant Andreev amplitude $R_A = 1$

→ $$\boxed{\frac{dI}{dV} = 2R_A = \frac{2e^2}{h} \text{ quantized at zero energy.}}$$

Law, Lee, Ng 2009



Γ: Coupling between tip & MZM



- The quantization is lost when the tip couples to other ordinary modes NOT charge neutral:

$R_A < 1$ on average → $\frac{dI}{dV}$ decreases.

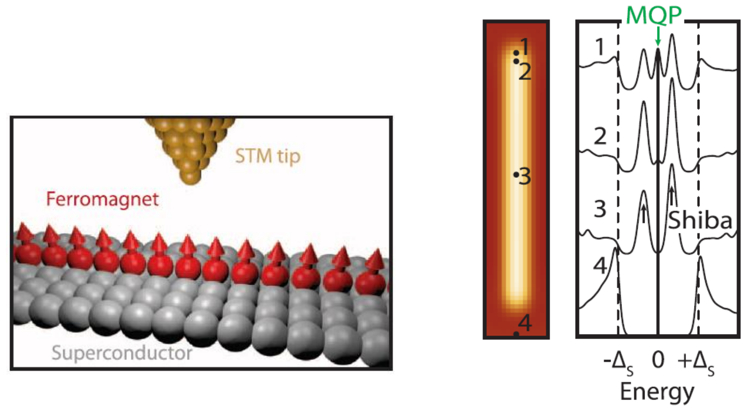
Flensberg 2010

Zero Bias Peak in Experiments

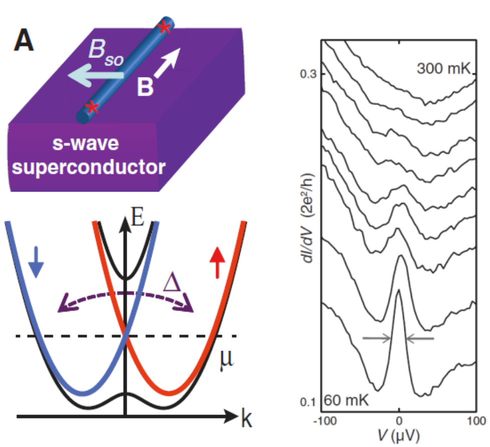
In experiments so far, the measured zero bias peak height is around

$$\frac{dI}{dV} < 0.1\,\frac{e^2}{h}$$

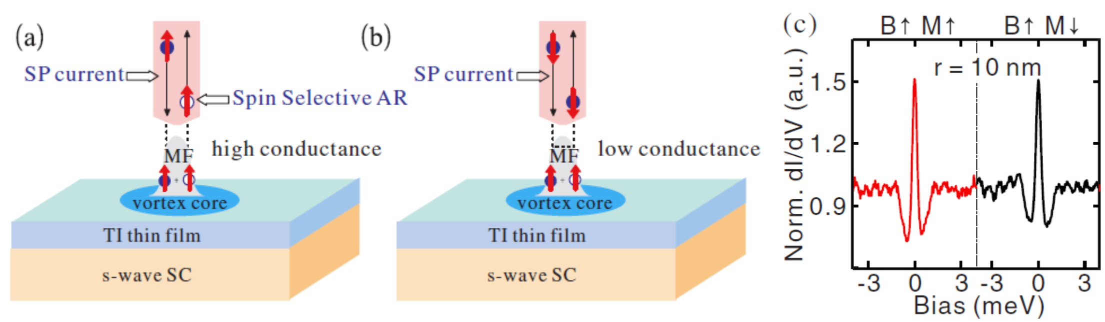
Probably due to normal modes coupling & finite temperature effect.



Fe atom chains-superconductor system, Yazdani group, Science 2014



Hybrid superconductor-nanowire system, Kouwenhoven group, Science 2012



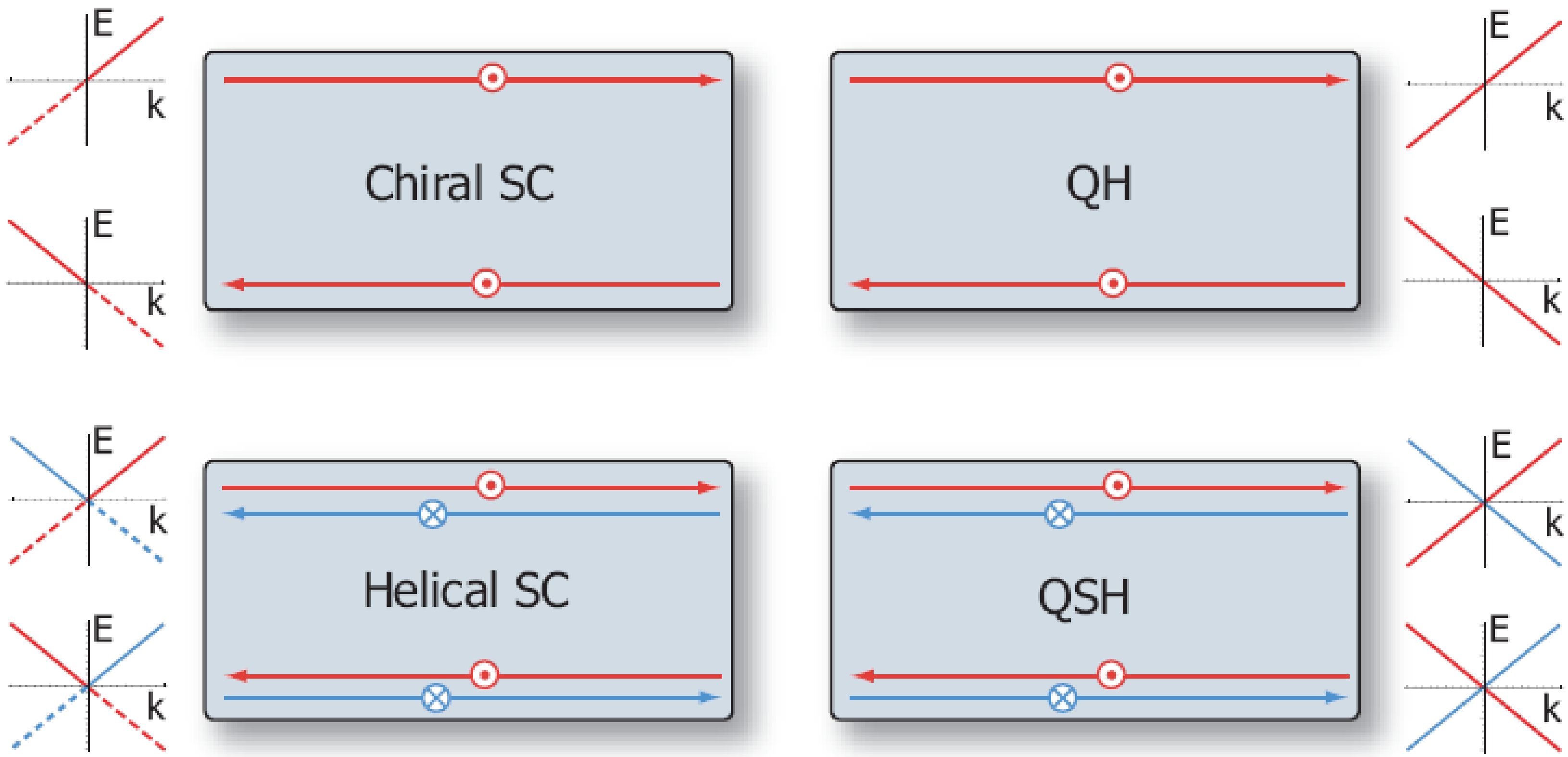
TI-SC thin film experiment, Jia group, 2016

Topological insulators and superconductors

Full pairing gap in the bulk, gapless Majorana edge and surface states

Chiral Majorana fermions

Chiral fermions



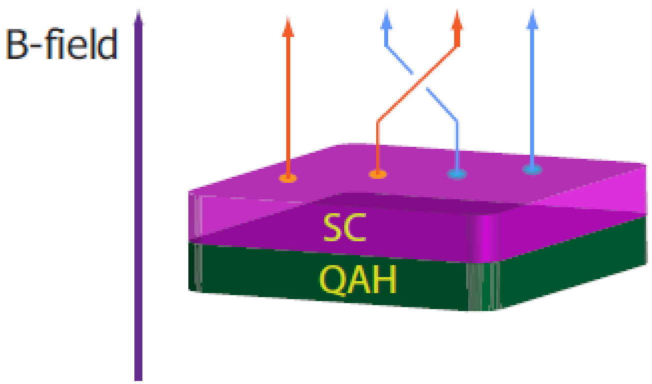
massless Majorana fermions

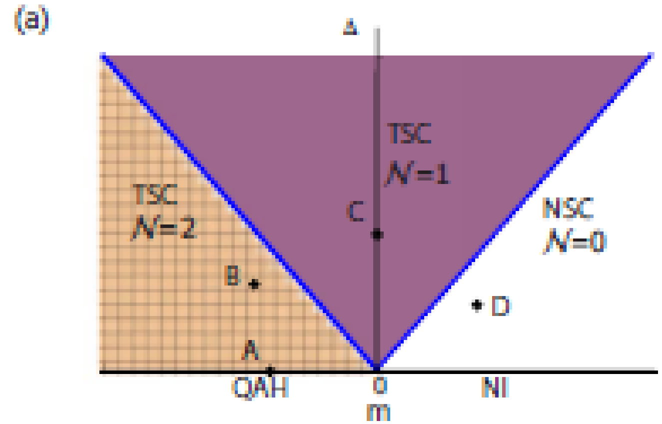
massless Dirac fermions

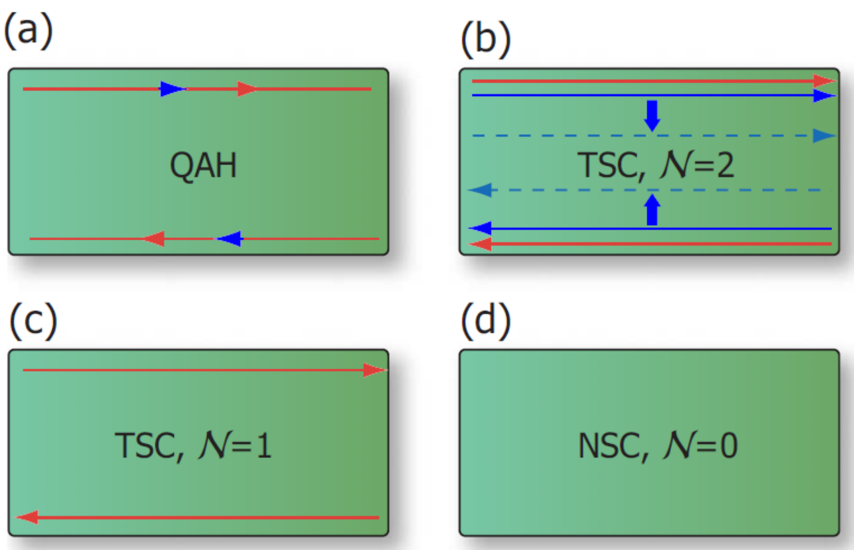
(Qi, Hughes, Raghu and Zhang, PRL, 2009)

Chiral topological superconductivity from QAHE

(Qi, Hughes and Zhang PHYSICAL REVIEW B 82, 184516 (2010))







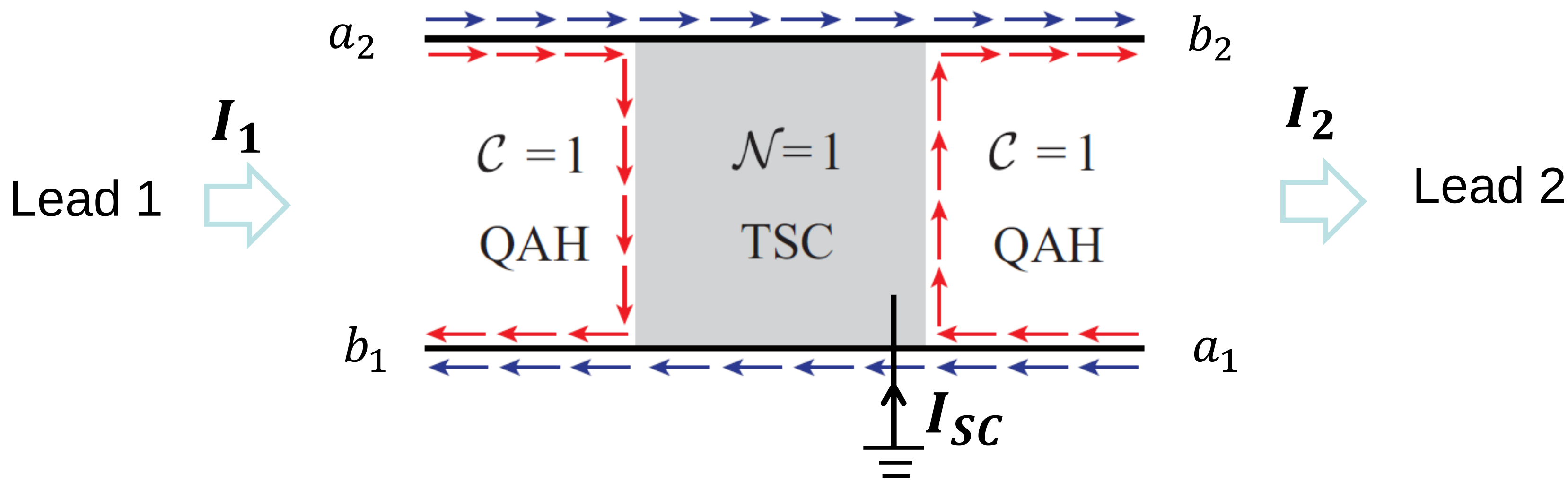
$$H_{\mathrm{QAH}} = \sum_{\mathbf{p}} \psi_{\mathbf{p}}^{\dagger} h_{\mathrm{QAH}}(\mathbf{p}) \psi_{\mathbf{p}},$$

$$h_{\mathrm{QAH}}(\mathbf{p}) = \begin{pmatrix} m(p) & A(p_x - ip_y) \\ A(p_x + ip_y) & -m(p) \end{pmatrix},$$

$$H_{\mathrm{BdG}} = \frac{1}{2} \sum_{\mathbf{p}} \Psi_{\mathbf{p}}^{\dagger} \begin{pmatrix} h_{\mathrm{QAH}}(\mathbf{p}) - \mu & i\Delta\sigma^{y} \\ -i\Delta^{*}\sigma^{y} & -h_{\mathrm{QAH}}^{*}(-\mathbf{p}) + \mu \end{pmatrix} \Psi_{\mathbf{p}},$$

As one sweeps the magnetic field, there is NECESSARILY an intermediate phase with chiral topological superconductivity!

Longitudinal Conductance of QAH-TSC junction



Electron states $a_{1,2}$, $b_{1,2}$ decomposes into Majoranas on the edge of TSC.

Scattering matrix (amplitude):

$$\begin{pmatrix} \hat{b}_{1;e} \\ \hat{b}^{\dagger}_{1;e} \\ \hat{b}_{2;e} \\ \hat{b}^{\dagger}_{2;e} \end{pmatrix} = \frac{1}{2} \begin{pmatrix} -1 & -1 & 1 & -1 \\ -1 & -1 & -1 & 1 \\ 1 & -1 & 1 & 1 \\ -1 & 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} \hat{a}_{1;e} \\ \hat{a}^{\dagger}_{1;e} \\ \hat{a}_{2;e} \\ \hat{a}^{\dagger}_{2;e} \end{pmatrix}$$

Chung, et.al. PRB 83, 100512(R) (2011)

Longitudinal Conductance of QAH-TSC junction

An electron incident from left (right) has quantum probabilities:

transmission $\mathcal{T} = 1/4$, Andreev transmission $\mathcal{T}_A = 1/4$,
reflection $\mathcal{R} = 1/4$, Andreev reflection $\mathcal{R}_A = 1/4$.

The *generalized Laudauer-Buttiker formula*:

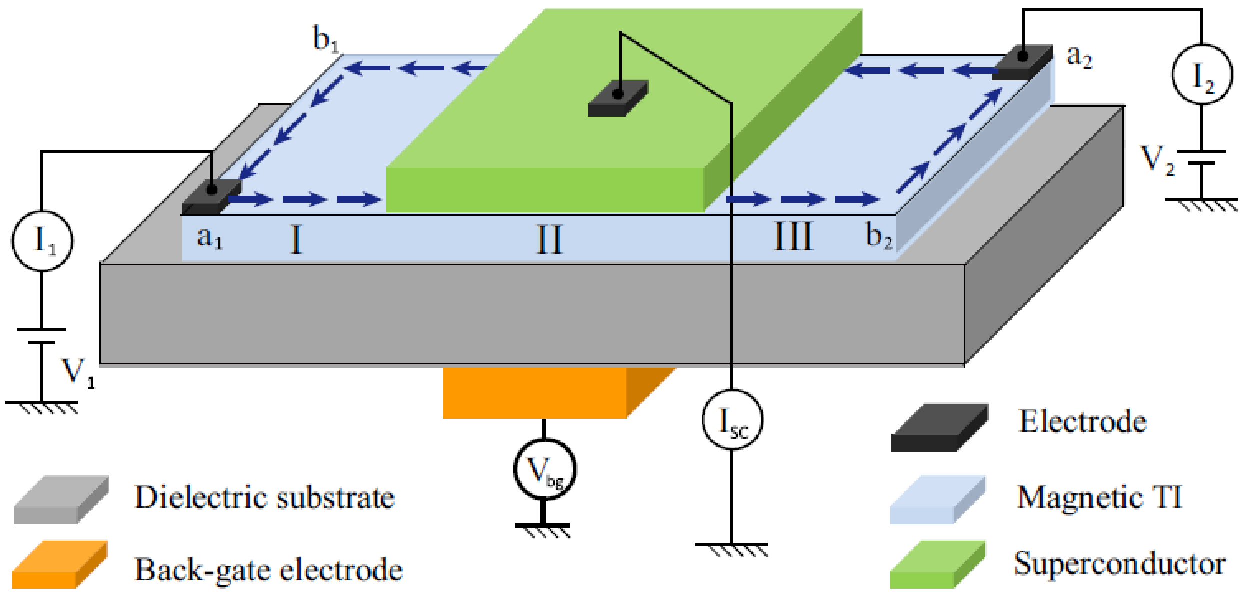
$$\begin{cases} I_1 = \frac{e^2}{h}[(1-\mathcal{R}+\mathcal{R}_A)(V_1-V_{\mathrm{SC}})-(\mathcal{T}-\mathcal{T}_A)(V_2-V_{\mathrm{SC}})] \\ -I_2 = \frac{e^2}{h}[\underbrace{(1-\mathcal{R}+\mathcal{R}_A)}_{1}(V_2-V_{\mathrm{SC}})-\underbrace{(\mathcal{T}-\mathcal{T}_A)}_{0}(V_1-V_{\mathrm{SC}})] \end{cases}$$

$$\Rightarrow \boxed{I = \frac{I_1+I_2}{2} = \frac{e^2}{2h}(V_1-V_2)\ .}$$

- When $I_{SC} = 0$, one has $I = I_1 = I_2$, and $\sigma_{12} = e^2/2h$.
- Grounding of TSC is not needed if the TSC is sufficiently large.

Realizing chiral Majorana and TSC with QAH+SC

Wang, et.al. PRB 92, 064520 (2015)

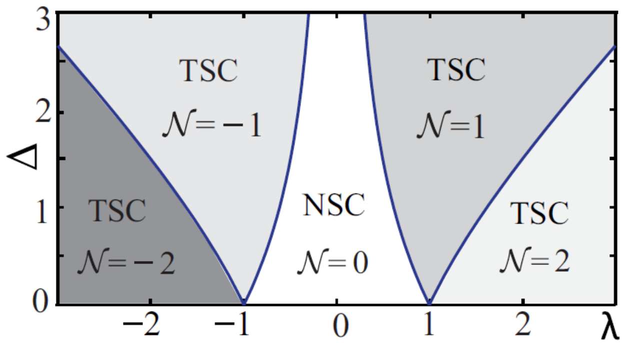


Experimental proposal

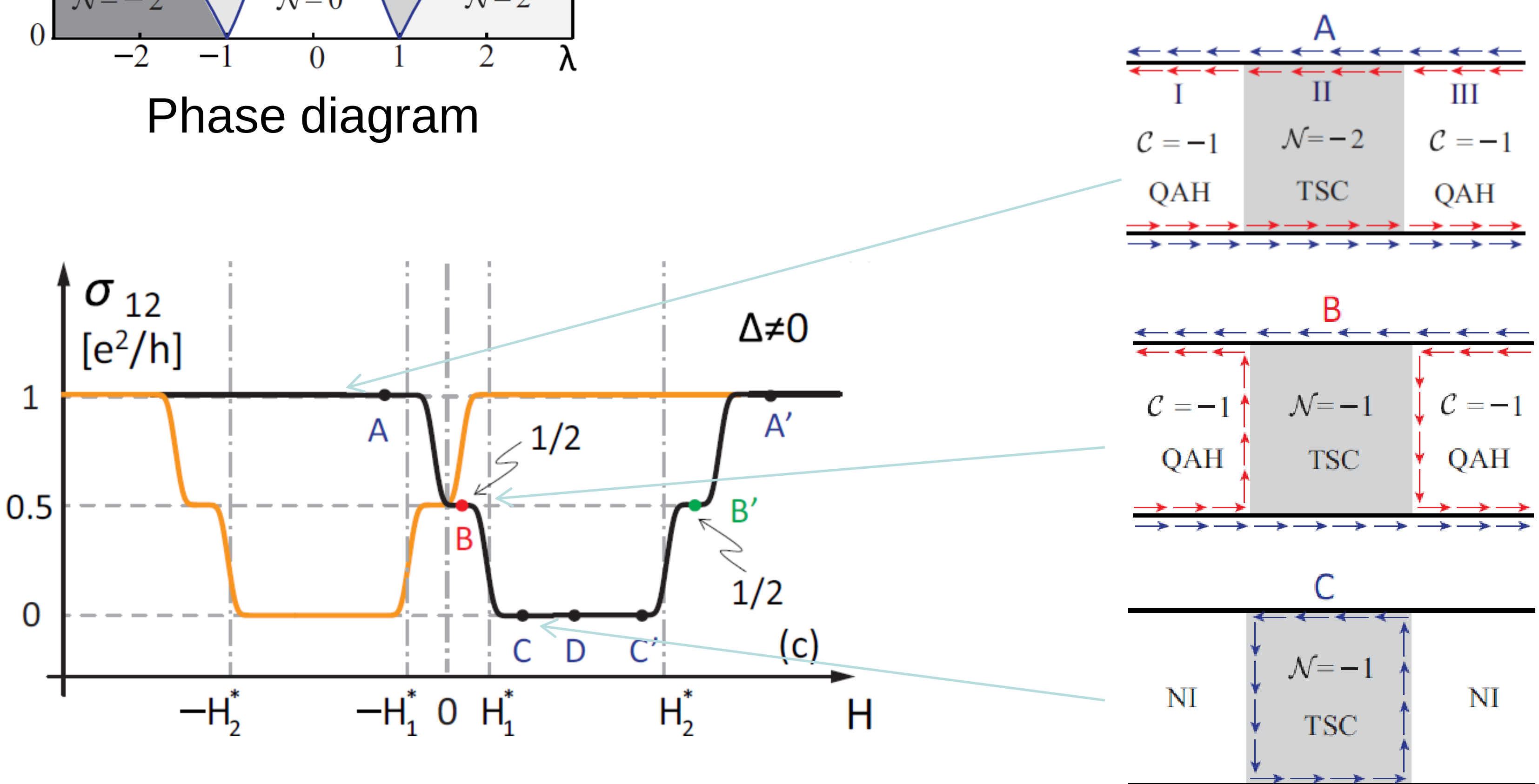
- S-wave SC covered on top of the middle region QAH sample
- Out-of-plane magnetic field applied
- Grounding of SC is not necessary if SC is large

Half conductance plateau

Exchange field λ can be tuned by magnetic field H (hysteresis), giving a half plateau in σ_{12} .



Phase diagram



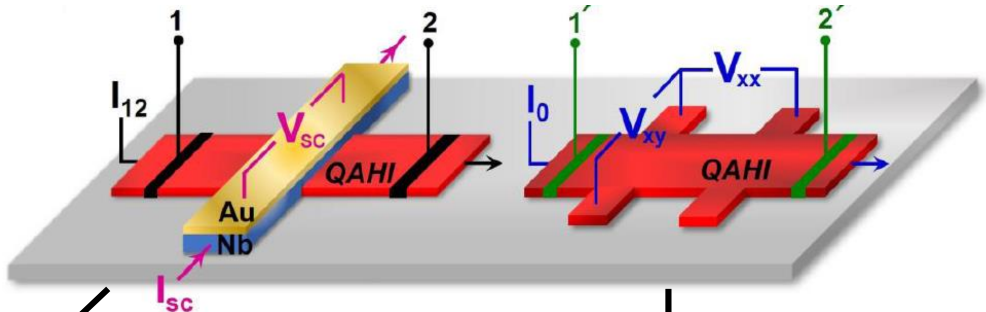
Longitudinal conductance

Experimental result

(K Wang+SC Zhang+K Liu+J Xia groups, Science)

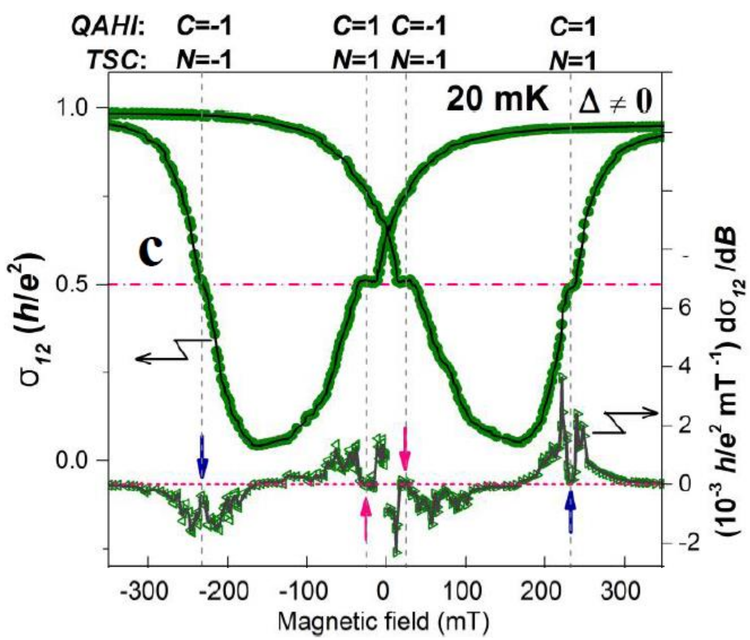
Sample: $(Cr_{0.12}Bi_{0.26}Sb_{0.62})_2Te_3$ grown on $GaAs(111)B$ supstrate, covered by Nb superconductor.

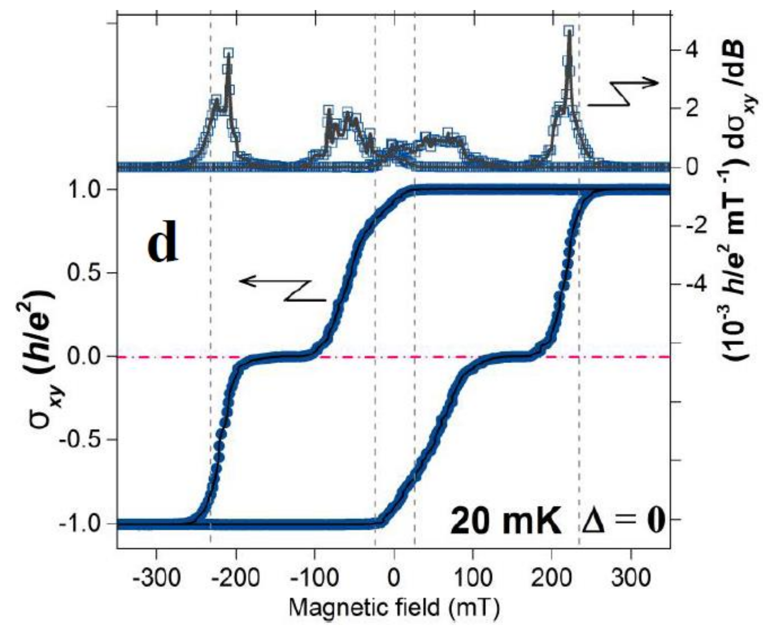
He, et.al.
Science 357, 294 (2017)



TSC measurement

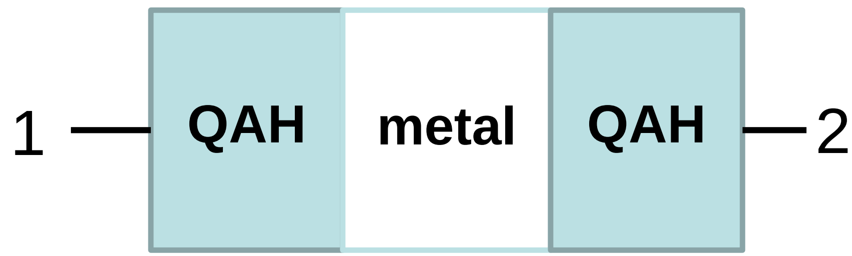
Hall measurement





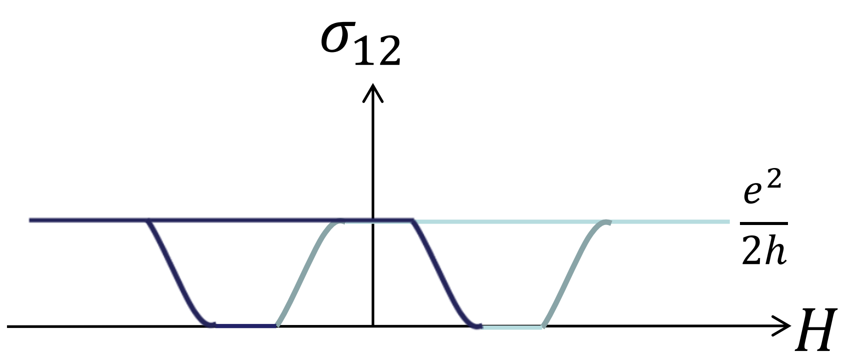
Exclusion of other explanations

The half-conductance plateau can also occur if the middle region is a metal, so that the two QAH form a series circuit:



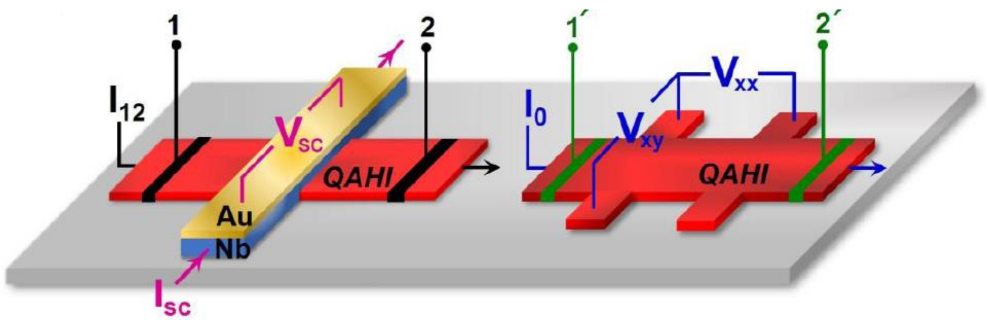
$$\sigma_{12} = \frac{e^2}{2h}$$

In this case, however, there is no integer plateau of $\sigma_{12} = e^2/h$.



Large magnetic field resistance

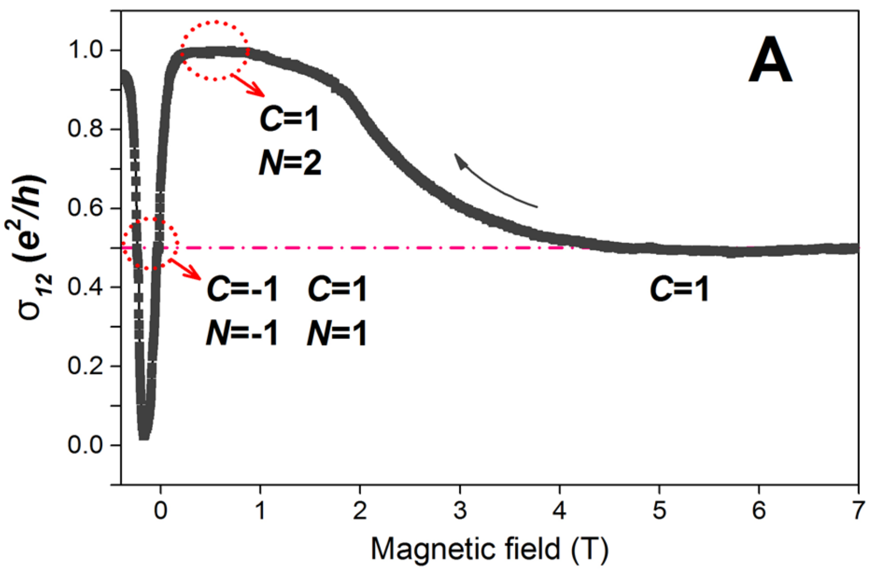
For magnetic field > *critical field of the superconductor*, the middle region becomes ***metallic***.



The system then becomes series connection of two QAH samples,

$$\Rightarrow \quad \sigma_{12} = \frac{e^2}{2h} \ .$$

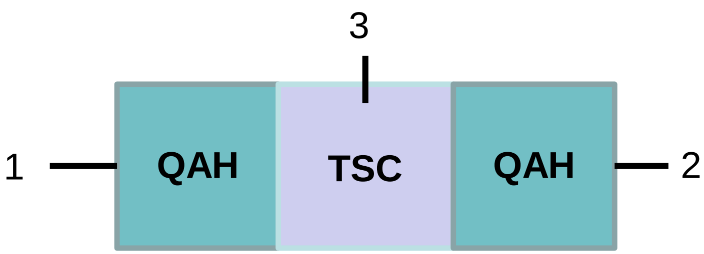
This is verified in the experiment.



Three-terminal conductance

The 3-terminal measurements can be used as a further verification.

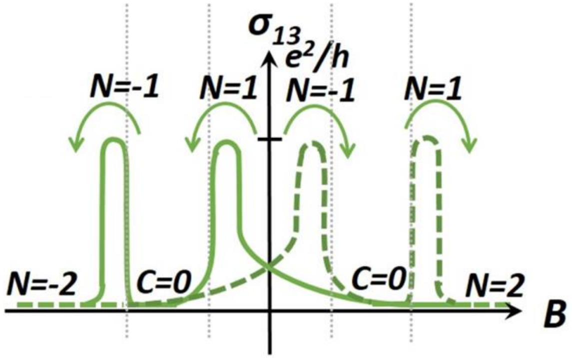
- The multi-terminal Landauer-Buttiker formula:



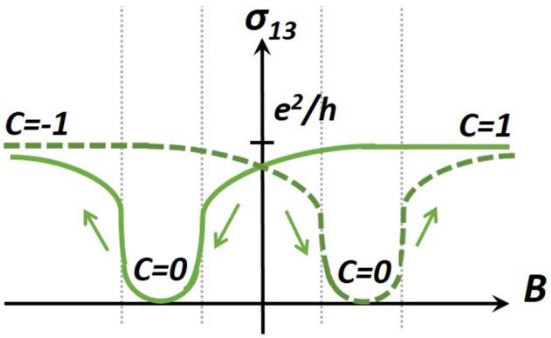
$$I_i = \frac{e^2}{h}\left[\left(k_i - R^i + R_A^i\right)\left(V_i - V_{SC}\right) - \sum_{j\neq i}\left(T^{ji} - T_A^{ji}\right)\left(V_j - V_{SC}\right)\right]$$

For middle region superconducting:

$\sigma_{13} = \sigma_{23} \leq e^2/h$ in N=1 TSC phase, $\sigma_{13} = \sigma_{23} \approx 0$ otherwise

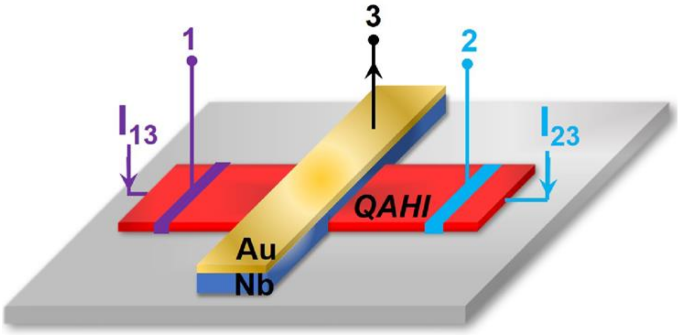


Prediction for middle region superconducting



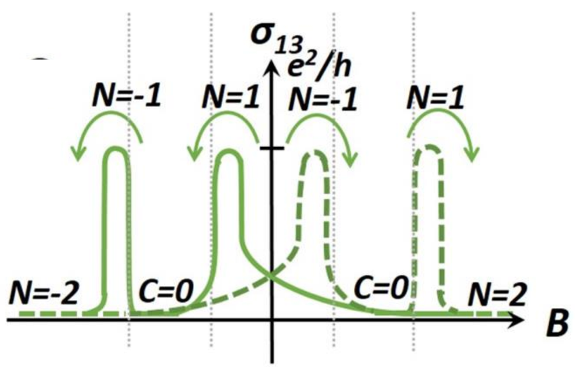
Prediction for middle region metallic

Three-terminal measurement

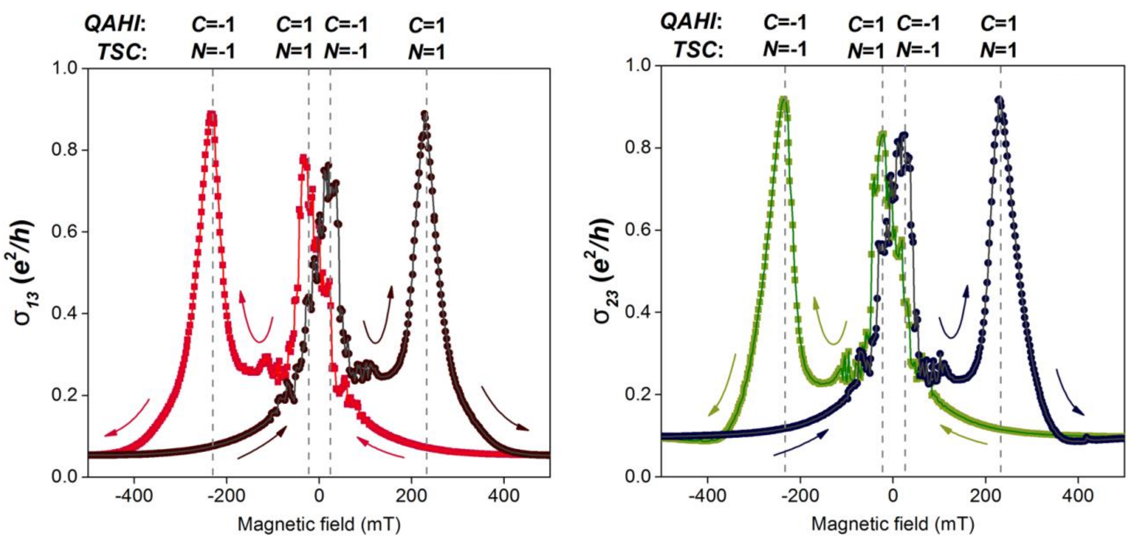


The 3-terminal measurements is also performed in the experiment, with terminal 3 implemented on Nb superconductor.

Wang & Zhang groups, Science



Theoretical prediction



Experimental measurements

Temperature Dependence of Half Plateau

The Majorana edge fermion takes the form:

$$\psi_k = u_k a_k + v_k a_k^\dagger$$

The scattering matrix is

$$\begin{pmatrix} a_{1,\mathbf{k}} \\ a^\dagger_{1,-\mathbf{k}} \\ a_{2,\mathbf{k}} \\ a^\dagger_{2,-\mathbf{k}} \end{pmatrix} = \frac{1}{2} \begin{pmatrix} r & r & t+1 & t-1 \\ r & r & t-1 & t+1 \\ t^*+1 & t^*-1 & -r^* & -r^* \\ t^*-1 & t^*+1 & -r^* & -r^* \end{pmatrix} \begin{pmatrix} b_{1,\mathbf{k}} \\ b^\dagger_{1,-\mathbf{k}} \\ b_{2,\mathbf{k}} \\ b^\dagger_{2,-\mathbf{k}} \end{pmatrix}$$

- At zero energy, $u_k = v_k = 1/\sqrt{2}$, $r = 1,\ t = 0$, and ψ_k is charge neutral.
- At energy $\epsilon,\ \ u_k \neq v_k,\ \ r = r(\epsilon)$ and $t = t(\epsilon) = c_1\epsilon + c_2\epsilon^2 + \cdots$.

The conductance at temperature T is

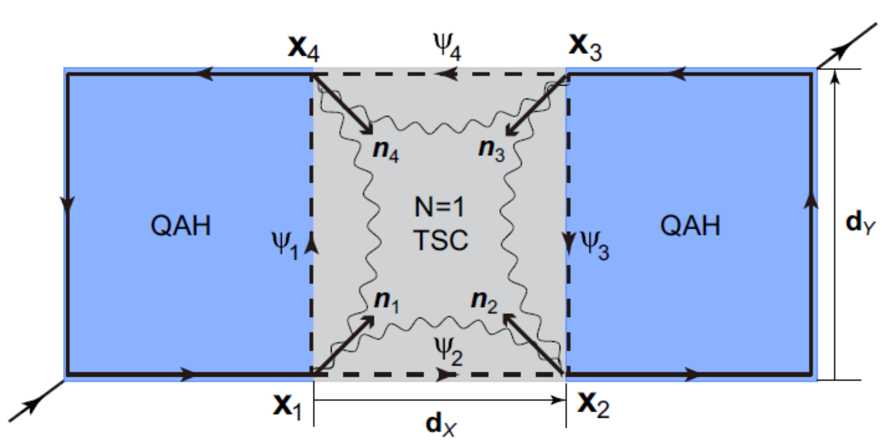
$$\sigma_{12} = \frac{e^2}{h}\int d\epsilon \left(-\frac{df}{d\epsilon}\right)\frac{1+t(\epsilon)}{2} \approx \frac{e^2}{2h}(1+\alpha T^2)$$

Here $f(\epsilon)$ is the Fermi-Dirac distribution.

Supercurrent Contribution

The bulk supercurrent is described by a boson field $\theta(\mathbf{x})$. The only allowed coupling of Majorana edge fermions with $\theta(\mathbf{x})$ is

$$H_{\text{eff}} = \frac{1}{2g}\int_{\mathcal{M}_{\text{sc}}} d^2\mathbf{x}\left[(\partial_t\theta)^2 + v_s^2(\nabla\theta)^2\right] - i v_F \sum_{i=1}^{4}\left[(\psi_i\psi_{i+1}\mathbf{n}_i\cdot\nabla\theta)_{\mathbf{x}_i} + \int_{\partial_i\mathcal{M}_{\text{sc}}} d\ell\,\psi_i\partial_\ell\psi_i\right]$$



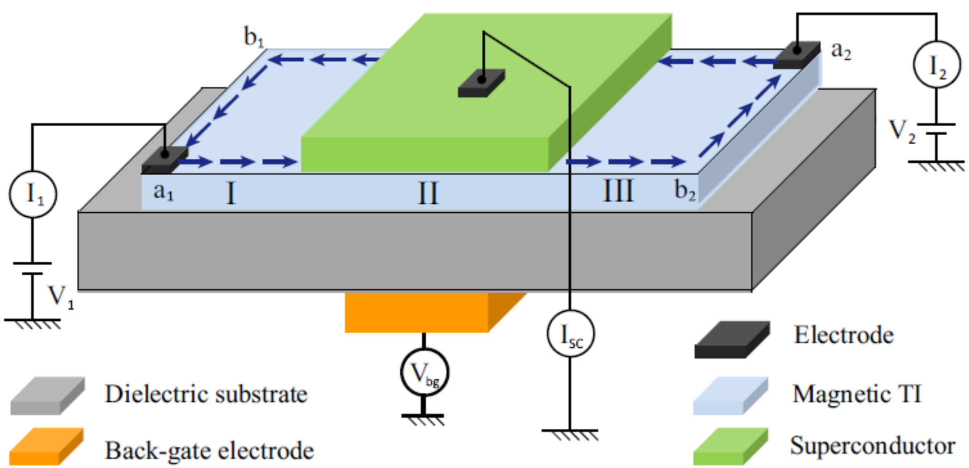
This leads to a **correction to the conductance**

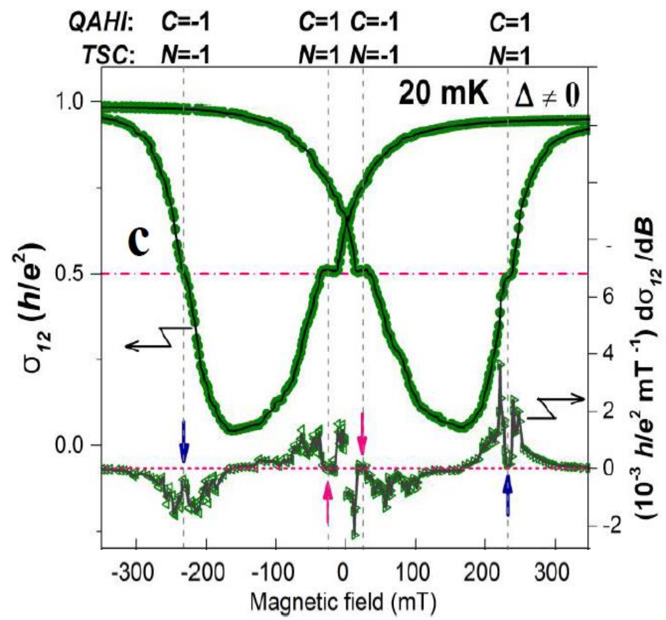
$$\delta\sigma_{12} = \frac{e^2}{2h}\frac{g\hbar}{16\pi^2 v_s}\sum_{ij}\langle \mathbf{n}_i\cdot\nabla\theta(\mathbf{x}_i)\mathbf{n}_j\cdot\nabla\theta(\mathbf{x}_j)\rangle \propto \frac{1}{L^3}\ ,$$

L is the size of TSC.

- For $L \sim 1\mu m$, $\delta\sigma_{12} \sim 10^{-6}e^2/h$.

After 80 years of searching, chiral Majorana fermion has been discovered!





Xiao-Liang Qi, Taylor L. Hughes, Shou-Cheng Zhang, “Chiral Topological Superconductor From the Quantum Hall State", Phys. Rev. B82, 184516 (2010).

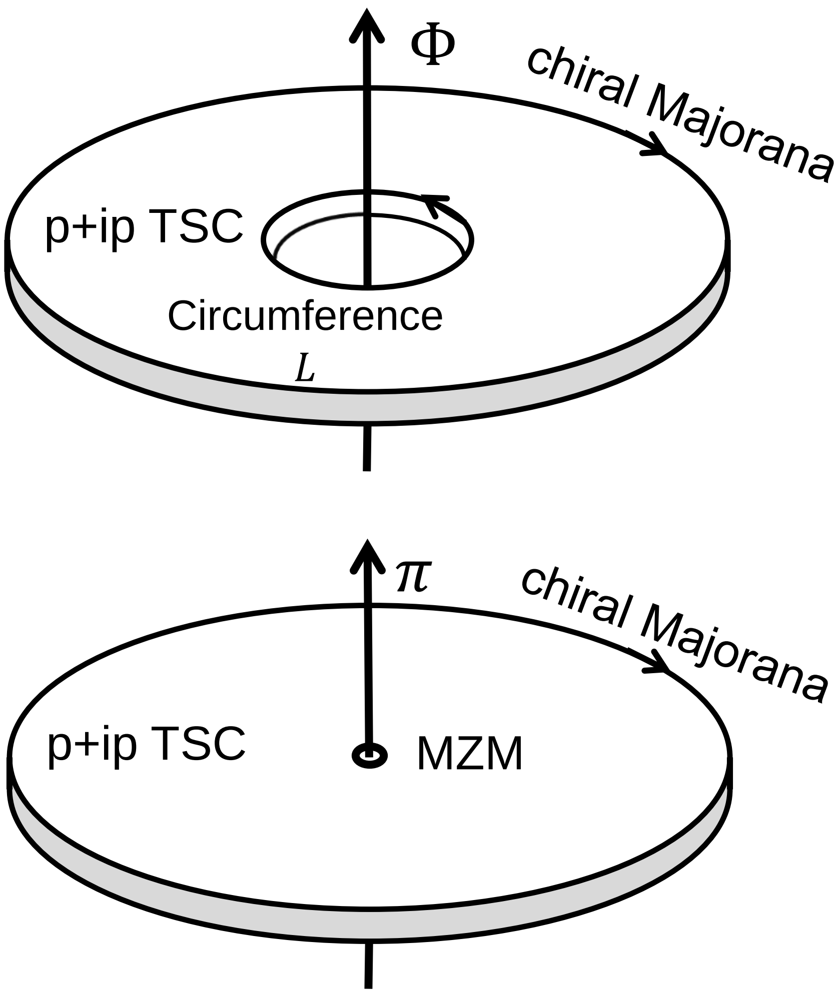
Suk Bum Chung, Xiao-Liang Qi, Joseph Maciejko, Shou-Cheng Zhang, ``Conductance and noise signatures of Majorana backscattering", Phys. Rev. B83, 100512(R) (2011).

Jing Wang, Quan Zhou, Biao Lian and Shou-Cheng Zhang, ``Chiral topological superconductor and half-integer conductance plateau from quantum anomalous Hall plateau transition", Physical Review B, 92, 064520 (2015).

Qing. Lin He, Lei Pan, Alexander. L Stern, Edward Burks, Xiaoyu Che, Gen Yin, Jing Wang, Biao Lian, Quan Zhou and Eun. Sang Choi, Koichi Murata, Xufeng Kou, Tianxiao Nie, Qiming Shao, Yabin Fan, Shou-Cheng Zhang, Kai Liu, Jing Xia and Kang L. Wang, ``Chiral Majorana edge state in a quantum anomalous Hall insulator-superconductor structure", Science 357, 294 (2017)

Chiral Majorana Edge State & Majorana Zero mode

A single chiral Majorana edge state implies the presence of ***Majorana zero mode*** (MZM) in a π flux superconducting vortex.

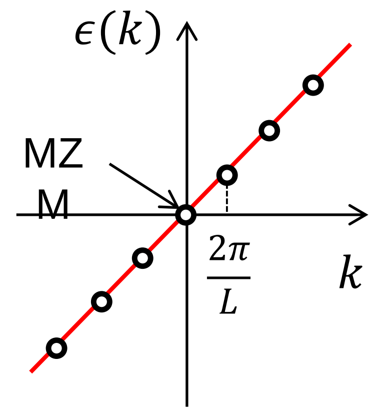


Momentum quantization:

$$k = \frac{2\pi n + \pi + \Phi}{L}, \qquad n \in \mathbb{Z}$$

2π spin rotation

magnetic flux



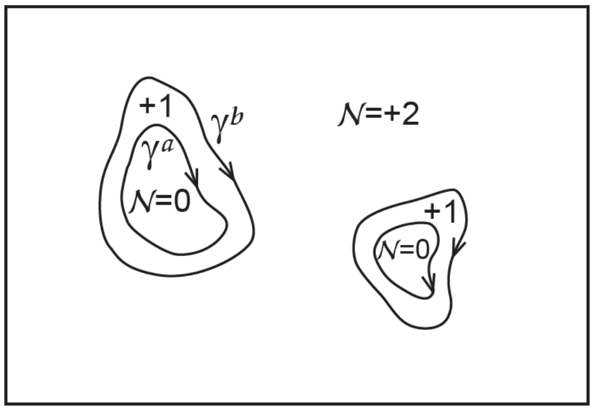
Majorana operators:

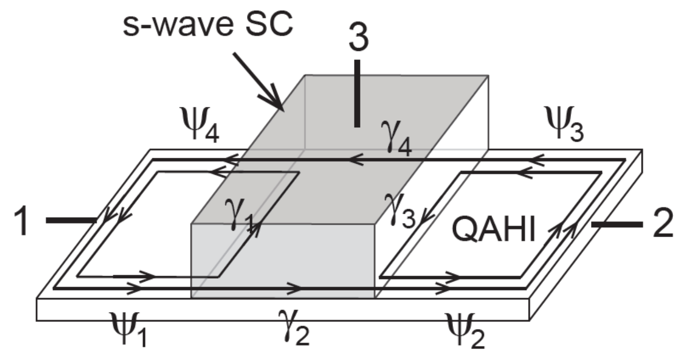
$$\gamma_k = \gamma_{-k}^\dagger \,, \qquad \gamma_0 = \gamma_0^\dagger$$

TSC in the presence of disorder

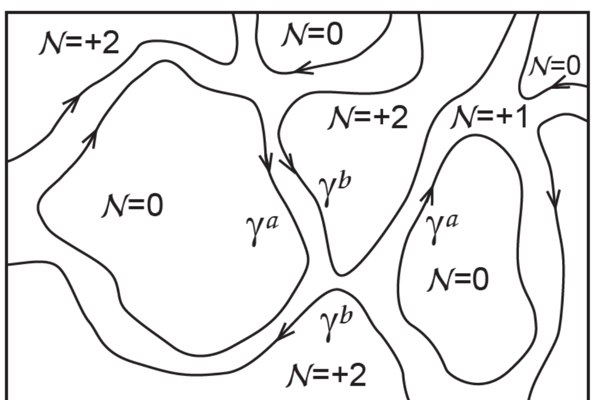
The chiral TSC phase is robust against disorders, as described by the percolation theory in the D symmetry class.

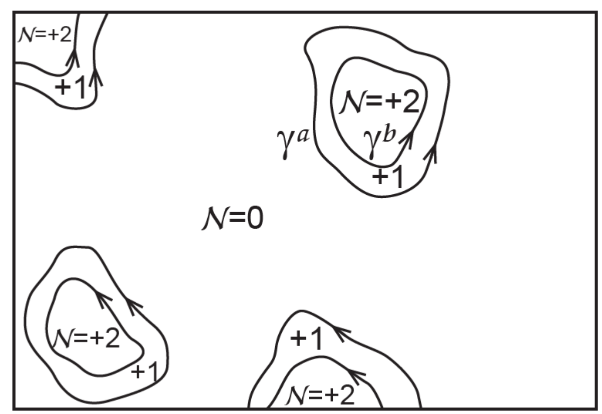
Zhang group,
arXiv:1709.05558





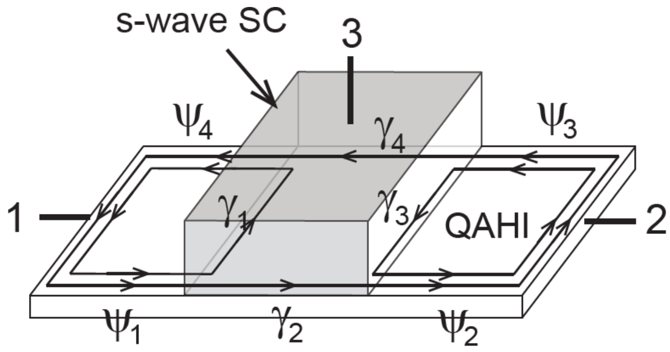
p+ip
TSC





Critical behavior of half plateau

The percolation theory determines the critical behaviors of the σ_{12} **half plateau & plateau transitions** due to chiral TSC.



Size L & temperature T dependence:

- $e^2/2h$ to e^2/h:

transition of middle region from TSC to QAHI

$$\frac{d\sigma_{12}}{dB} \propto L^{1/\nu_D} \text{ or } T^{-p'/2\nu_D} \ , \ \nu_D \approx 1$$

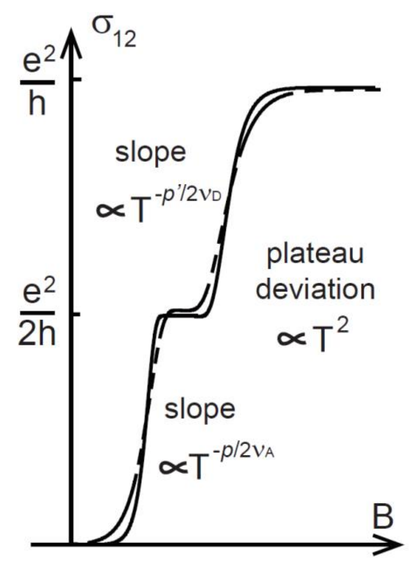
- 0 to $e^2/2h$:

transition of left & right region from NI to QAHI

$$\frac{d\sigma_{12}}{dB} \propto L^{1/\nu_A} \text{ or } T^{-p/2\nu_A} \ , \ \nu_A \approx 7/3$$

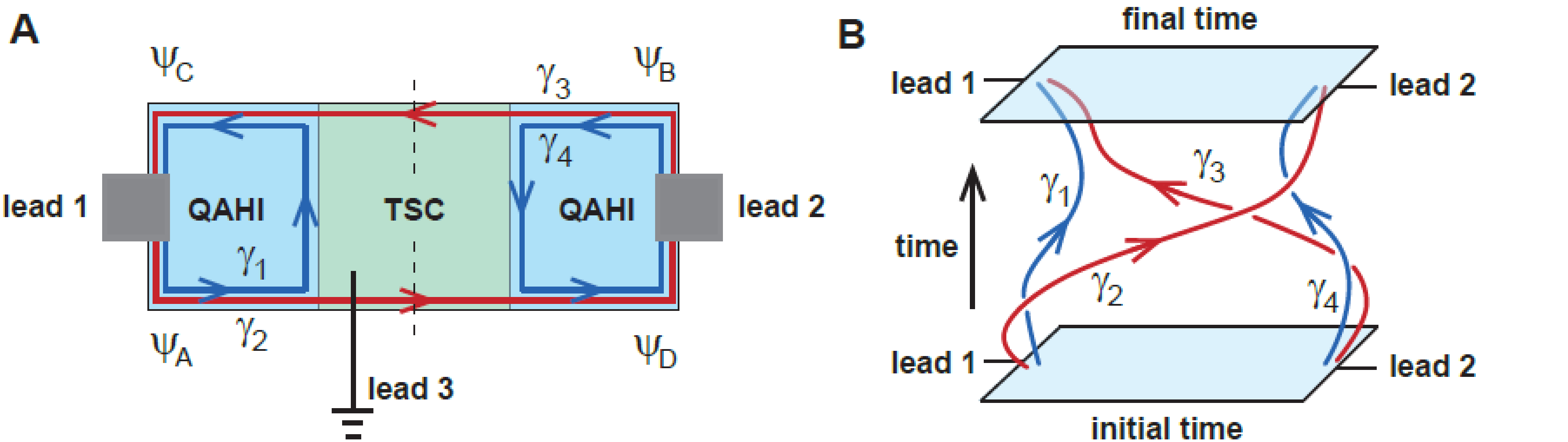
- deviation of half plateau:

$$\delta\sigma_{12} \propto T^2$$

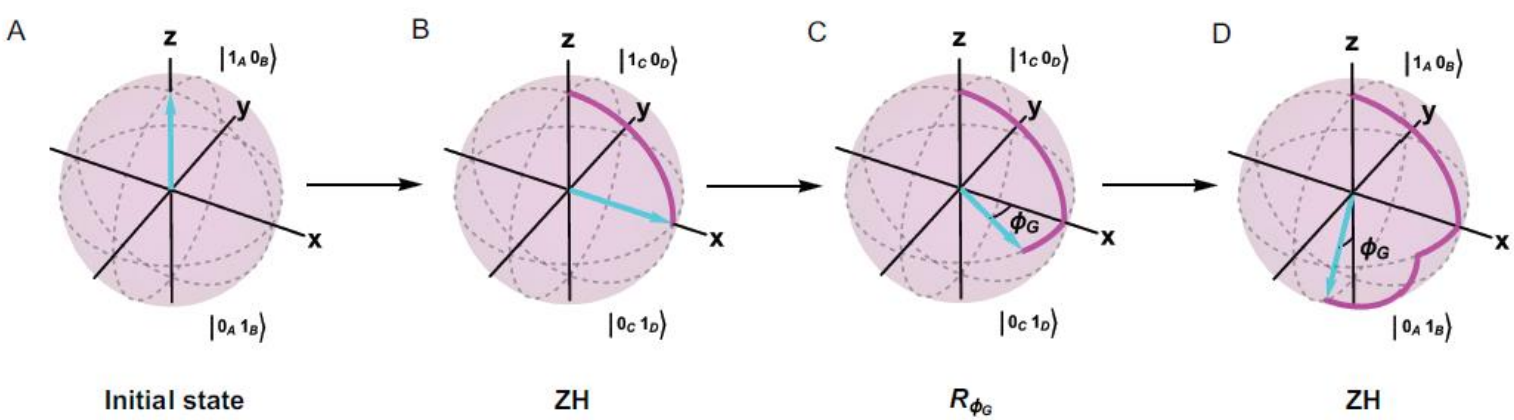


Zhang group, to appear

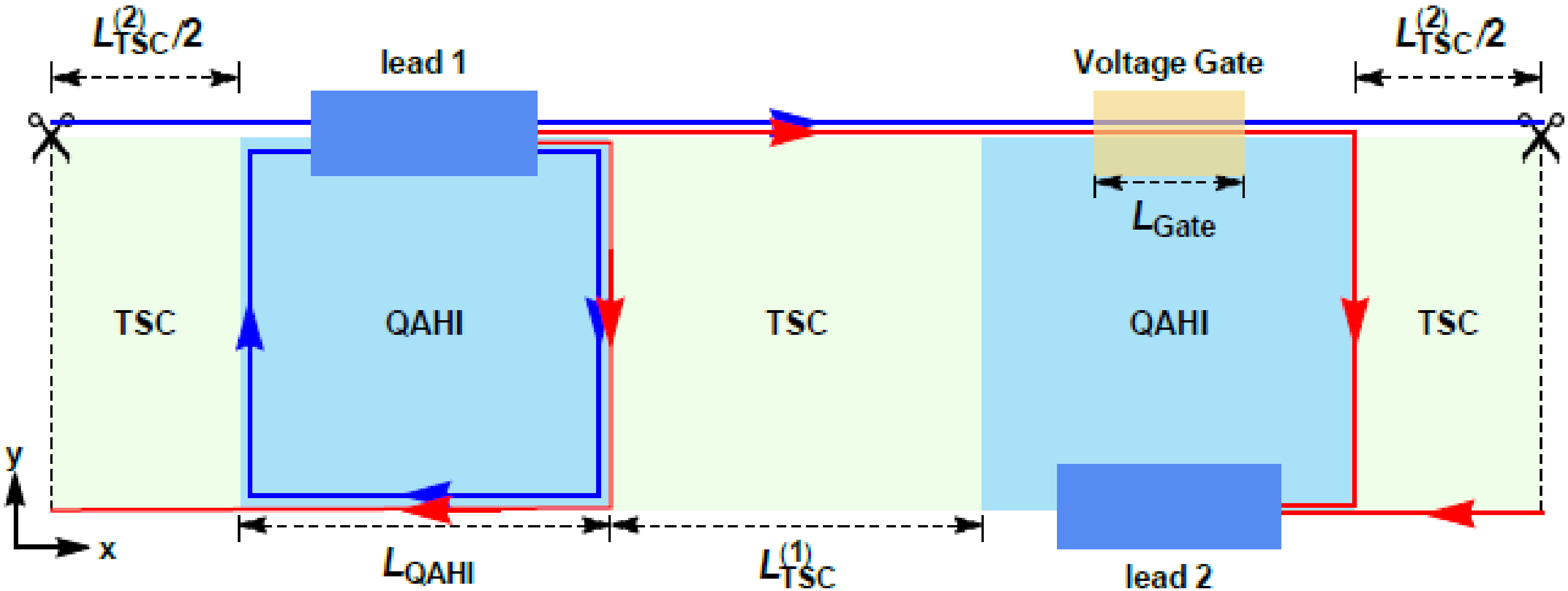
Nonabelian braiding of chiral Majorana fermion



We can split one qubit into two chiral Majorana fermions. Natural propagation leads to non-abelian braiding. (Zhang group, arXiv:1712.06156)

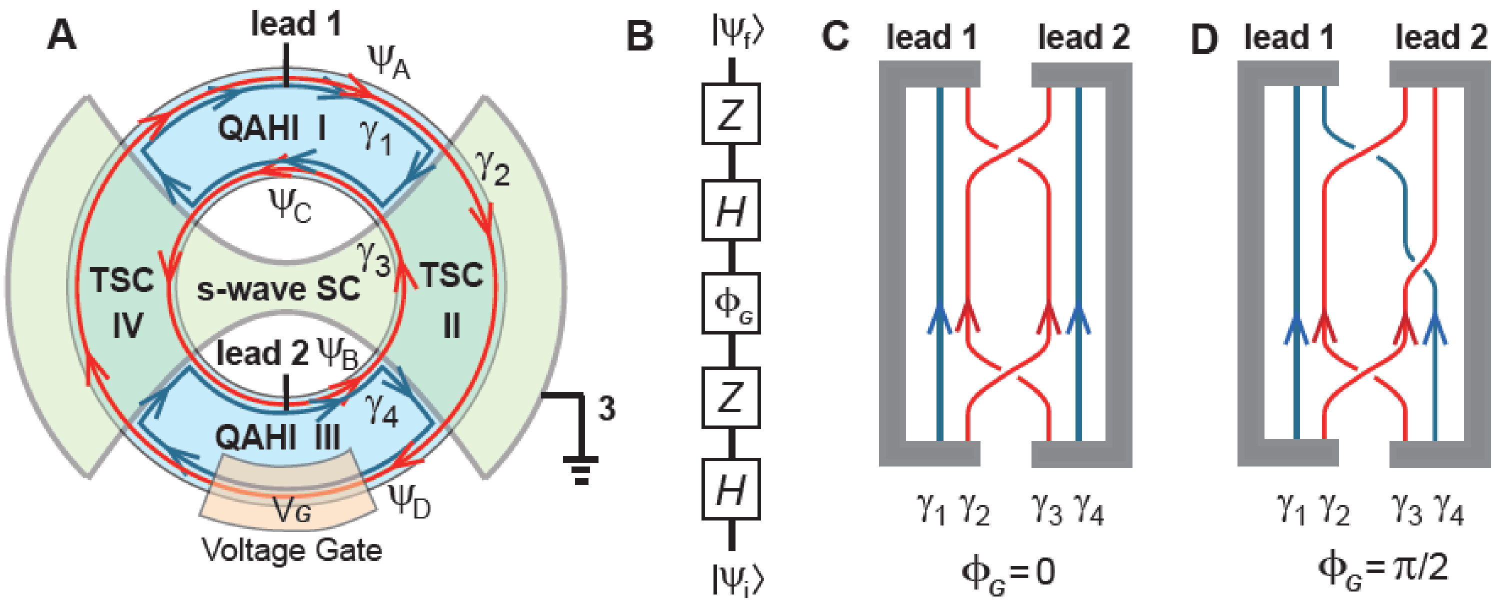


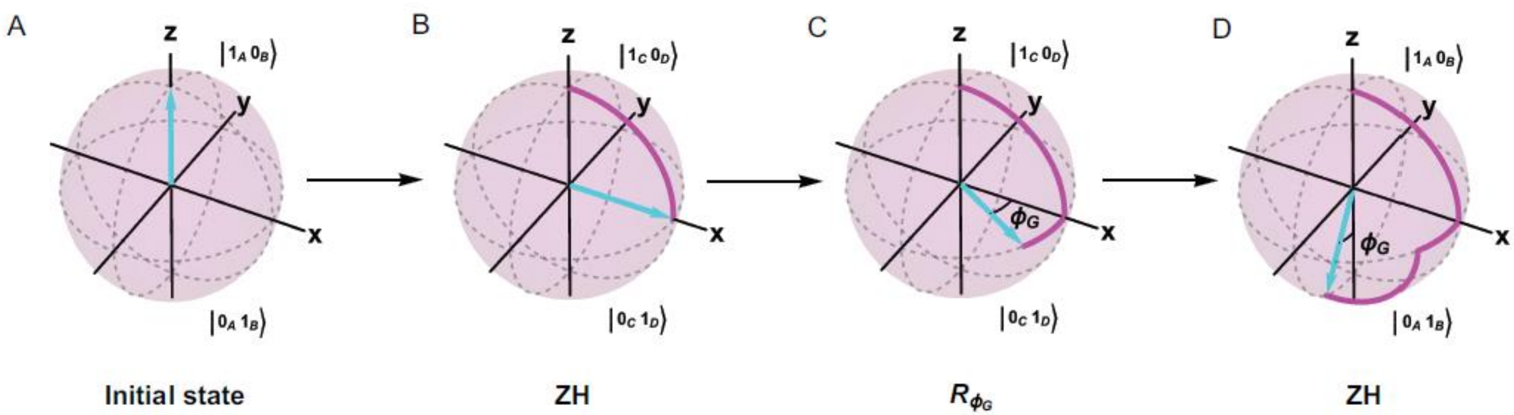
Voltage gate tuning of the phase



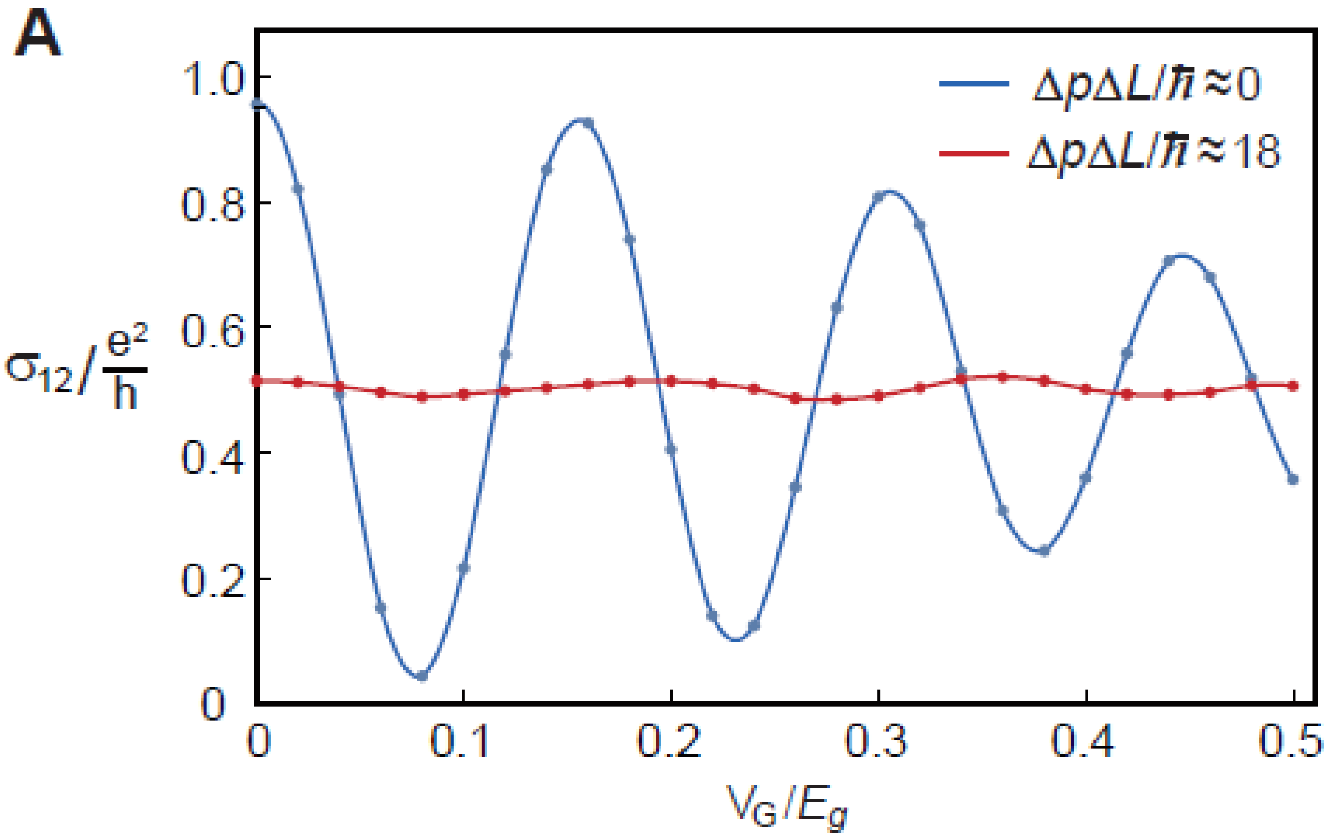
A gate voltage applied to the QAH edge states introduces an additional phase. Applying periodic boundary condition to the TSC region leads to Corbino geometry.

Coherent braiding of chiral Majorana fermion demonstrated via Corbino geometry



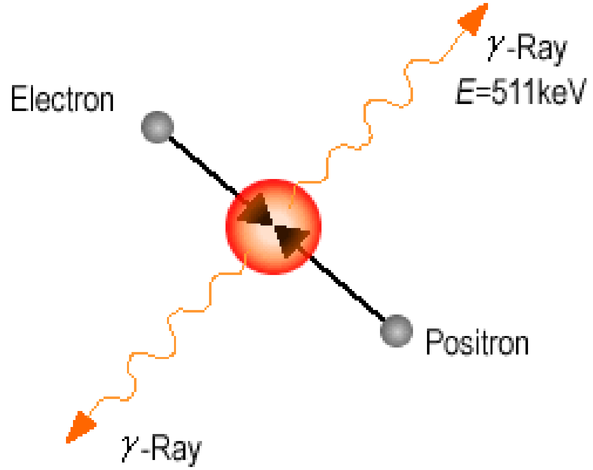


Theoretical prediction of quantum oscillation



Angel particle

- In Dan Brown's book "Angels and Demons", the weight of the positron matter is $M(e^+) \sim 5 * 10^{-6}\ g$ yet the energy released is comparable to tons of TNT!



- Angel particle: we discovered a perfect world, with only angels, no demons! Angel particles may finally make quantum computers possible, bringing great benefit to humanity.