Braiding Majorana zero modes in spin space: from worldline to worldribbon

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KITS

Acknowledgements

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- Xiaopeng Li (Fudan University)
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  J. D. Sau (University of Maryland)
- Fundings:

Students:

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Li Chen (HUST)

Ying-Ping He (PKU)

Jeffrey Ting-Fung Poon (PKU)

Outline

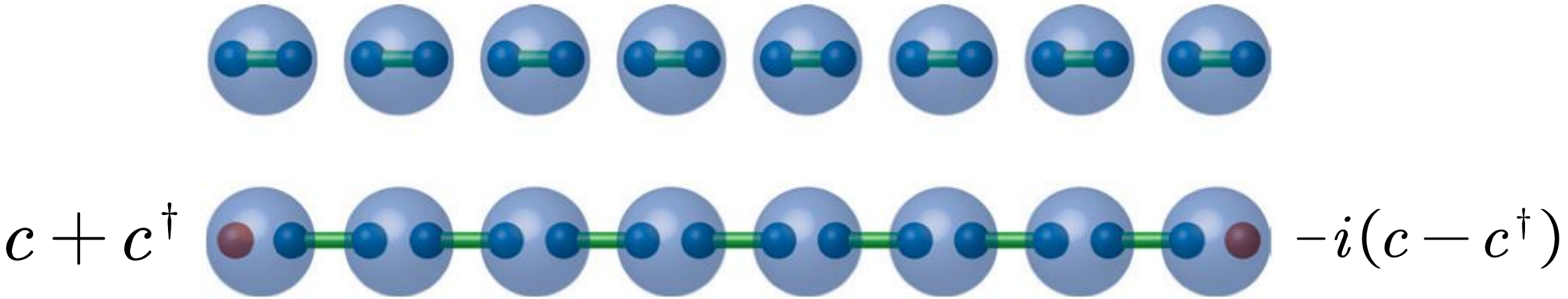
- The recent progresses in detecting and braiding Majorana zero modes
- Majorana spin properties and their applications in detecting and braiding
- Spin-statistics theorem and ribbon equation: from worldline to worldribbon
- Braiding Majorana zero modes in spin space

Majorana fermions in condensed matter

Unpaired Majorana fermions in quantum wires

A Yu Kitaev

2001 Phys.-Usp. 44 131

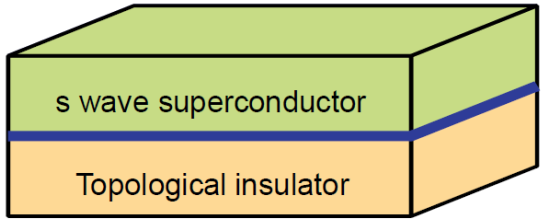


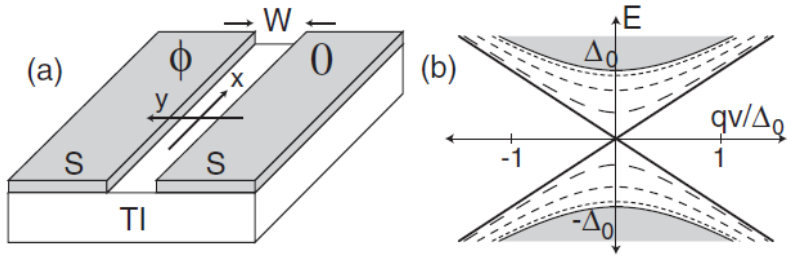
Proximity induced topological superconductors

- Theory

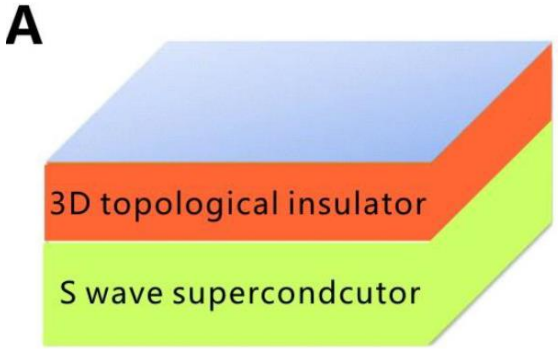
$$\mathcal{H} = -iv\tau^z\sigma\cdot\nabla - \mu\tau^z + \Delta_0(\tau^x\cos\phi + \tau^y\sin\phi).$$

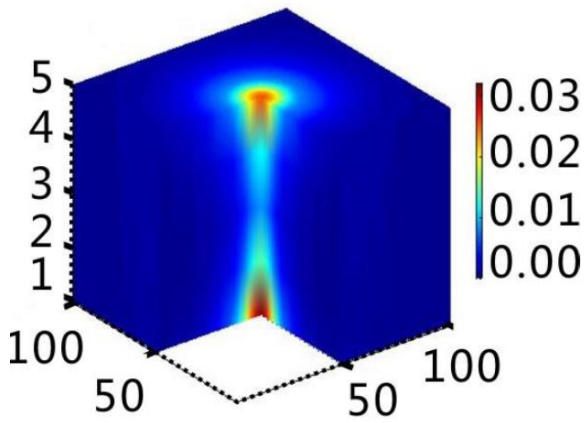
Liang Fu and C. L. Kane (2008)

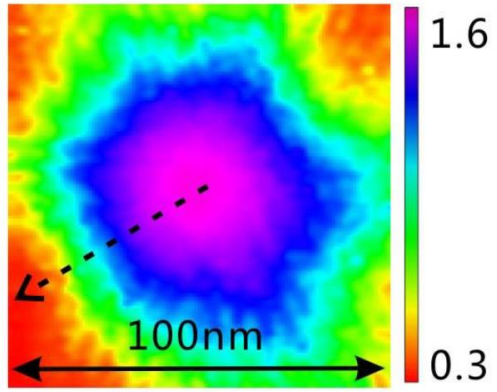


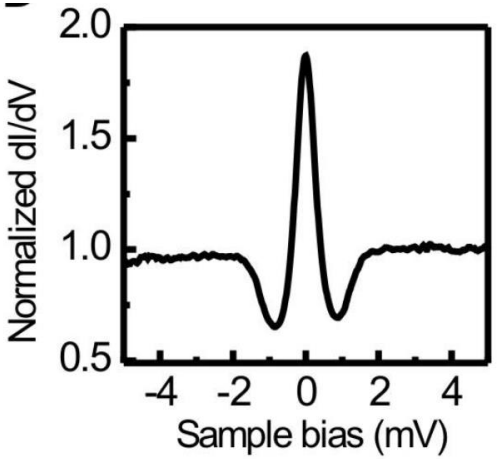


- Experimental observation





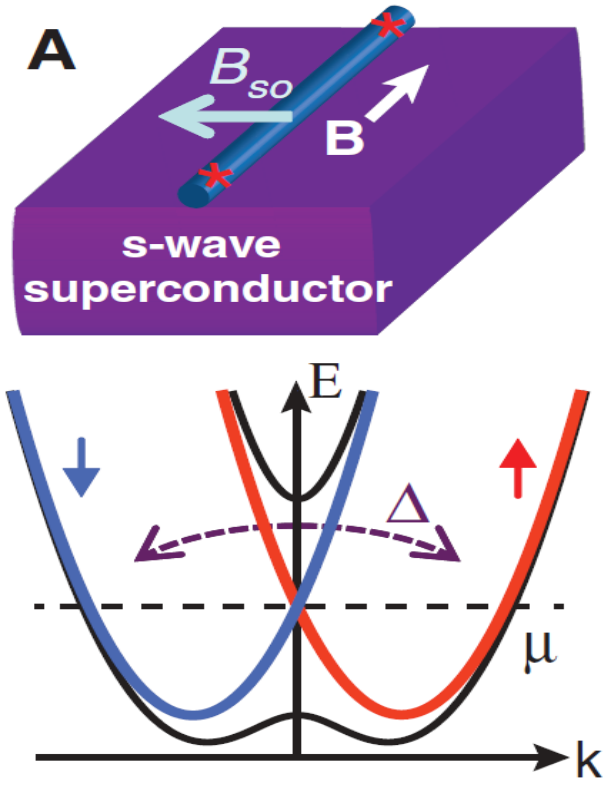


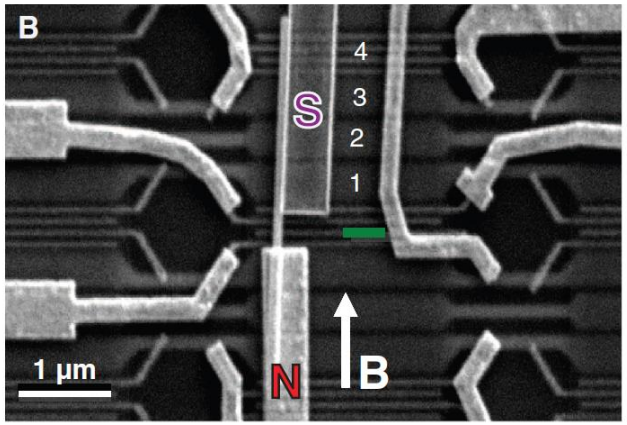


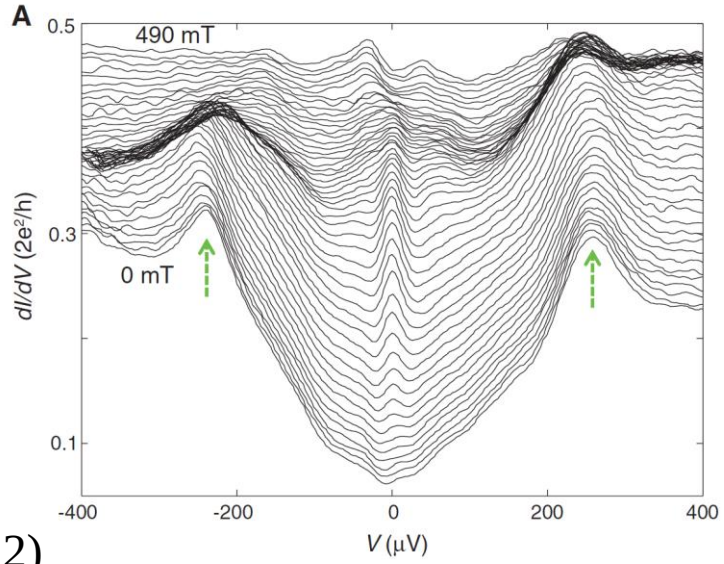
Jin-Peng Xu …, Jin-Feng Jia, PRL (2015)

Proximity induced topological superconductors

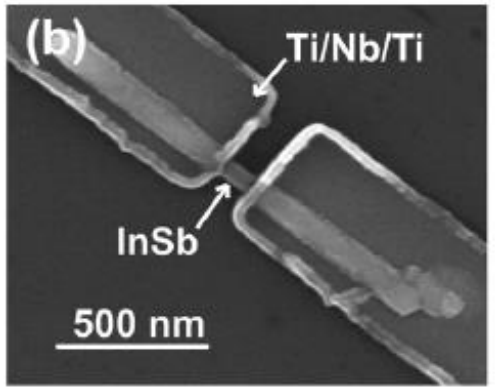
Theory: Sau *et al*, PRL (2010), Lutchyn *et al*, PRL (2010), Oreg *et al*, PRL (2010), Alice, PRB (2010), …..



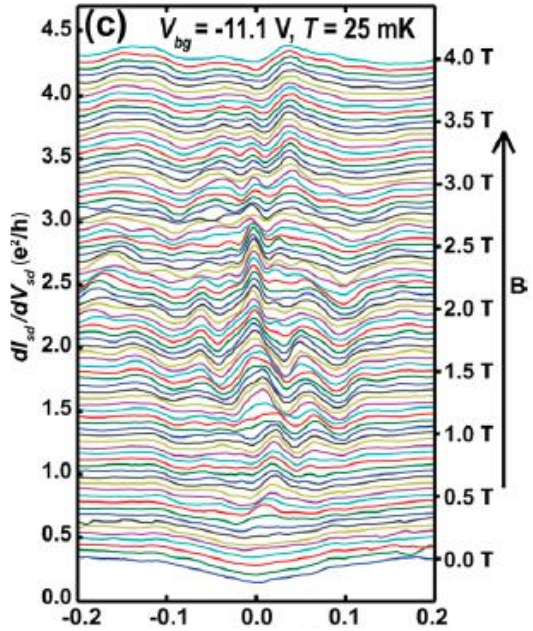




Mourik et al, … Kouwenhoven (2012)

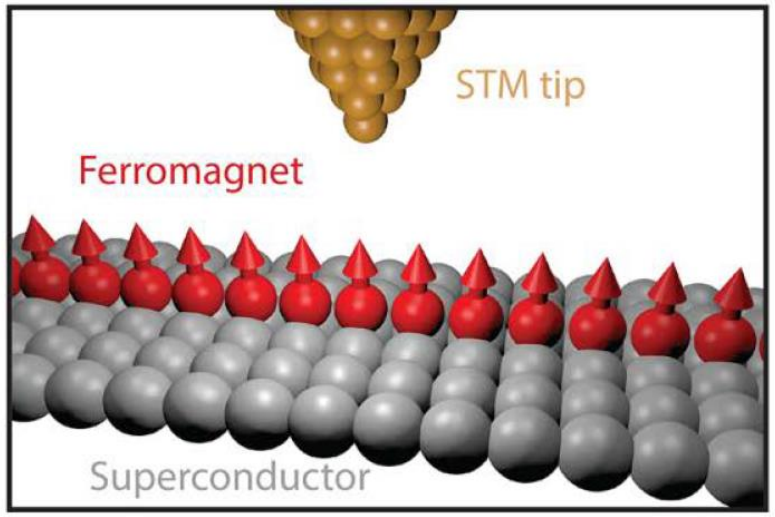


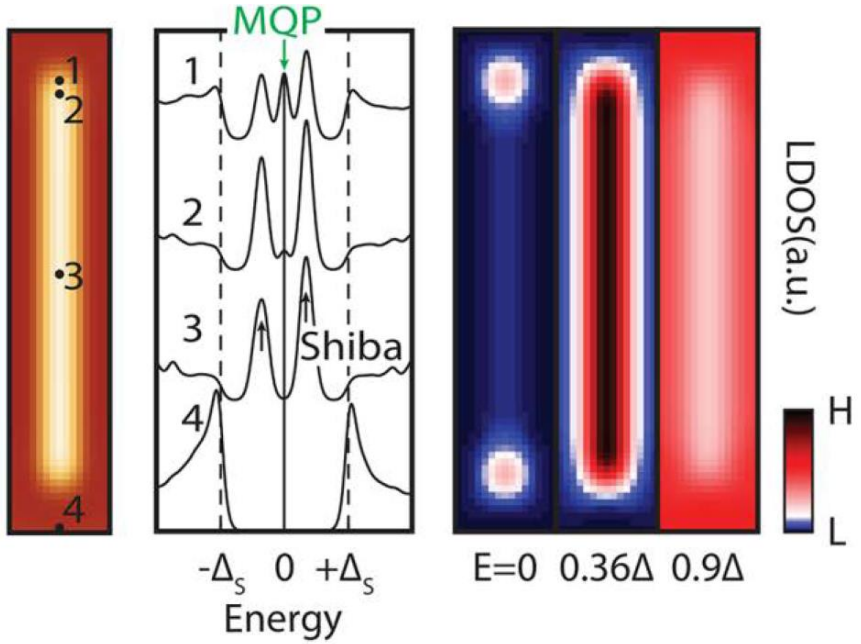
M. T. Deng, … H. Q. Xu (2012)



Proximity induced topological superconductors

- Ferromagnetic chain + spin-orbit coupled superconductors





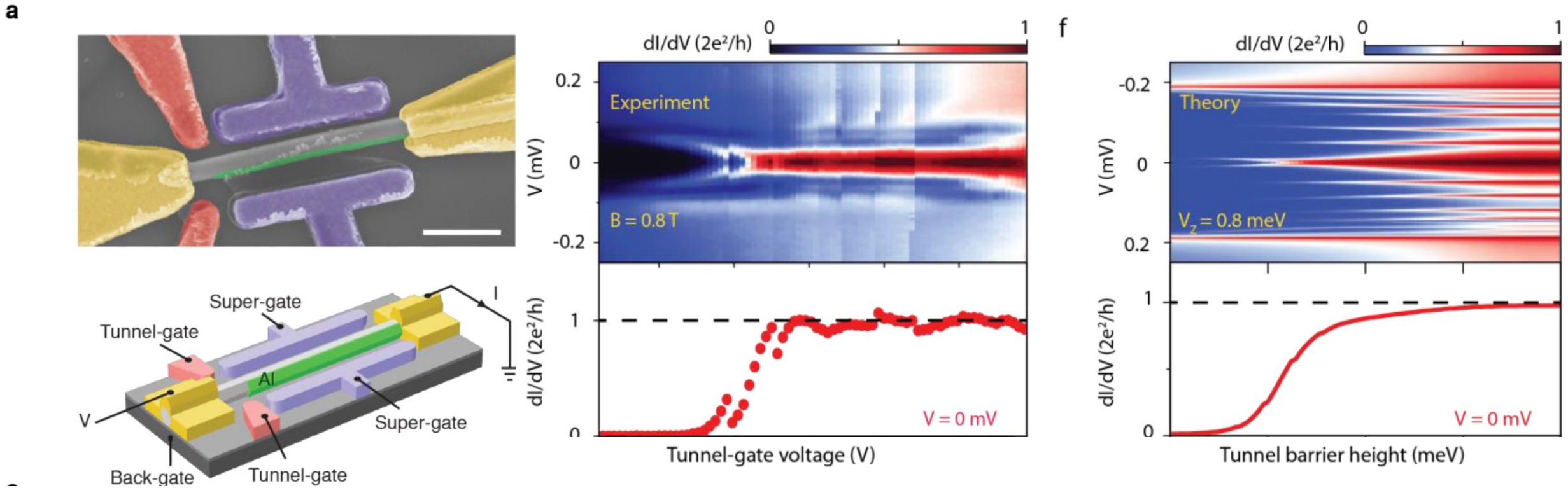
Stevan Nadj-Perge, et. al, Science (2015)

Detecting Majorana: zero bias peak

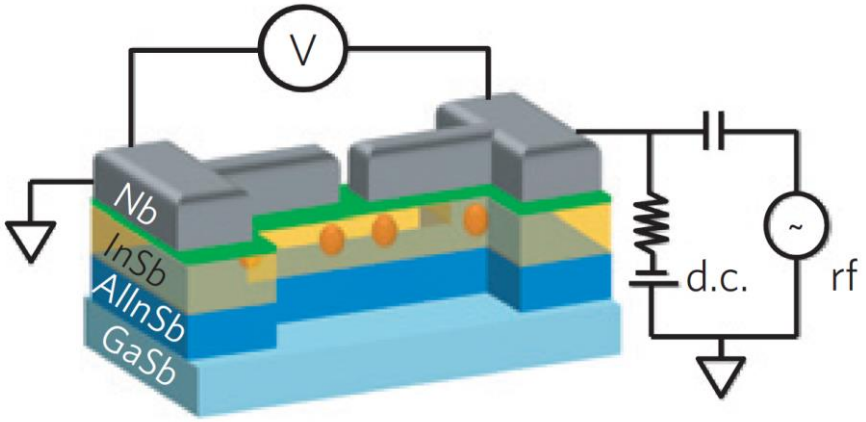
Quantized Majorana Conductance

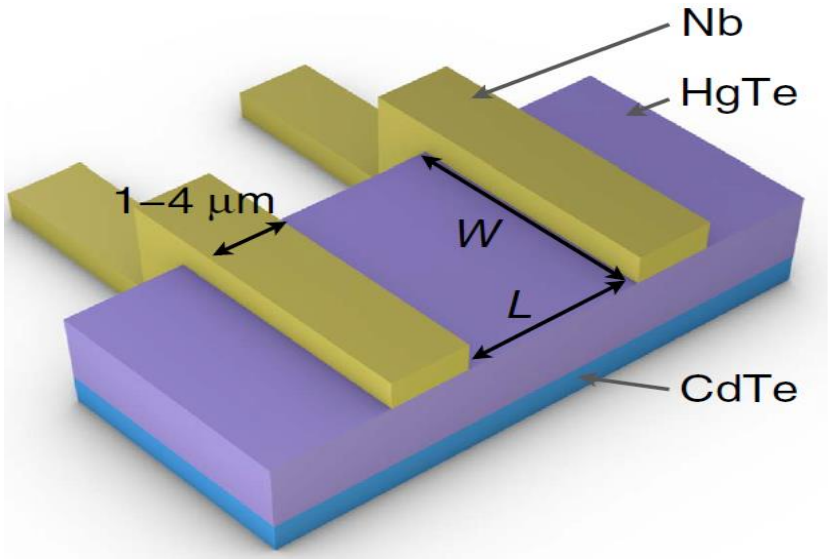
Hao Zhang[1]*, Chun-Xiao Liu[2]*, Sasa Gazibegovic[3]*, Di Xu[1], John A. Logan[4], Guanzhong Wang[1], Nick van Loo[1], Jouri D.S. Bommer[1], Michiel W.A. de Moor[1], Diana Car[3], Roy L. M. Op het Veld[3], Petrus J. van Veldhoven[3], Sebastian Koelling[3], Marcel A. Verheijen[3,7], Mihir Pendharkar[5], Daniel J. Pennachio[4], Borzoyeh Shojaei[4,6], Joon Sue Lee[6], Chris J. Palmstrøm[4,5,6], Erik P.A.M. Bakkers[3], S. Das Sarma[2], Leo P. Kouwenhoven[1,8]

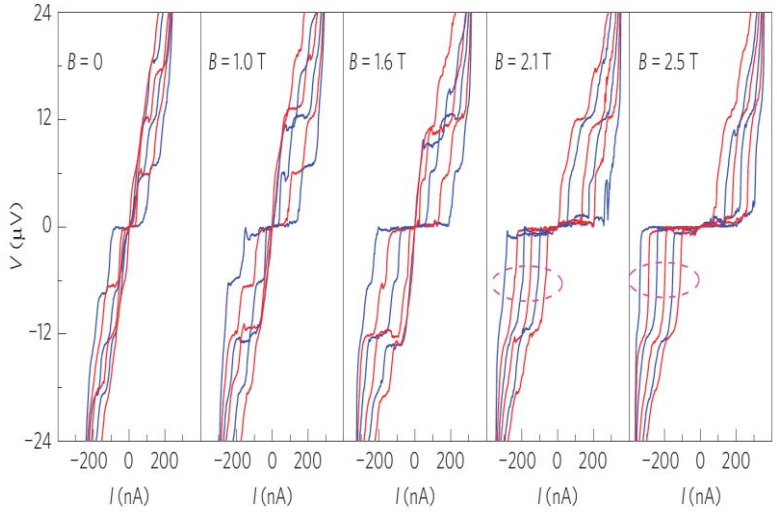
arXiv.1710.10701



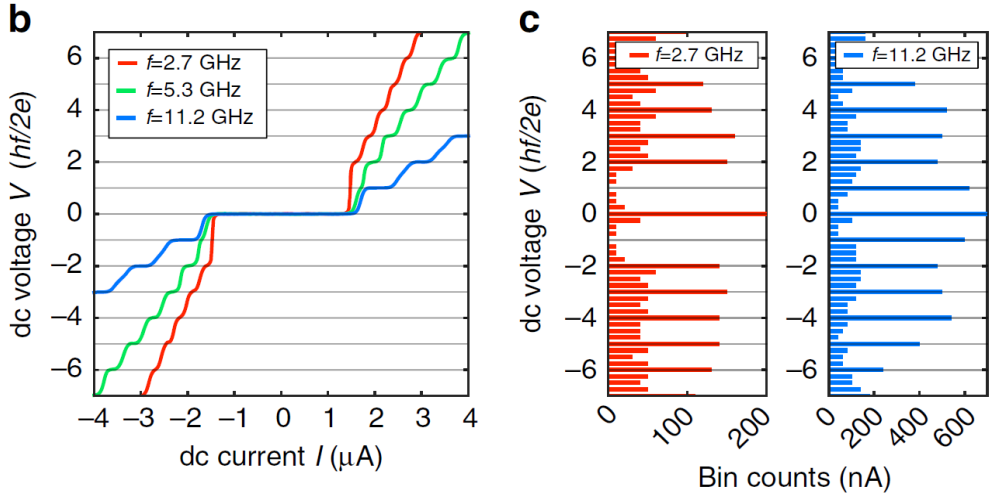
Detecting Majorana:4π Josephson effect





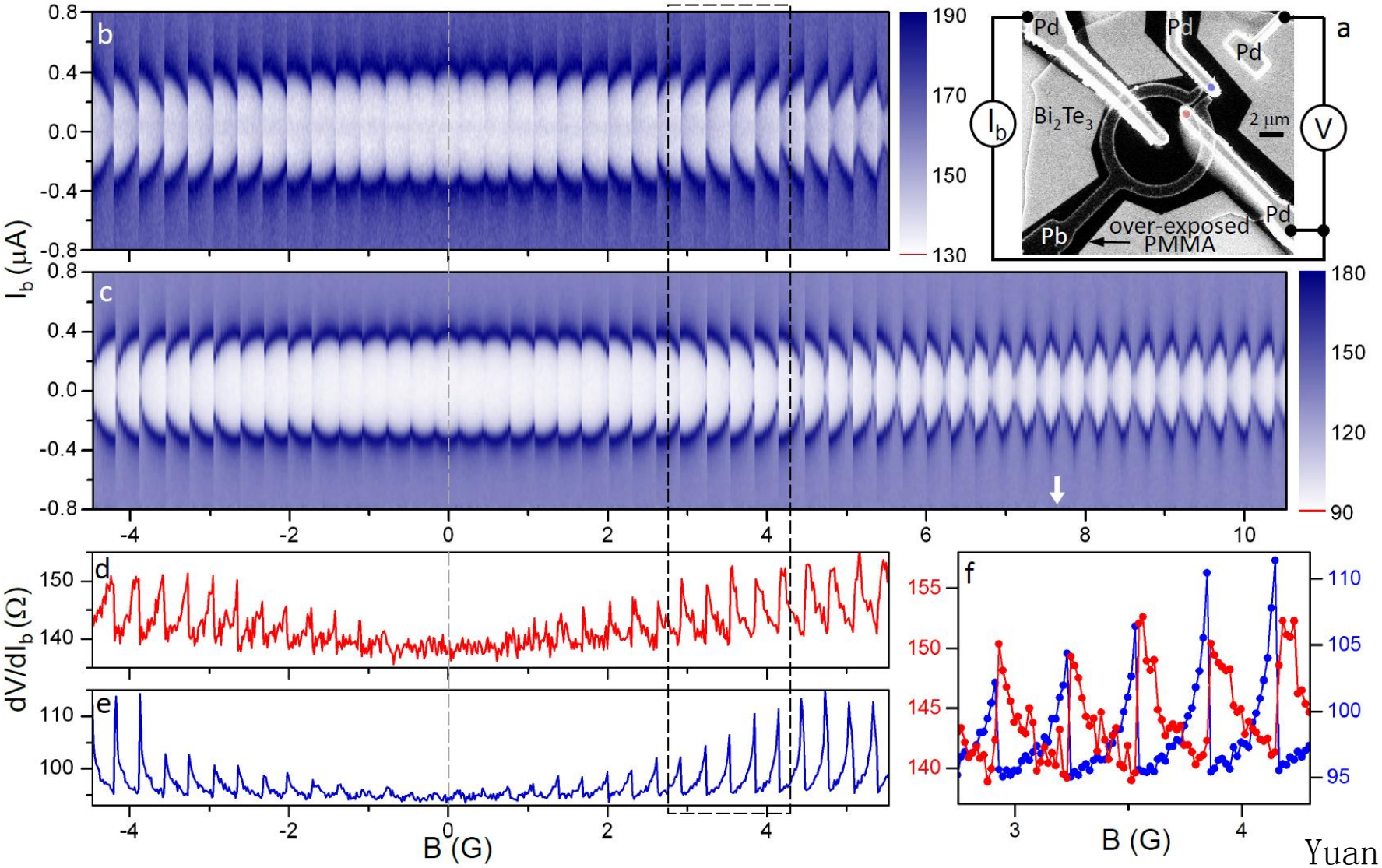


Leonid K. Rokhinson, *et. al*, 2012



J. Wiedenmann, *et. al*, (2016)

Unusual Josephson effect



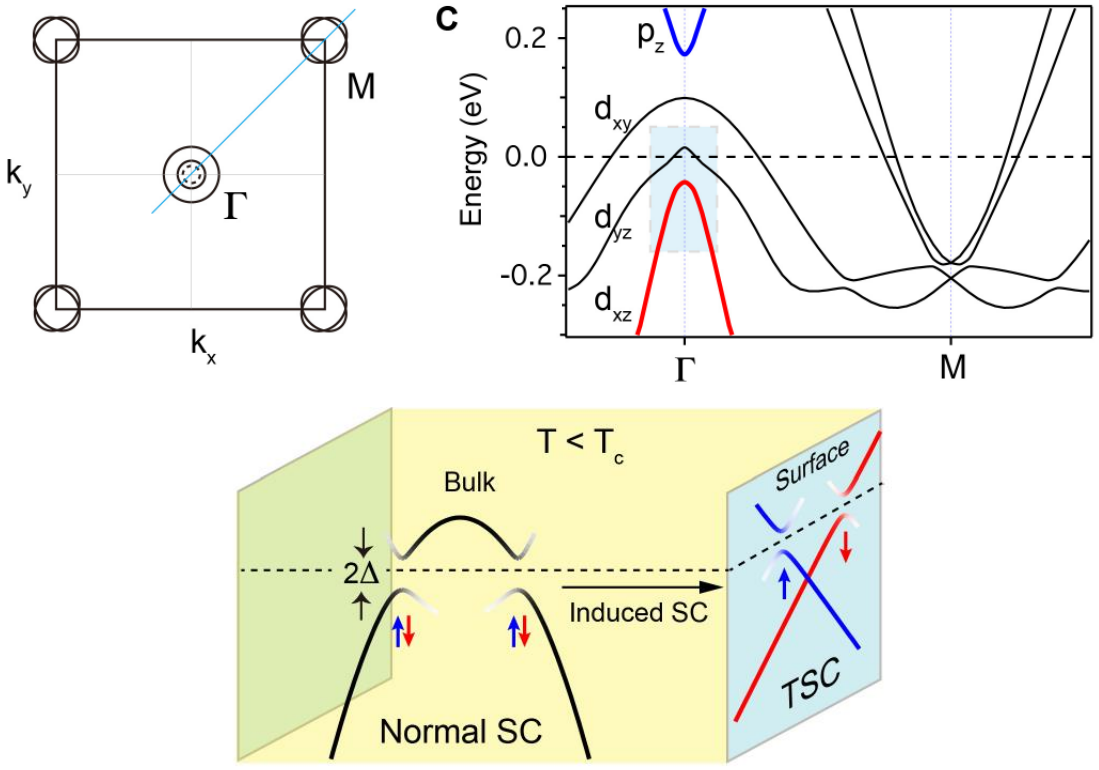
Yuan Pang, …, Li Lu, 2015

Intrinsic or proximity?

Observation of topological superconductivity on the surface of iron-based superconductor

arXiv.1706.05163

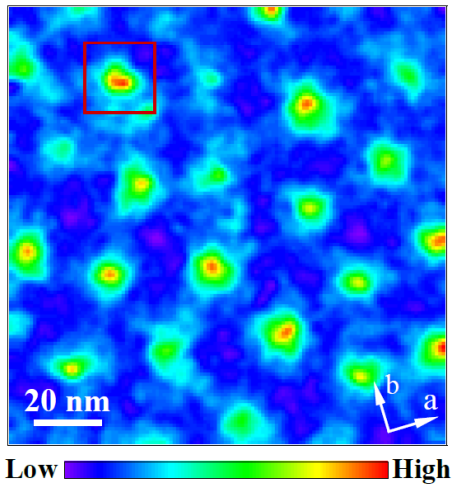
Peng Zhang[1*], Koichiro Yaji[1], Takahiro Hashimoto[1], Yuichi Ota[1], Takeshi Kondo[1], Kozo Okazaki[1], Zhijun Wang[2], Jinsheng Wen[3], G. D. Gu[4], Hong Ding[5*], and Shik Shin[1*]

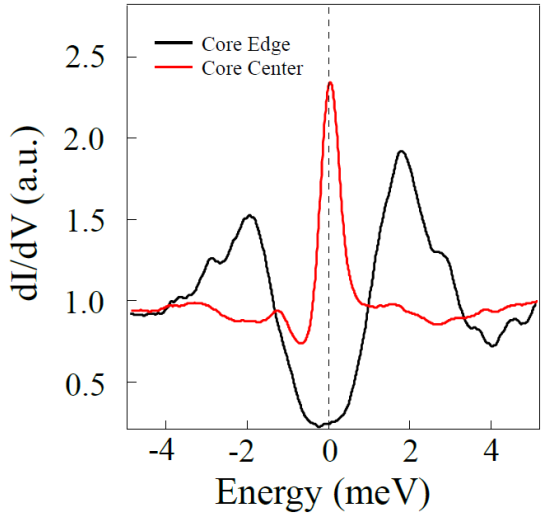


Observation of pristine Majorana bound state in iron-based superconductor

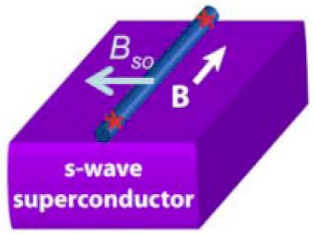
arXiv.1706.06074

Dongfei Wang[1,2†], Lingyuan Kong[1,2†], Peng Fan[1,2†], Hui Chen[1], Yujie Sun[1], Shixuan Du[1,3], John Schneeloch[4], Ruidan Zhong[4], Genda Gu[4], Liang Fu[5], Hong Ding[1,2,3*], and Hongjun Gao[1,2,3*]

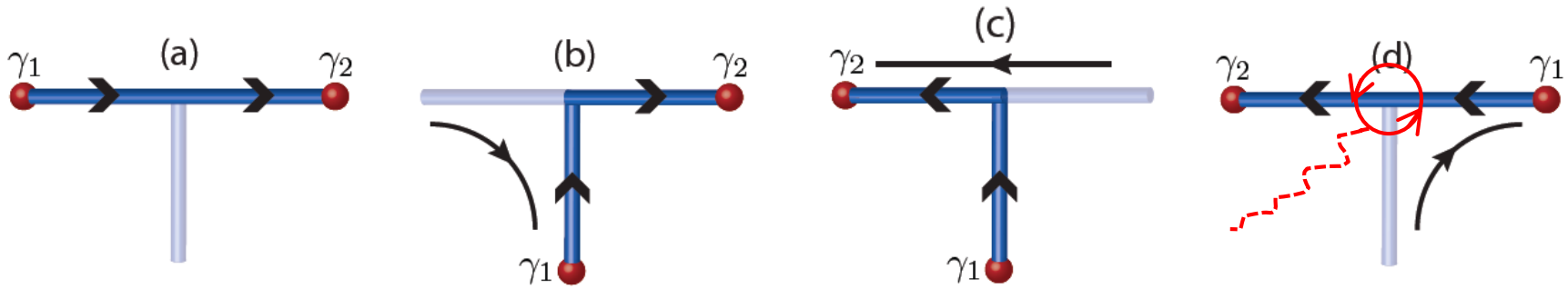




1D system (J. Alicea et al., Nature Phys. 2011)



Kouwenhoven group, Science (2012)
H. Q. Xu group, Nano Lett. (*2012*)
A. Das et al, Nature Physics (2012)
A. Yazdani group, Science (2014)
C. M. Marcus group: Science (2016)



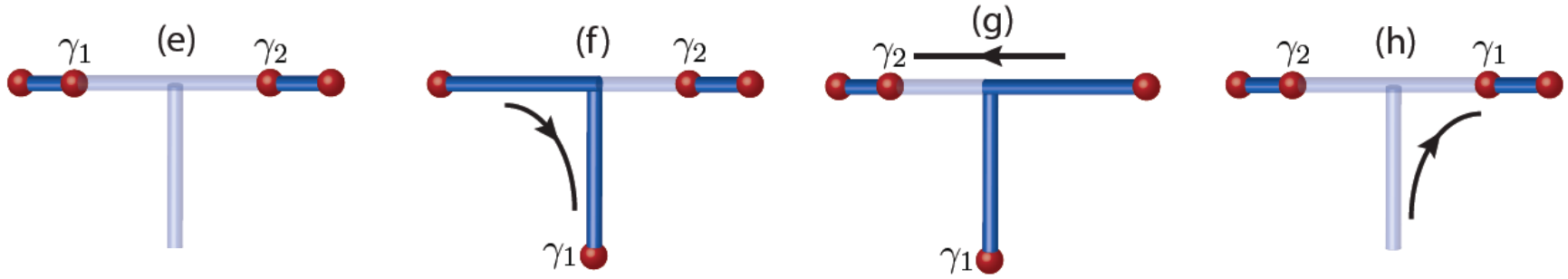
t

$$U_{12} = e^{\frac{\pi}{4}\gamma_1\gamma_2}$$

$$\gamma_1 \longrightarrow \gamma_2$$

$$\gamma_2 \longrightarrow -\gamma_1$$

Four Majorana modes



$|00\rangle$

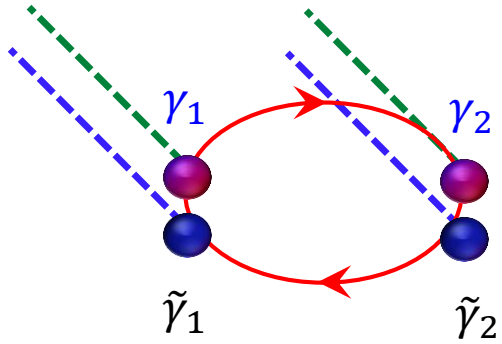
t

$(|00\rangle + i|11\rangle)/\sqrt{2}$

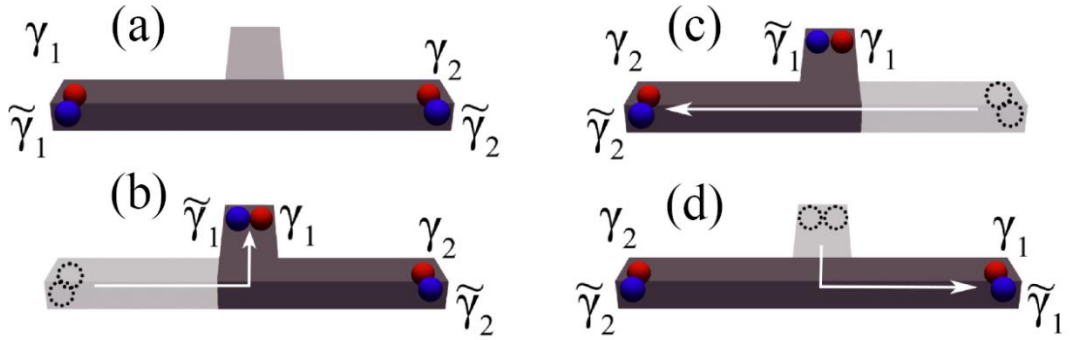
Non-Abelian braiding of Majorana Kramers' pairs

Definition: the braiding operation for exchanging two Majorana Kramers' pairs, but without local operations of two Majorana modes in a single Kramers' pair.

1) General case



2) 1D version



Braiding operation:

$$U_{12}|0\tilde{0}\rangle \longrightarrow |0\tilde{0}\rangle, \qquad U_{12}|1\tilde{1}\rangle \longrightarrow |1\tilde{1}\rangle$$

$$U_{12}|0\tilde{1}\rangle \longrightarrow e^{i\phi}|0\tilde{1}\rangle, \qquad U_{12}|1\tilde{0}\rangle \longrightarrow e^{-i\phi}|1\tilde{0}\rangle$$

Condition: $U_{12}^4 = 1$, then: $\phi = 0, \pi/2$, or π.

Consider the special case with two decoupled time-reversed copies: $\phi \equiv \pi/2$.

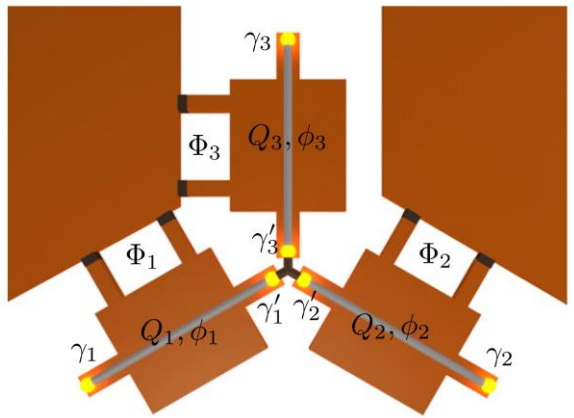
So, generically: $U_{12}=e^{\frac{\pi}{4}\gamma_1\gamma_2}e^{\frac{\pi}{4}\tilde{\gamma}_1\tilde{\gamma}_2}$

XJL, Chris L. M. Wong, and K. T. Law, PRX 4, 021018 (2014);
Gao, He, and XJL, PRB 94.224509 (2016).

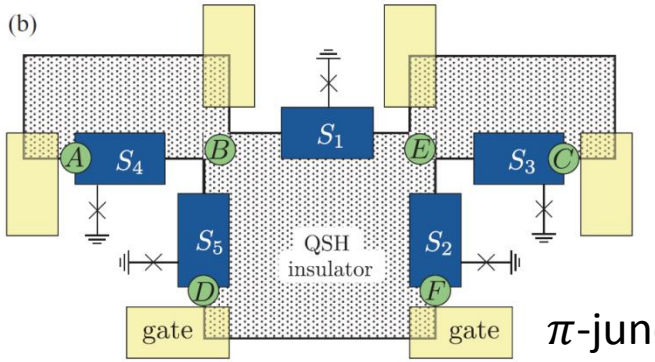
Typical challenges

1. Complexity of moving Majorana through junction, non-adiabatic effects.

Coulomb-assisted Majorana braiding



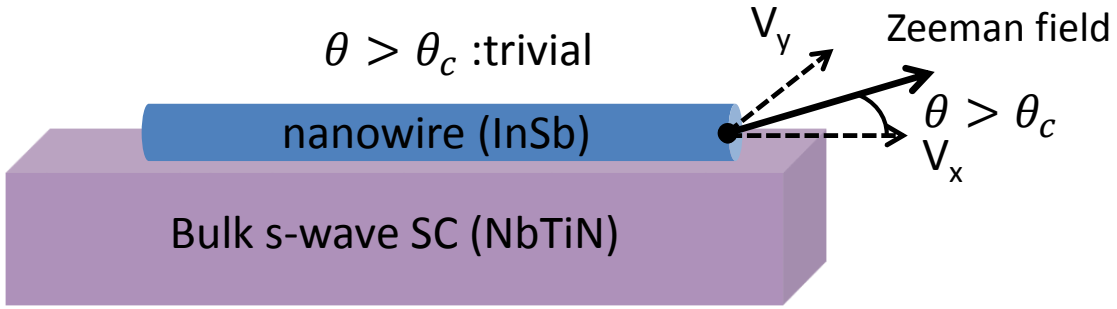
CWJ Beenakker group: *New Journal of Physics* **14** (2012) 035019



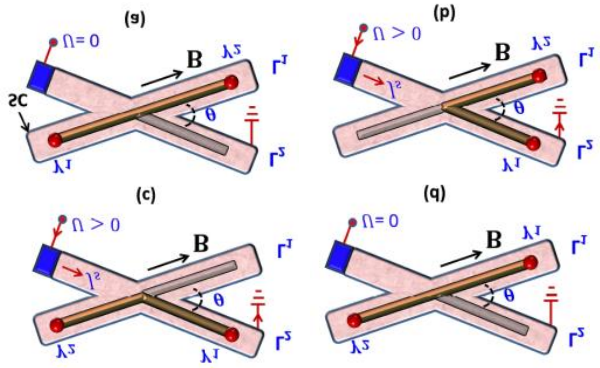
π-junction manipulation

PRB 87, 241405 (R) (2013)

2. Misalignement of nanowire with external **B** field.



XJL, Phys. Rev. Lett., 109,106404 (2012)
XJL and A. M. Lobos, Phys. Rev. B 87, 060504 (R) (2013)



supercurrent-assisted Majorana braiding

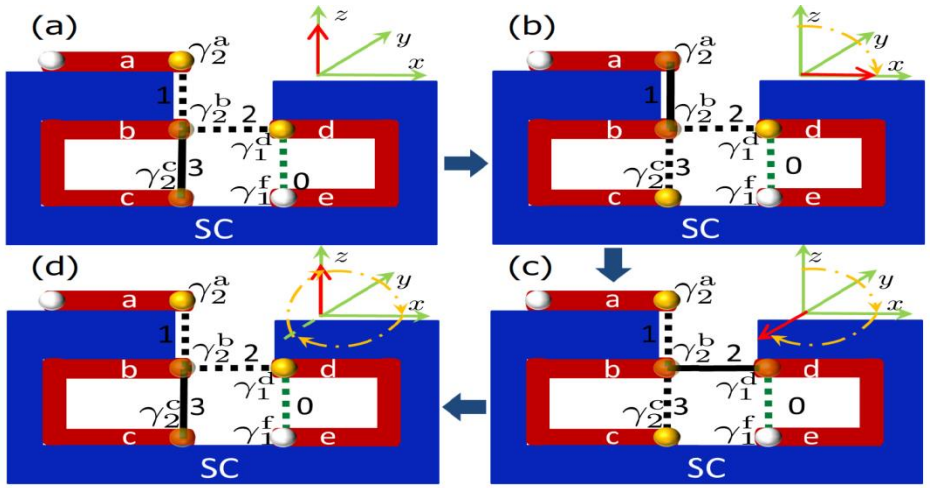
3. Multi-channel, multiple low-energy modes, quasiparticle poisoning……

Other improvements

Parallel junctions: external magnetic field can align with all topological superconducting wires. Braiding by measurements.

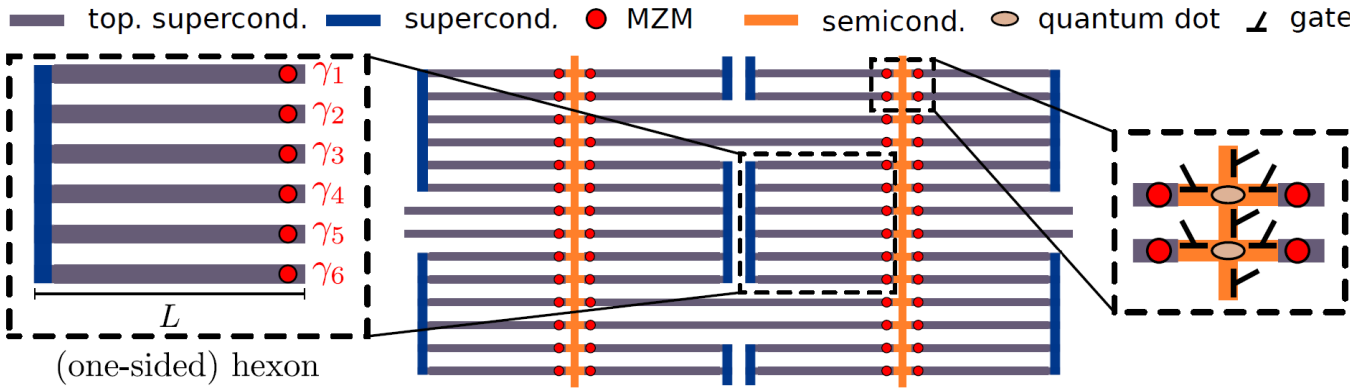
Majorana spintronics

X. Liu, X. Li, D.-L. Deng, XJL, and S. D. Sarma, Phys. Rev. B **94**, 014511 (2016).



T. Karzig, C. Knapp, R. M. Lutchyn, P. Bonderson, M. B. Hastings, C. Nayak, J. Alicea, K. Flensberg, S. Plugge, Y. Oreg, C. M. Marcus, and M. H. Freedman, PRB **95**, 235305 (2017)

Scalable designs for quasiparticle

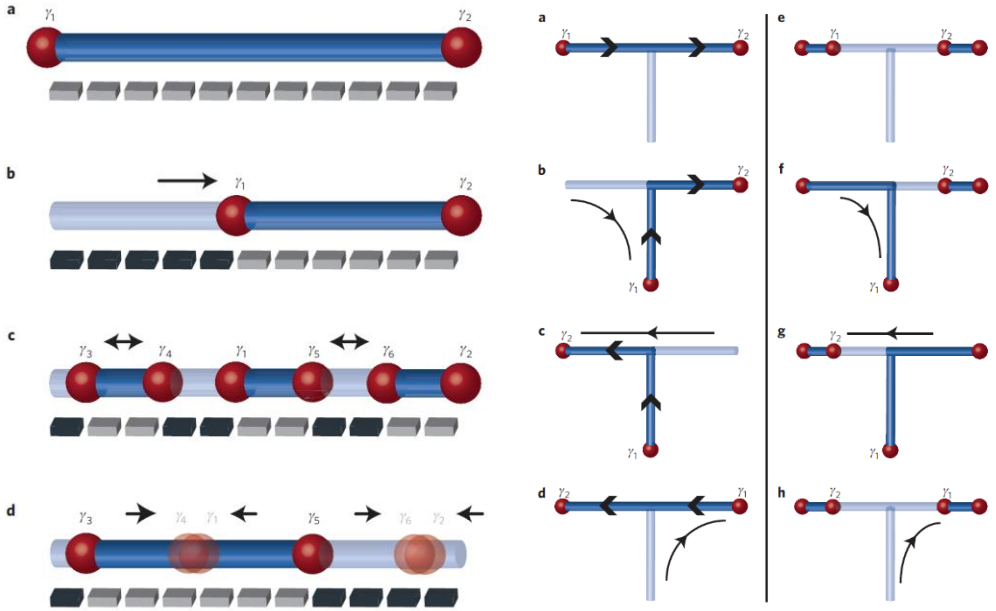


Braiding

ARTICLES
PUBLISHED ONLINE: 13 FEBRUARY 2011 | nature physics

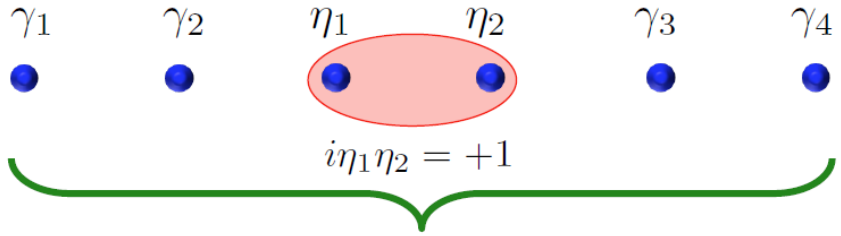
Non-Abelian statistics and topological quantum information processing in 1D wire networks

Jason Alicea[1]*, Yuval Oreg[2], Gil Refael[3], Felix von Oppen[4] and Matthew P. A. Fisher[3,5]



Braiding without Braiding: Teleportation-Based Quantum Information Processing with Majorana Zero Modes

Sagar Vijay[1,2] and Liang Fu[1]



$$|\psi_i\rangle \sim |\phi\rangle \otimes |i\eta_1\eta_2 = +1\rangle$$

Measurements

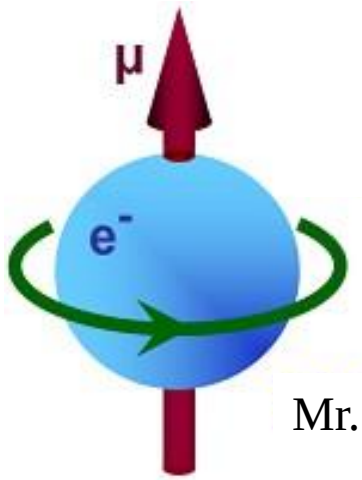
I. $i\gamma_2\eta_1 = +1$

II. $i\eta_1\gamma_3 = +1$

III. $i\eta_1\eta_2 = +1$

$$|\psi_f\rangle \sim \hat{U}_{23}|\phi\rangle \otimes |i\eta_1\eta_2 = +1\rangle$$

Why considering spin-property of MFs?



Mr. Electron

Two parts to his personality !

CHARGE

SPIN

- *Electrons* are the basic particles in the devices which can support Majorana fermions. So Majorana fermion must has spin degree of freedom.

Spinless ≠ Spin is useless

How spin appear in MFs

- MF: equal weight superposition of electron and hole

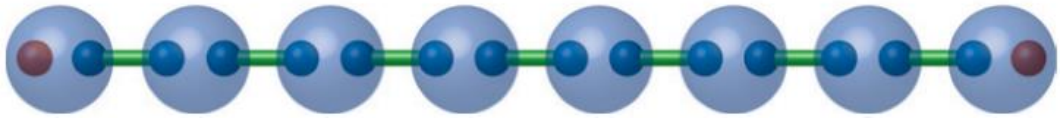
$$\gamma = c_{\sigma} + c^{\dagger}_{\sigma'} \qquad \langle c\, c \rangle \neq 0$$

- Majorana fermions satisfy

$$\gamma^{\dagger} = c^{\dagger}_{\sigma} + c_{\sigma'} = c_{\sigma} + c^{\dagger}_{\sigma'} = \gamma$$

$$\sigma = \sigma'$$

- This requires



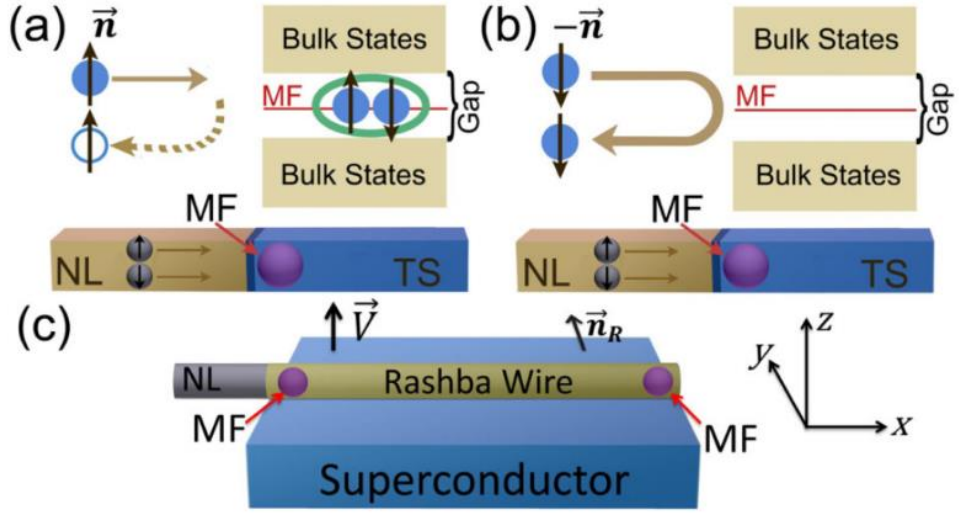
$$\langle c_{\sigma} c_{\sigma} \rangle \neq 0$$

Spin-triplet nature of Majorana zero modes

PRL 112, 037001 (2014) PHYSICAL REVIEW LETTERS week ending 24 JANUARY 2014

Selective Equal-Spin Andreev Reflections Induced by Majorana Fermions

James J. He,[1] T. K. Ng,[1] Patrick A. Lee,[2] and K. T. Law[1,*]

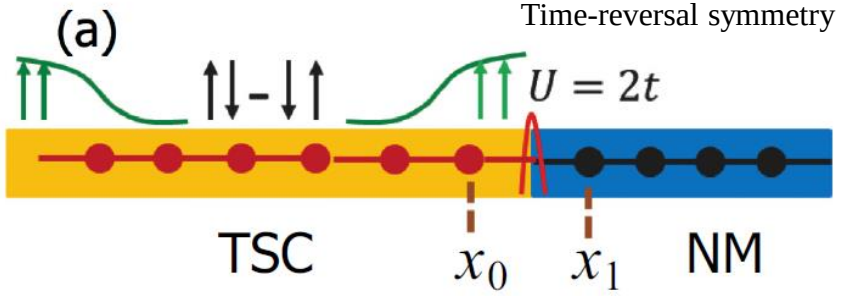


PHYSICAL REVIEW B 92, 014513 (2015)

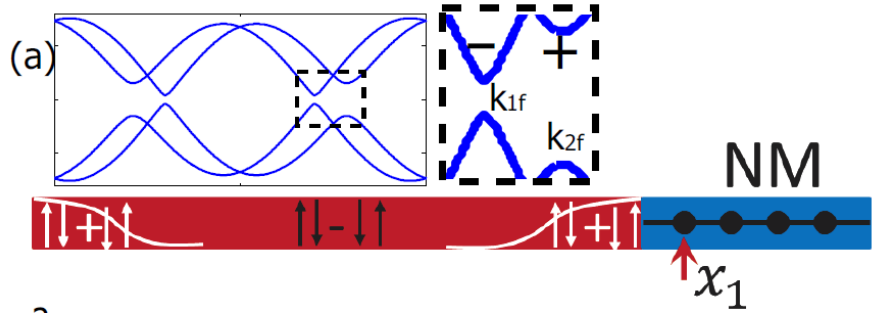
Universal spin-triplet superconducting correlations of Majorana fermions

Xin Liu,* Jay D. Sau, and S. Das Sarma

Time-reversal symmetry broken



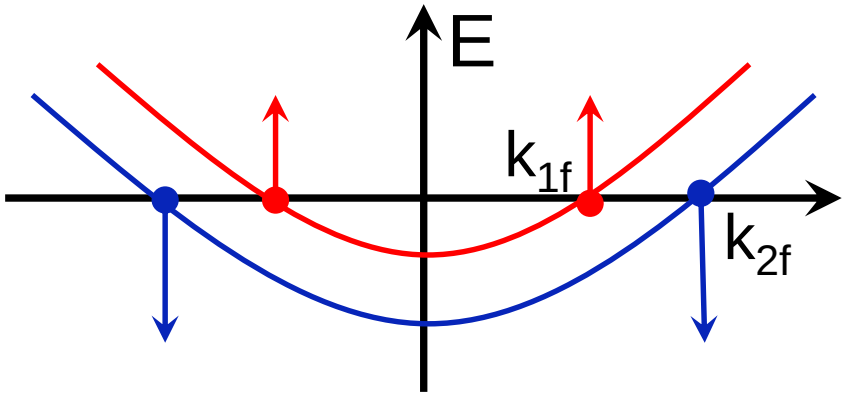
Time-reversal symmetry respected



Manipulating MZMs by spin degree of freedom

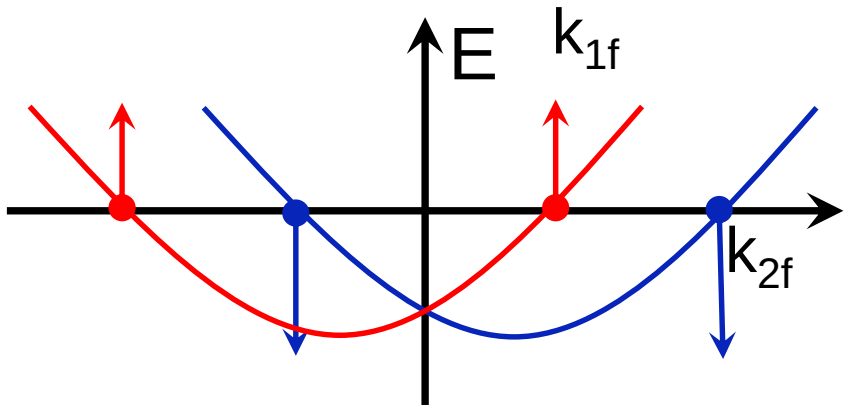
- Magnetic field or magnetization (Zeeman field)

$$H_Z = M \cdot \sigma$$



- Spin-orbit coupling

$$H_{so} = -\frac{\hbar}{4m_0^2c^2}\vec{\sigma} \cdot \vec{p} \times (\nabla V_0)$$

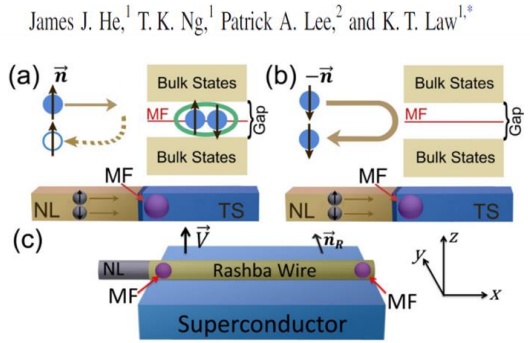


Detecting and braiding with spin degree of freedom

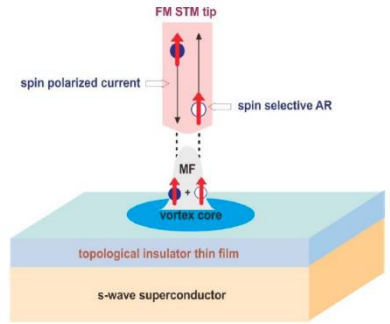
- Zero bias peak

Selective Equal-Spin Andreev Reflections Induced by Majorana Fermions

James J. He,[1] T. K. Ng,[1] Patrick A. Lee,[2] and K. T. Law[1,*]



Hao-hua Sun, . . . Jinfeng Jia (2016)

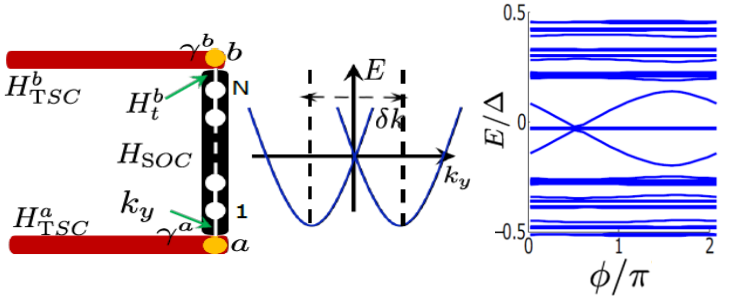


- 4π Josephson effect

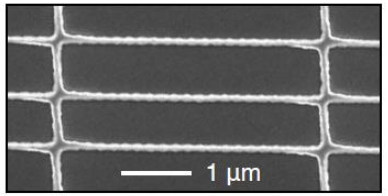
Majorana spintronics

(2016)

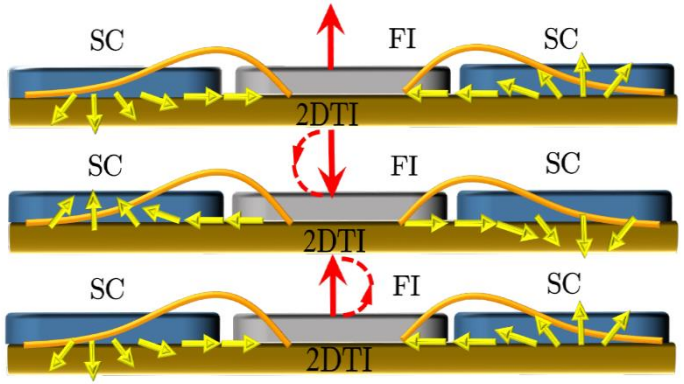
Xin Liu,[1,2] Xiaopeng Li,[2] Dong-Ling Deng,[2] Xiong-Jun Liu,[3,4] and S. Das Sarma[2]



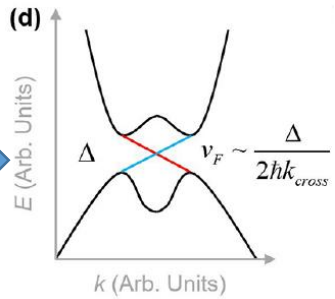
S. Vaitiekenas, …, C. M. Marcus (2018)

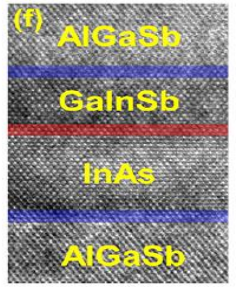


- Braiding



Lingjie Du, …, Rui-Rui Du (2017)





Spin-statistics theorem

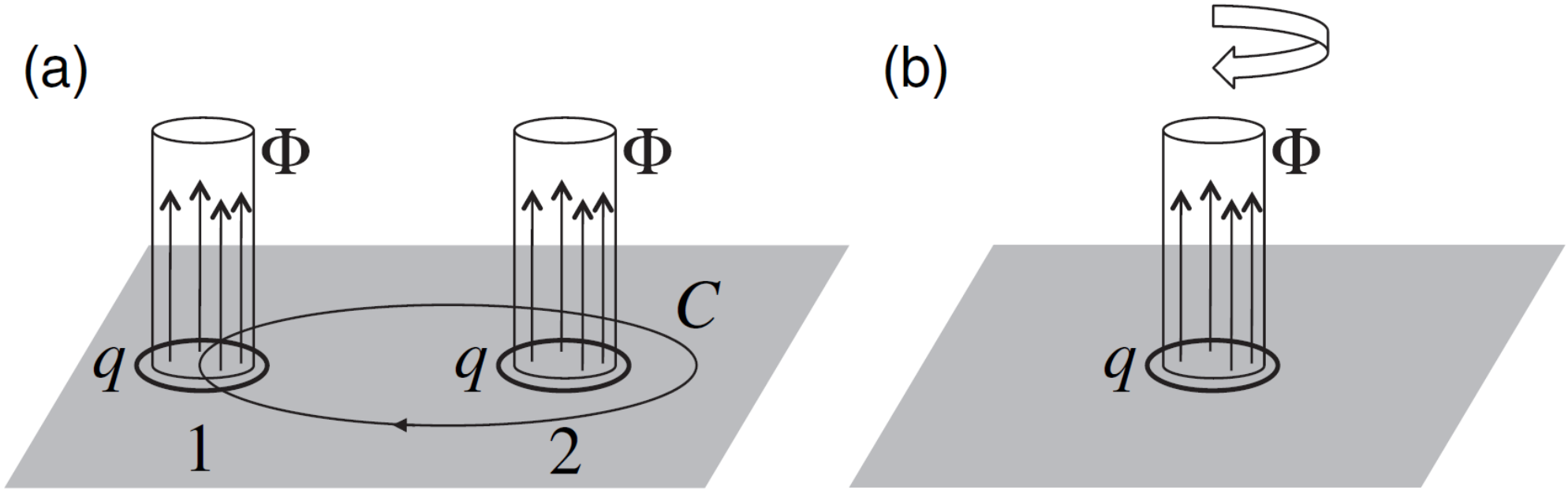
- The relation between spin and statistics from flux-charge composite

Phase from exchange

$$exp\left[i\left(\frac{q\Phi}{2}+\frac{q\Phi}{2}\right)\right]$$

Phase from self-rotation

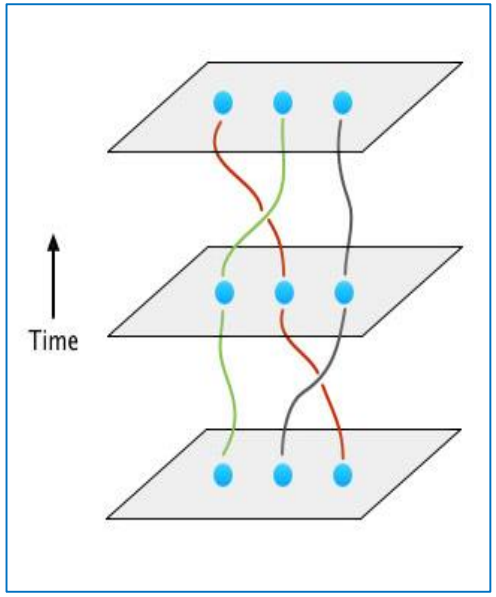
$$exp\left[i(q\Phi)\right]=exp\left[-i2\pi J\right]$$

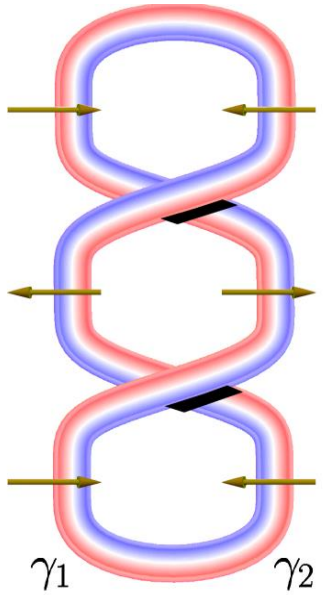


Jiannis K. Pachos (2013)

From worldline to worldribbon

- Worldline: track the history of particles in spacetime



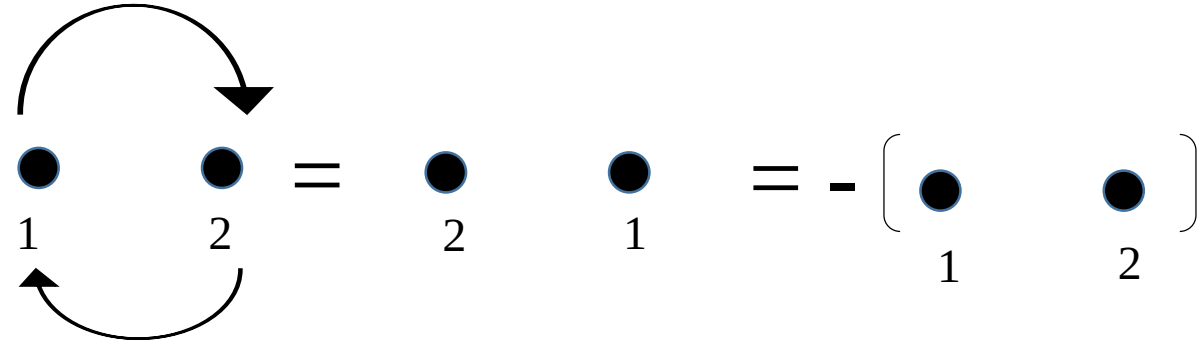


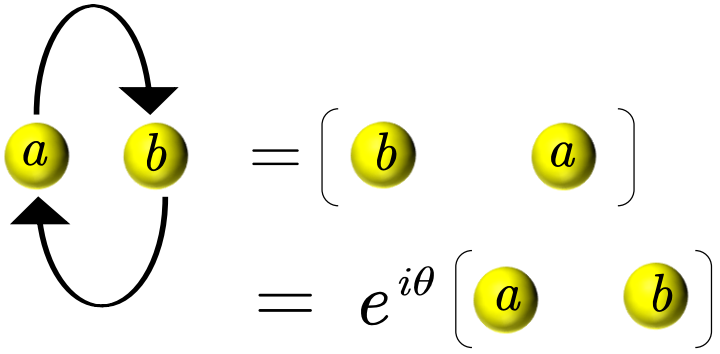
- Worldribbon: track the history of particles in both spacetime and internal degree of freedom

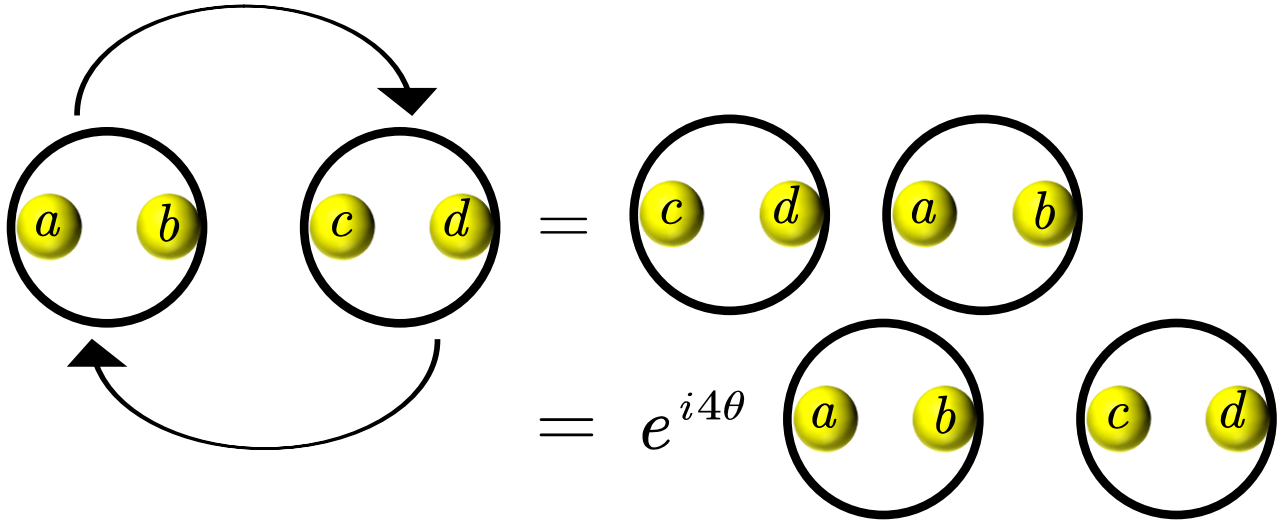
- Ribbon equation:

$$(R_{ab}^{c})^{k} = e^{i\pi k s_a} e^{i\pi k s_b} e^{-i\pi k s_c}$$

An intuitive picture for non-abelian statistics







$$e^{i4\theta} = -1 \;\Rightarrow\; \theta = \pi/4 \;\Rightarrow\; J = 1/8$$

Majorana form and robust full braiding

- Proximity induced topological superconductor and Majorana form

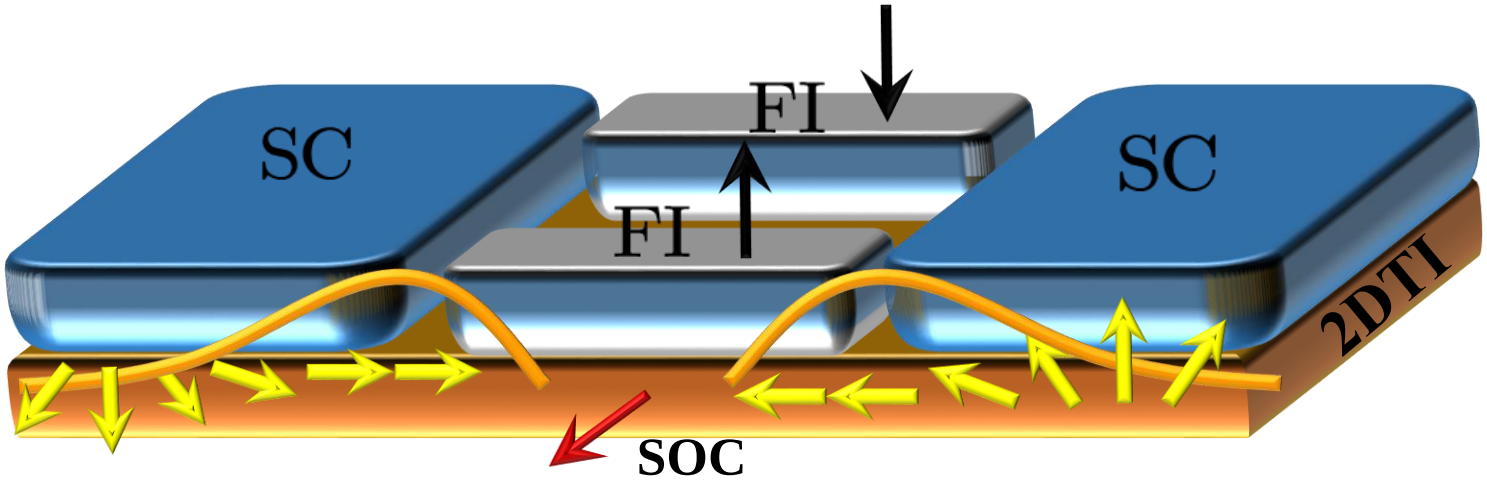
$$\hat{H} = \begin{pmatrix} h(\hat{\boldsymbol{p}}) + \boldsymbol{m}(\boldsymbol{r}) \cdot \sigma & \Delta_{\mathrm{SC}}(\boldsymbol{r}) \\ \Delta_{\mathrm{SC}}^{\dagger}(\boldsymbol{r}) & -h(\hat{\boldsymbol{p}}) + \boldsymbol{m}(\boldsymbol{r}) \cdot \sigma \end{pmatrix} \qquad \Psi(\boldsymbol{r}) = (\psi_{\mathrm{e}}(\boldsymbol{r}), i\sigma_y \psi_{\mathrm{e}}^{*}(\boldsymbol{r})^{\mathrm{T}}$$

- The accumulated phase through closed adiabatic evolution

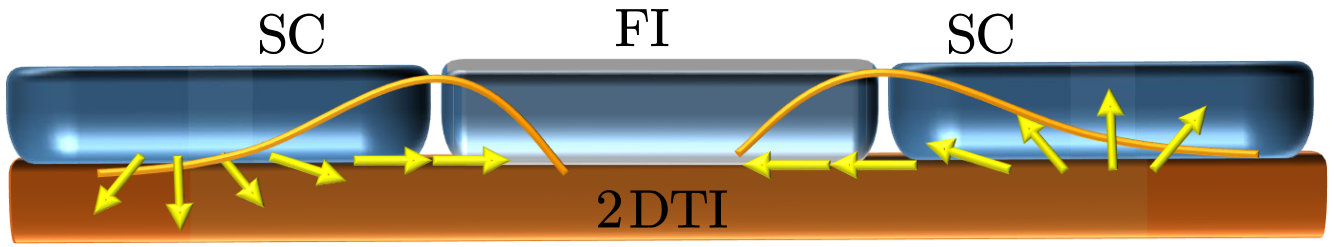
dynamic phase + Berry phase + monodromy phase

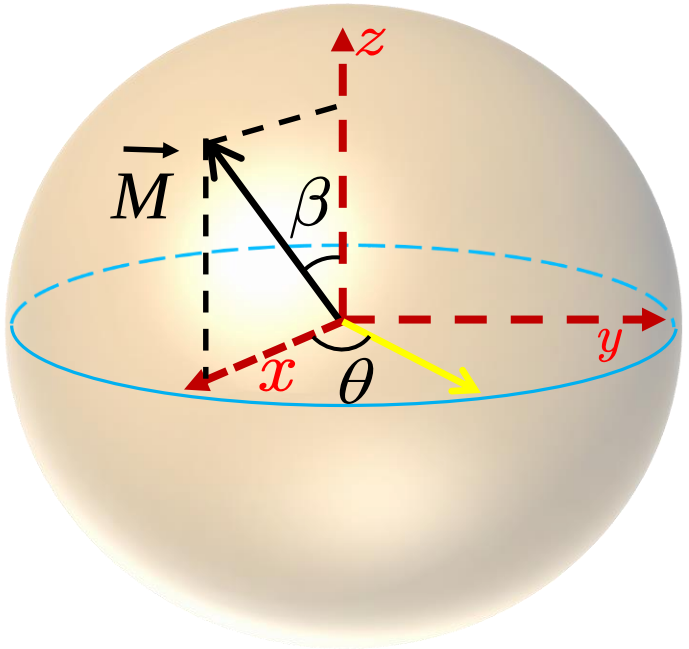
- E=0
- Berry connection: $\mathrm{Im}\,\langle\Psi|\,\partial_{\mathrm{t}}\,|\Psi\rangle = 0$
- Monodromy phase = nπ

Majorana spin property in 2DTI based system



$$\hat{H} = (v\hat{p}\sigma_z - \mu)\tau_z + \boldsymbol{M}(x)\cdot\boldsymbol{\sigma} + \Delta(x)\tau_x$$



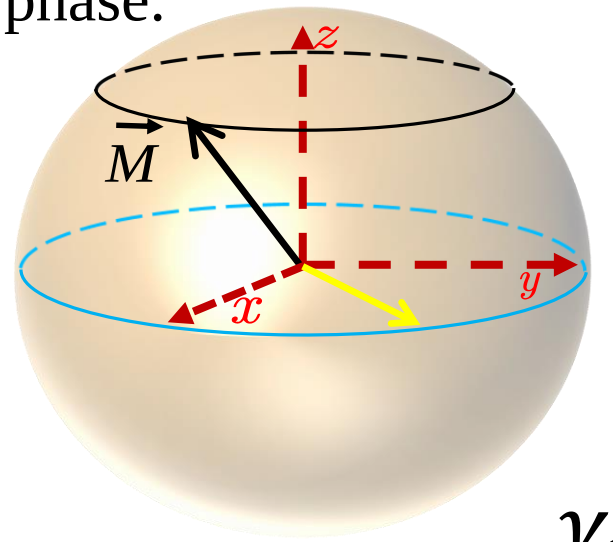


Majorana spin ⊥ SO axis

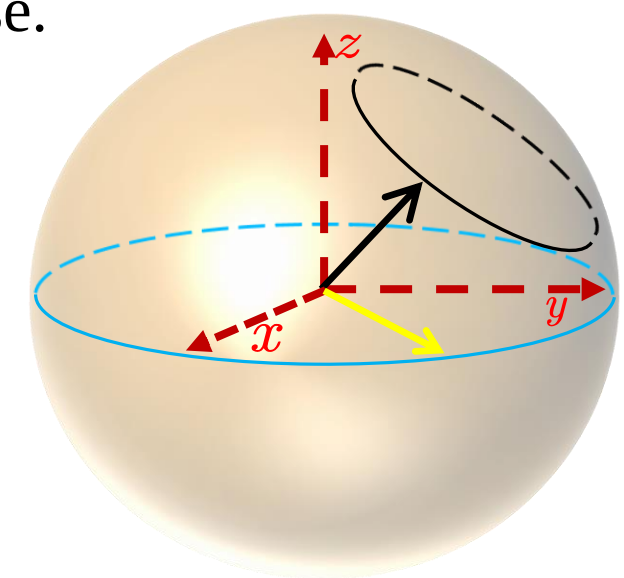
"Braiding Majorana zero modes in spin space: from worldline to worldribbon"
Xun-Jiang Luo, *et .al.,* arXiv:1803.02173

full braiding vs no braiding

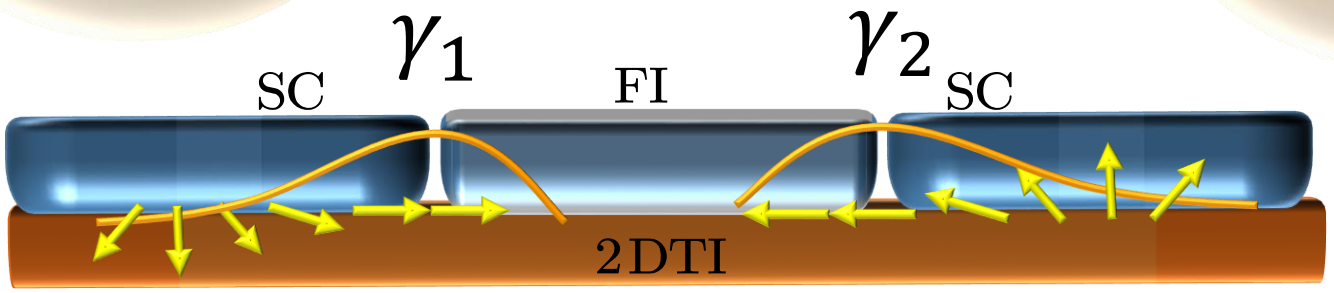
- The Majorana spin will acquire a π quantum phase.



- The Majorana spin will acquire 0 quantum phase.



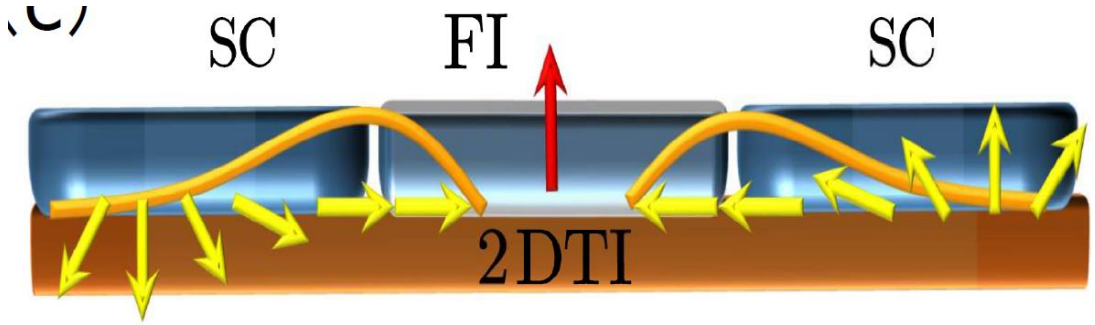
$\gamma_{1,2} \longrightarrow -\gamma_{1,2}$
full braiding



$\gamma_{1,2} \longrightarrow \gamma_{1,2}$
no braiding

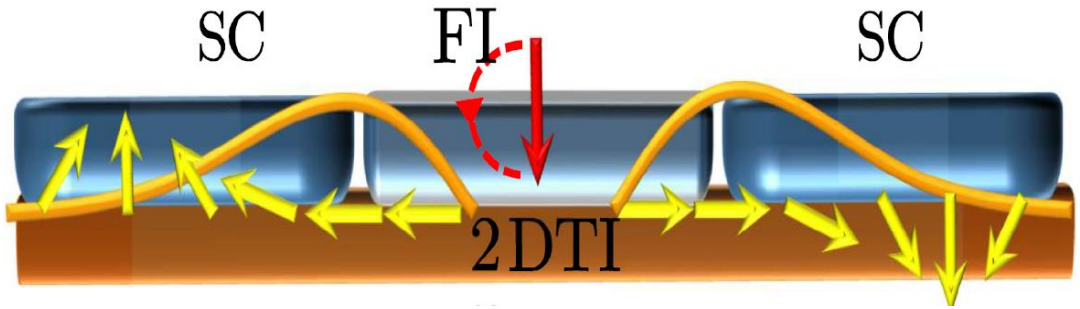
$$(R^c_{\gamma_1\gamma_2})^2 = \begin{pmatrix} e^{-i\frac{\pi}{2}} & 0 \\ 0 & e^{i\frac{\pi}{2}} \end{pmatrix} \qquad i\gamma_1\gamma_2 = \pm 1$$

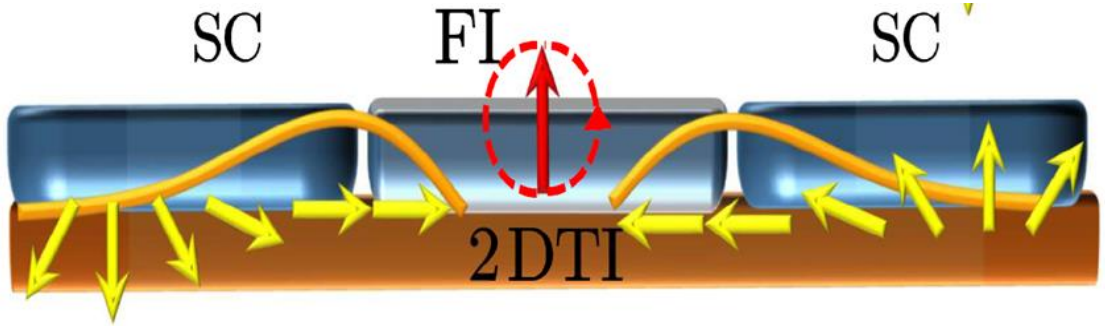
Single braiding



$$\Psi_{1,2}(-\boldsymbol{M}) = T\Psi_{1,2}(\boldsymbol{M}), T = i\sigma_y K$$

$$R^{c}_{\gamma_1\gamma_2} = \begin{pmatrix} e^{-i\frac{\pi}{4}} & 0 \\ 0 & e^{i\frac{\pi}{4}} \end{pmatrix}$$





$$\left(R^{c}_{\gamma_1\gamma_2}\right)^2 = \begin{pmatrix} e^{-i\frac{\pi}{2}} & 0 \\ 0 & e^{i\frac{\pi}{2}} \end{pmatrix}$$

Two theorems

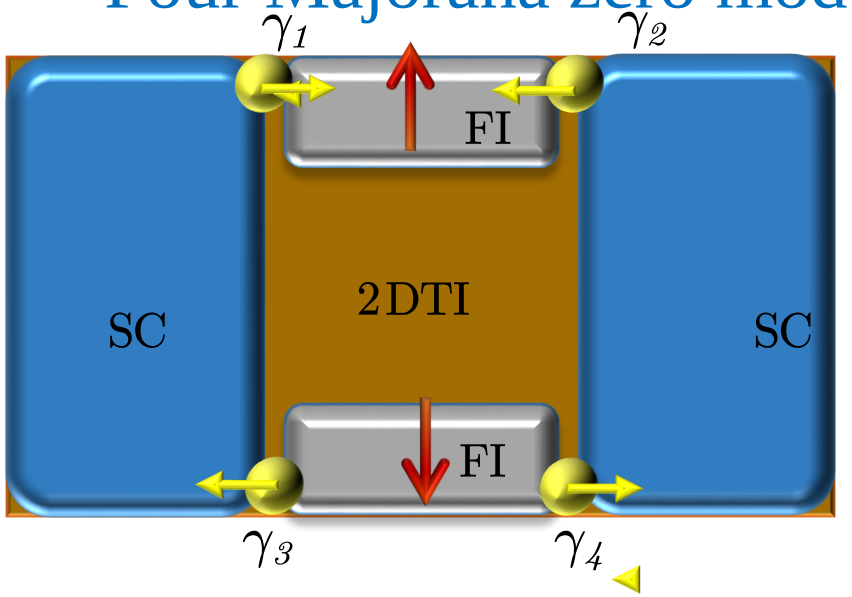
Theorem 1: *The adiabatic spin evolution of each MZM, following an arbitrary closed path in varying the direction of* $\boldsymbol{m}$ *without closing bulk gap, accumulates a geometric phase quantized to* $n\pi$*, which leads to* n *times full braiding of* γ_1 *and* γ_2 *in fusion space.*

Theorem 2: *The adiabatic evolution of MZMs* γ_1 *and* γ_2 *following an arbitrary magnetization winding path, with the initial and final Zeeman term satisfying* $\boldsymbol{m}_{\mathrm{i}} = -\boldsymbol{m}_{\mathrm{f}}$*, reverses the spin of each MZM, which corresponds to a single braiding of* γ_1 *and* γ_2*.*

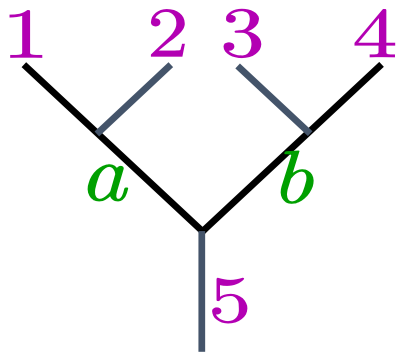
"Braiding Majorana zero modes in spin space: from worldline to worldribbon"
Xun-Jiang Luo, *et .al.,* arXiv:1803.02173

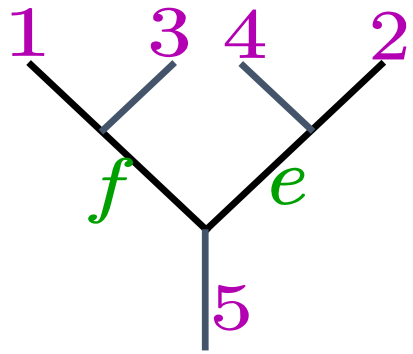
The Majorana qubit

- Four Majorana zero modes build one qubit



The total parity is conserved: $P = \gamma_1\gamma_2\gamma_3\gamma_4$

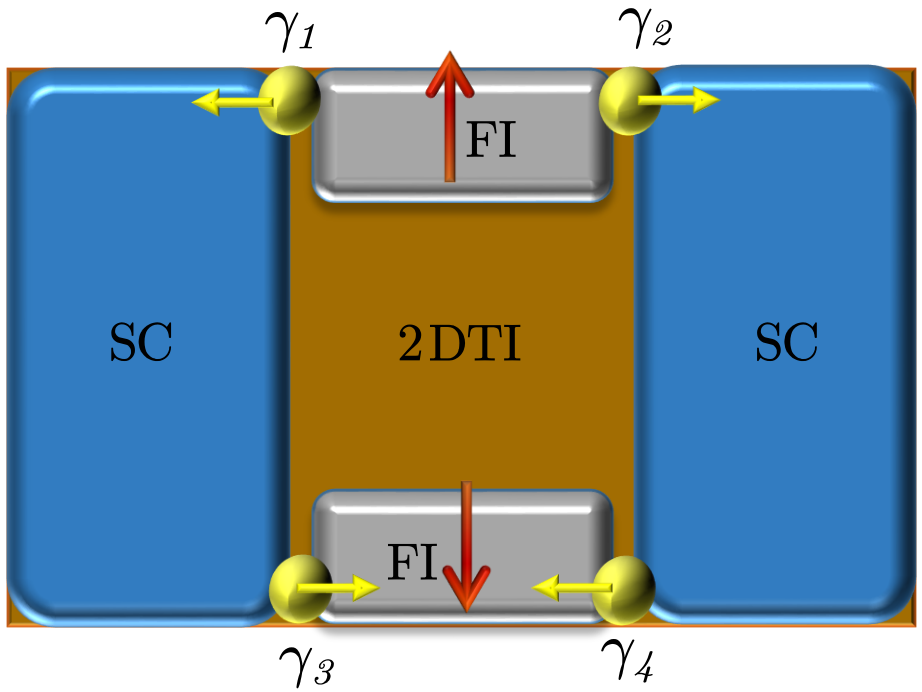




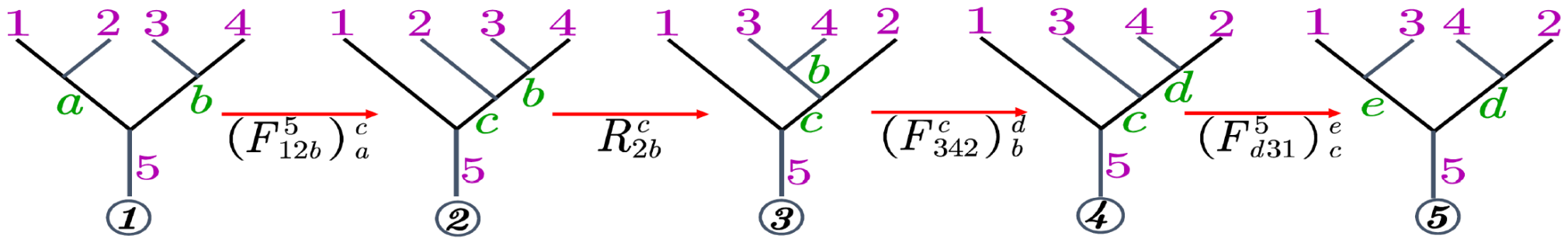
- The qubit states for different fusion space

basis	fermion operators	fusion states
$\lvert i\gamma_1\gamma_2, i\gamma_3\gamma_4\rangle$	$f_{\mathrm{u}} = (\gamma_3 + i\gamma_4)/2,$ $f_{\mathrm{d}} = (\gamma_1 + i\gamma_2)/2$	$\lvert 00\rangle_{\mathrm{FI}},$ $\lvert 11\rangle_{\mathrm{FI}} = f_{\mathrm{d}}^\dagger f_{\mathrm{u}}^\dagger \lvert 00\rangle_{\mathrm{FI}}$
$\lvert i\gamma_1\gamma_3, i\gamma_4\gamma_2\rangle$	$d_{\mathrm{L}} = (\gamma_1 + i\gamma_3)/2$ $d_{\mathrm{R}} = (\gamma_4 + i\gamma_2)/2$	$\lvert 00\rangle_{\mathrm{SC}}$ $\lvert 11\rangle_{\mathrm{SC}} = d_{\mathrm{L}}^\dagger d_{\mathrm{R}}^\dagger \lvert 00\rangle_{\mathrm{SC}}$

The transformation between the two fermionic space



$$\begin{pmatrix} |00\rangle_{\mathrm{FI}} \\ |11\rangle_{\mathrm{FI}} \end{pmatrix} = \hat{T} \begin{pmatrix} |00\rangle_{\mathrm{SC}} \\ |11\rangle_{\mathrm{SC}} \end{pmatrix}, \quad \hat{T} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ i & -i \end{pmatrix}$$

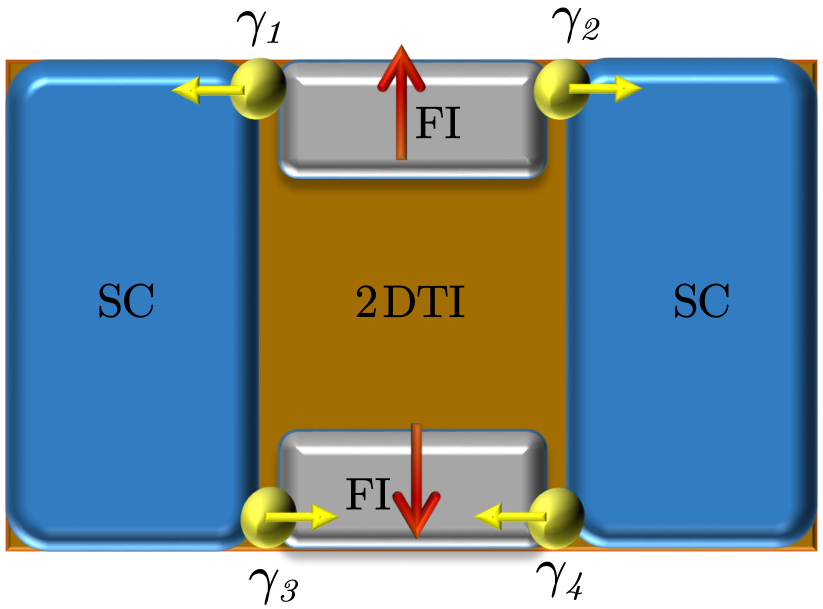


Braiding operations in two basis

- FI basis

$$R^{c}_{\gamma_1\gamma_2} = \begin{pmatrix} e^{-i\frac{\pi}{4}} & 0 \\ 0 & e^{i\frac{\pi}{4}} \end{pmatrix}$$

$$\left(R^{c}_{\gamma_1\gamma_2}\right)^2 = \begin{pmatrix} e^{-i\frac{\pi}{2}} & 0 \\ 0 & e^{i\frac{\pi}{2}} \end{pmatrix}$$



- SC basis

$$\exp(i\frac{\sigma_x}{4}\pi) = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & i \\ i & 1 \end{pmatrix}$$

$$\exp(-i\frac{\sigma_x}{2}\pi) = i\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$|00\rangle_{SC} \rightarrow |11\rangle_{SC}$

charge pumping

Fractional charge and quantized current in the quantum spin Hall state Nat. Phys., (2008)

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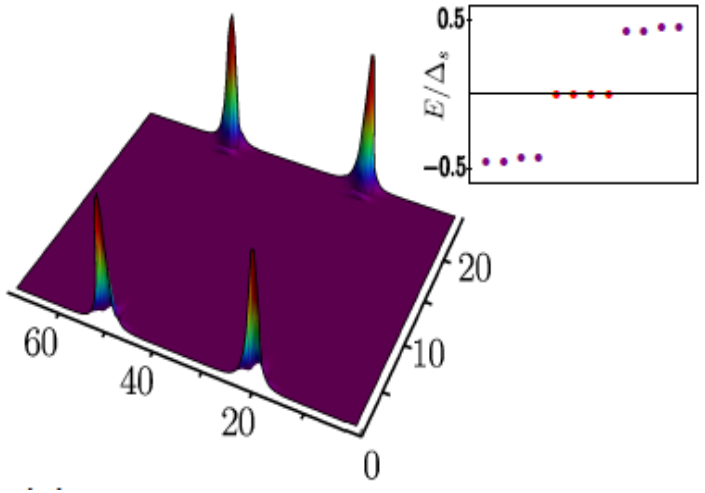
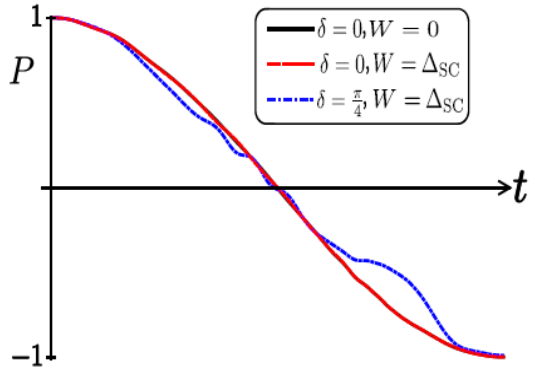
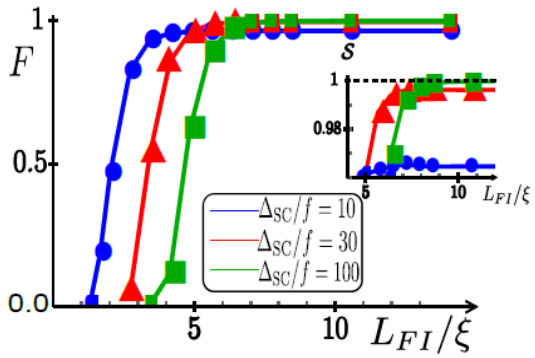
*e-mail: sczhang@stanford.edu

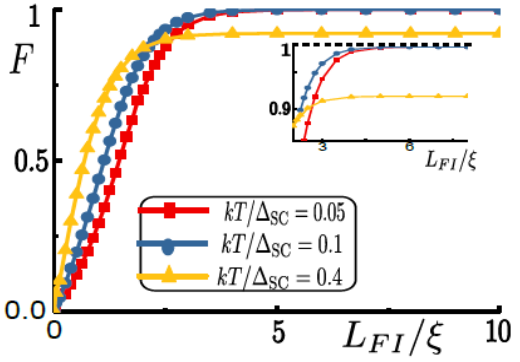
Numerical results

- Numerical simulation in the presence of (static and dynamical) disorder and finite size effects





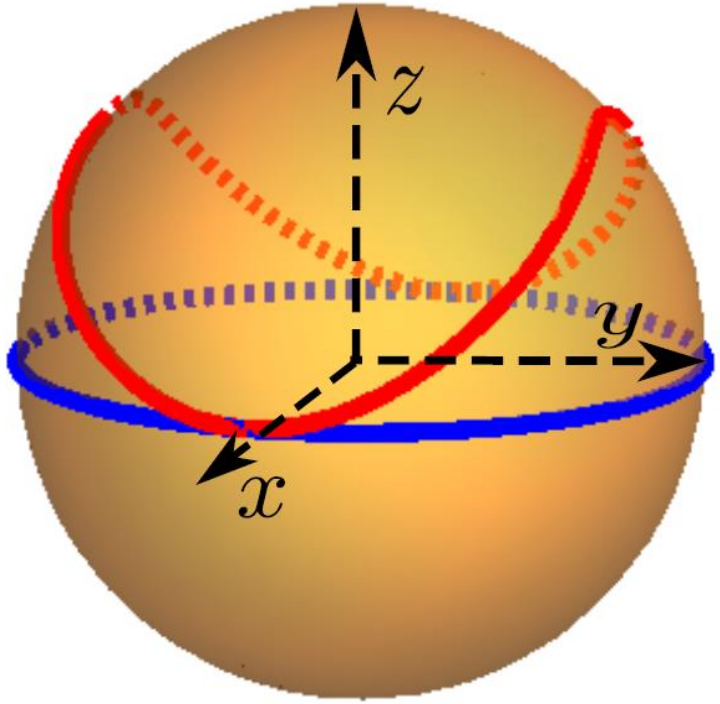




After a full braiding

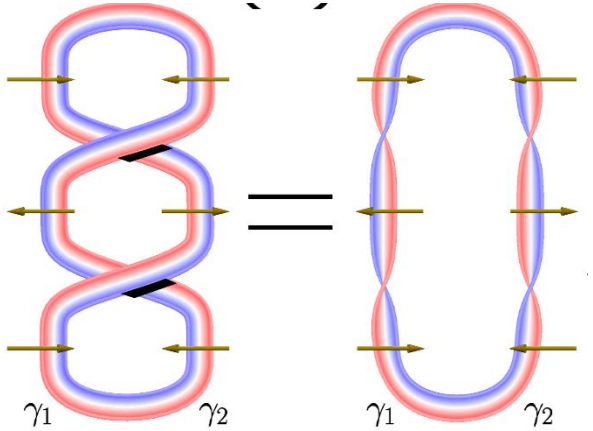
$$\gamma_{1,2} \longrightarrow -\gamma_{1,2}$$
$$\gamma_{3,4} \longrightarrow \gamma_{3,4}$$

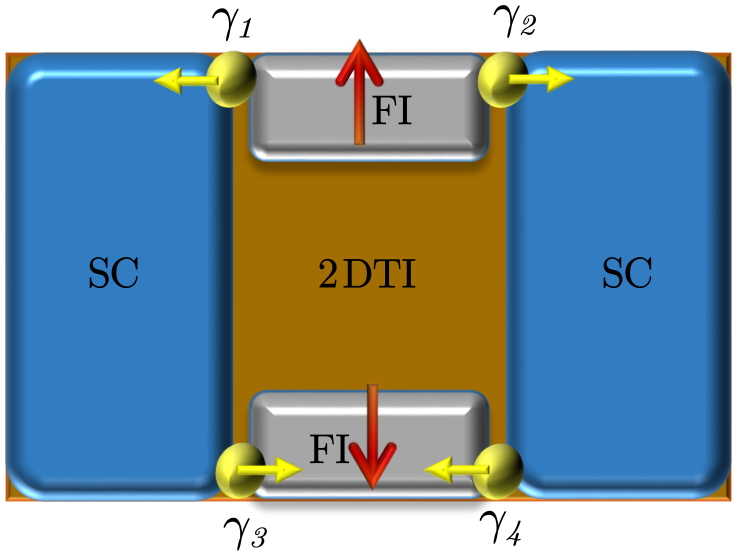
Two magnetization trajectories

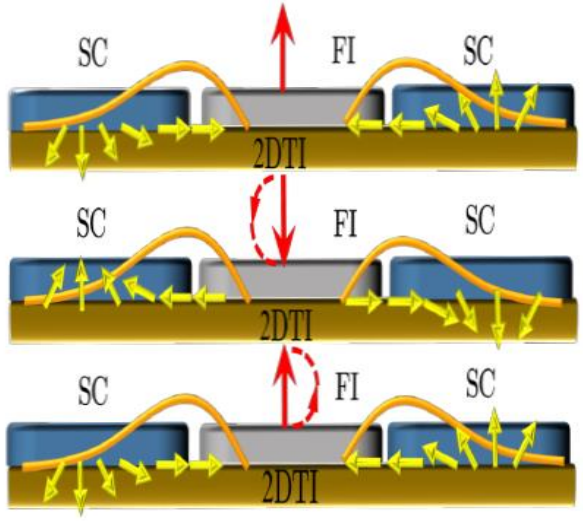


Conclusions

- The Majorana zero modes in solid have universal spin property
- The detecting and braiding MZMs have spin counterparts
- Worldlines should be extended to worldribbons when considering Majorana internal degree of freedom
- Braiding Majorana zero modes can be achieved in Majorana spin space







Thanks for your attention