Interplay of Topology and Geometry in Fractional Quantum Hall Liquids

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Interface and phase transition between Moore-Read and Halperin 331 fractional quantum Hall states: Realization of chiral Majorana fermion

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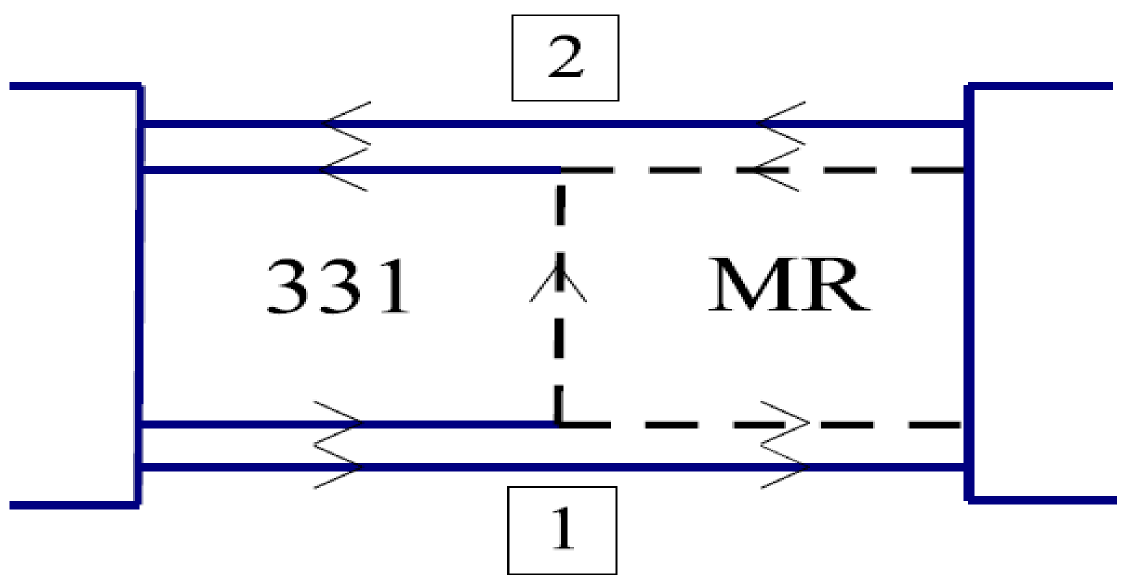
We consider an interface separating the Moore-Read state and Halperin 331 state in a half-filled Landau level, which can be realized in a double quantum well system with varying interwell tunneling and/or interaction strengths. In the presence of electron tunneling and strong Coulomb interactions across the interface, we find that all charge modes localize and the only propagating mode left is a chiral Majorana fermion mode. Methods to probe this neutral mode are proposed. A quantum phase transition between the Moore-Read and Halperin 331 states is described by a network of such Majorana fermion modes. In addition to a direct transition, they may also be separated by a phase in which the Majorana fermions are delocalized, realizing an incompressible state which exhibits quantum Hall charge transport and bulk heat conduction.



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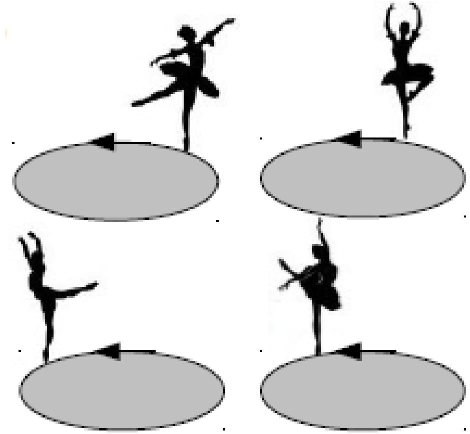
- Quantum Hall effects gave birth to the notion of **topological phase** and **topological order**, which are concepts now widely used in all branches of physics.
- **Key understanding** (or post-diction): Hall conductance is a topological quantum number/invariant (**Thouless**).
- Many **universal** predictions based on topology, including fractional charge and statistics of quasiparticles in fractional quantum Hall liquids, and universal chiral Luttinger liquid behavior at edges. Limited success in confronting experiments. Further experimental and theoretical work needed.

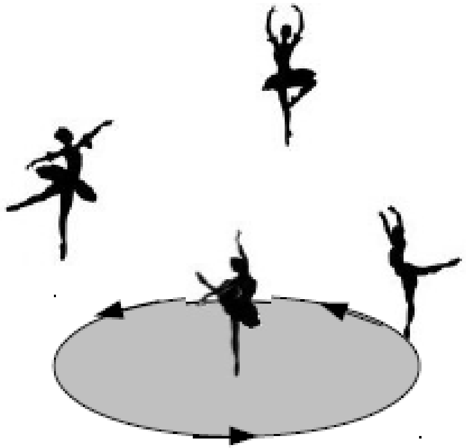
Key Messages of This Talk

- There is very interesting **non-universal** physics in fractional quantum Hall liquids.
- Some of the non-universal physics is associated with geometry, not just topology (**Haldane 11**).
- Geometry of fractional quantum Hall liquids can be directly measured experimentally (KY 13).
- Elementary neutral excitations of fractional quantum Hall liquids are quanta of geometry fluctuation, or spin-2 **gravitons** (**Haldane, Son**).
- These gravitons can be excited and probed using acoustic waves, acting like gravitational waves (KY16).

Integer (“trivial”) Dancing Pattern



Laughlin (Abelian) Dancing Pattern

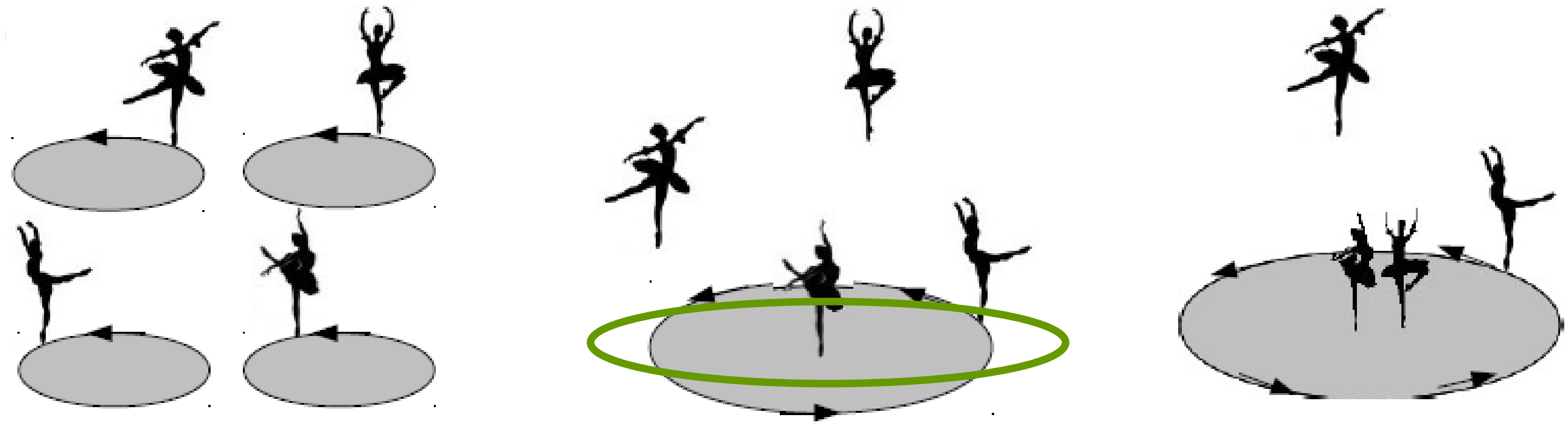


Moore-Read Dancing Pattern

Integer (“trivial”) Dancing Pattern

Laughlin (Abelian) Dancing Pattern

Moore-Read Dancing Pattern



Fractional quantum Hall dancing pattern has both topological and geometrical aspects. The latter determined by energetics; sensitive to specifics of electron-electron interaction, and in particular effective mass anisotropy. Described by **area-preserving** diffeomorphism (Haldane 11).

Contrary to common belief, Laughlin state contains a hidden (variational) parameter describing geometry (anisotropy) of correlation hole around each electron.

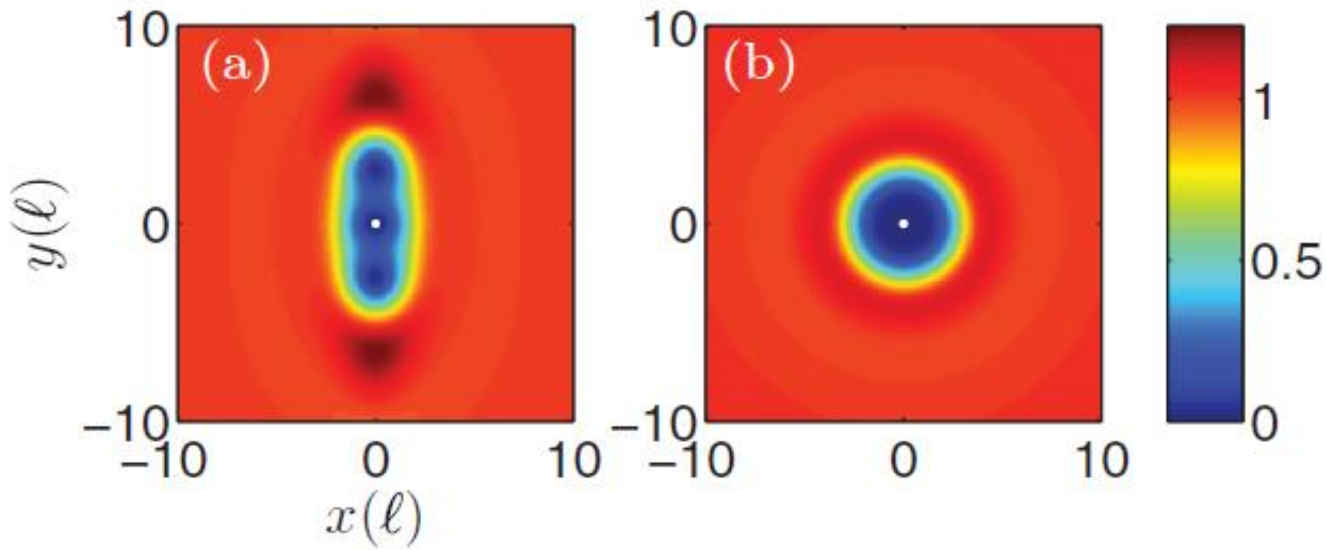
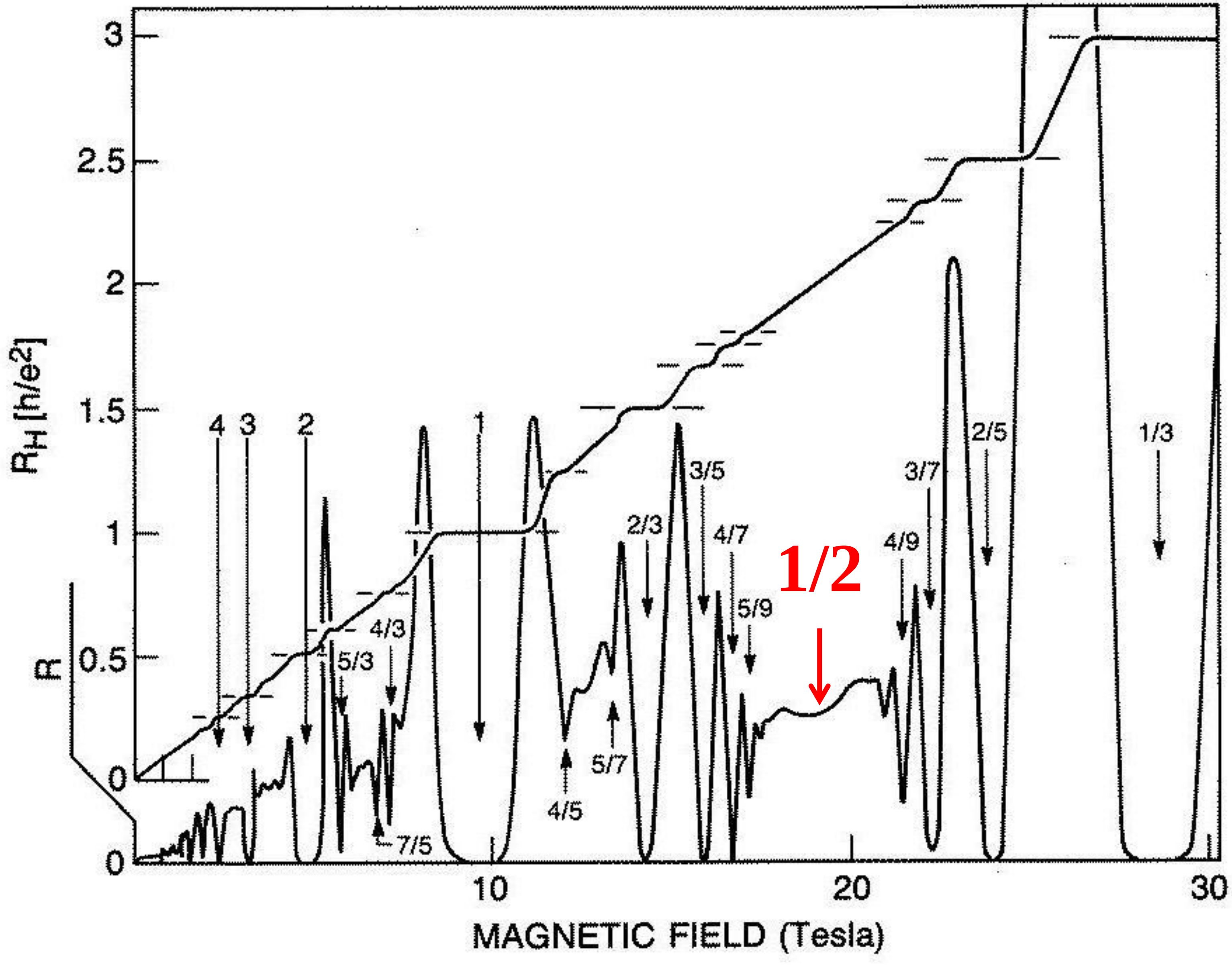


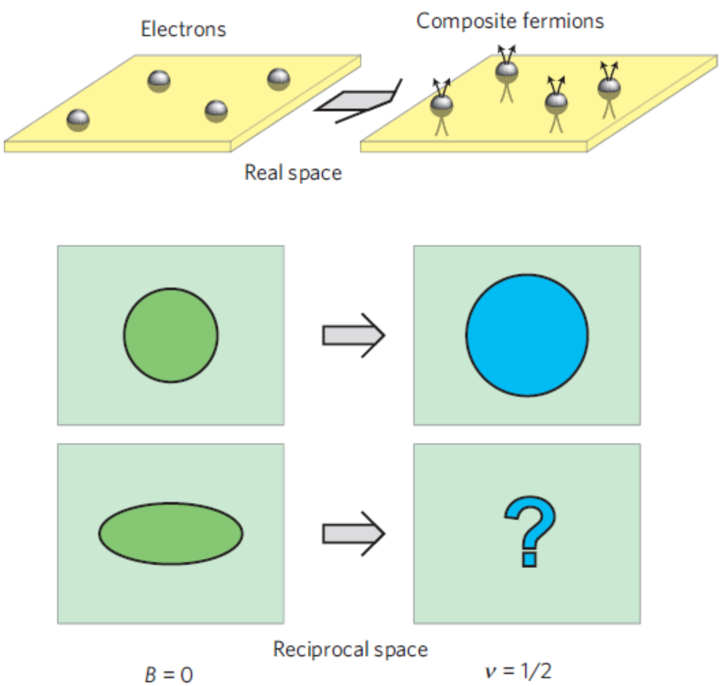
FIG. 4. (Color online) The pair correlation function for $\nu = 1/3$ Laughlin state with $\gamma = 1/2$ (a) and $\gamma = 0$ (b) of an $N = 10$ system.

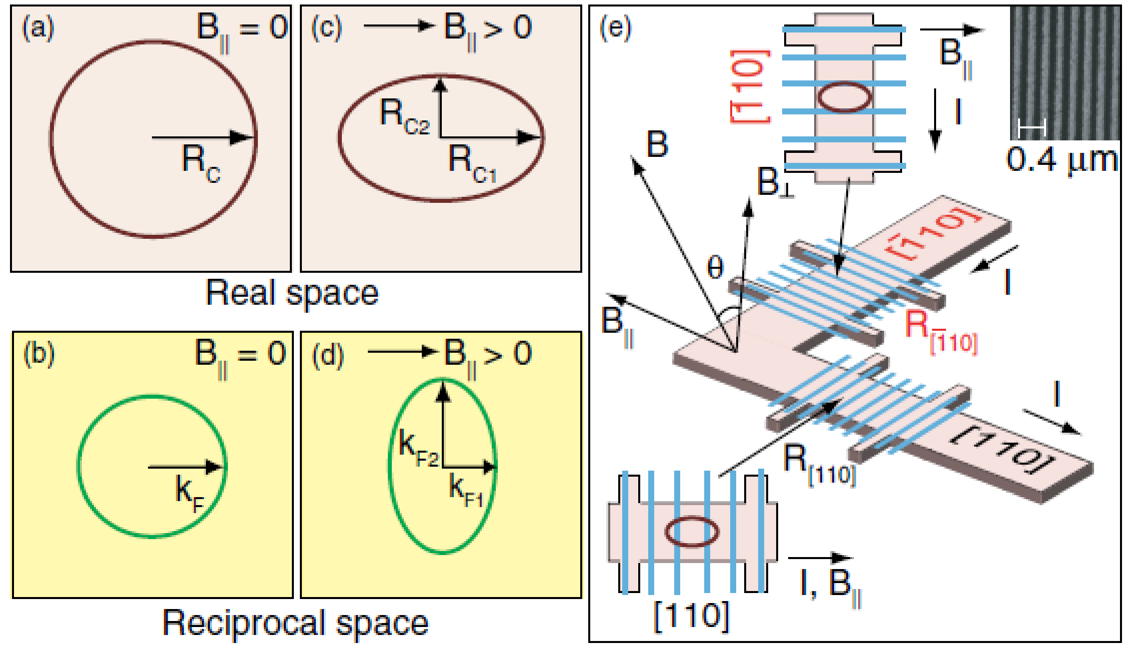
From R. Qiu, D. Haldane, X. Wan, KY and S. Yi 12. Related work by B. Yang et al. 12, H. Wang, R. Narayanan, X. Wan and F. C. Zhang 12.



R. Willett and J. Eisenstein

Measuring Geometry Experimentally (Shayegan group13,17)





$B_{\parallel}$ or strain affects electron dispersion: $T = \frac{1}{2}(m^{-1})^{\mu\nu}\Pi_\mu\Pi_\nu = \frac{g^{\mu\nu}\Pi_\mu\Pi_\nu}{2m_0}$

Key finding: CF Fermi surface much less anisotropic than electron, contrary to naïve Chern-Simons mean field prediction. Relation between the two unclear.

KY13: CF Fermi surface reflects Haldane's geometry!

Luttinger's theorem: Fermi surface can change shape but not area; area-preserving diffeomorphism!

Exactly Solvable Model of Geometry Change (KY 13)

Bare interaction: $V = \sum_{i<j} v(\mathbf{r}_i - \mathbf{r}_j) = \sum_{i<j} \sum_{\mathbf{q}} v_{\mathbf{q}} e^{i\mathbf{q}\cdot(\mathbf{r}_i - \mathbf{r}_j)}$

Interaction Projected to the *n*th Landau Level

(*a* is effective mass anisotropy):

$$\tilde{V} = \sum_{i<j} \sum_{\mathbf{q}} v_{\mathbf{q}} e^{i\mathbf{q}\cdot(\mathbf{R}_i - \mathbf{R}_j)} F_n(\mathbf{q}, a), \qquad F_0(\mathbf{q}, a) = e^{-(aq_x^2 + q_y^2/a)\ell^2/2}.$$

$$v(r) = v(0)e^{-r^2/(2s^2)}, \quad g = \sqrt{(a\ell^2 + s^2)/(\ell^2/a + s^2)}$$

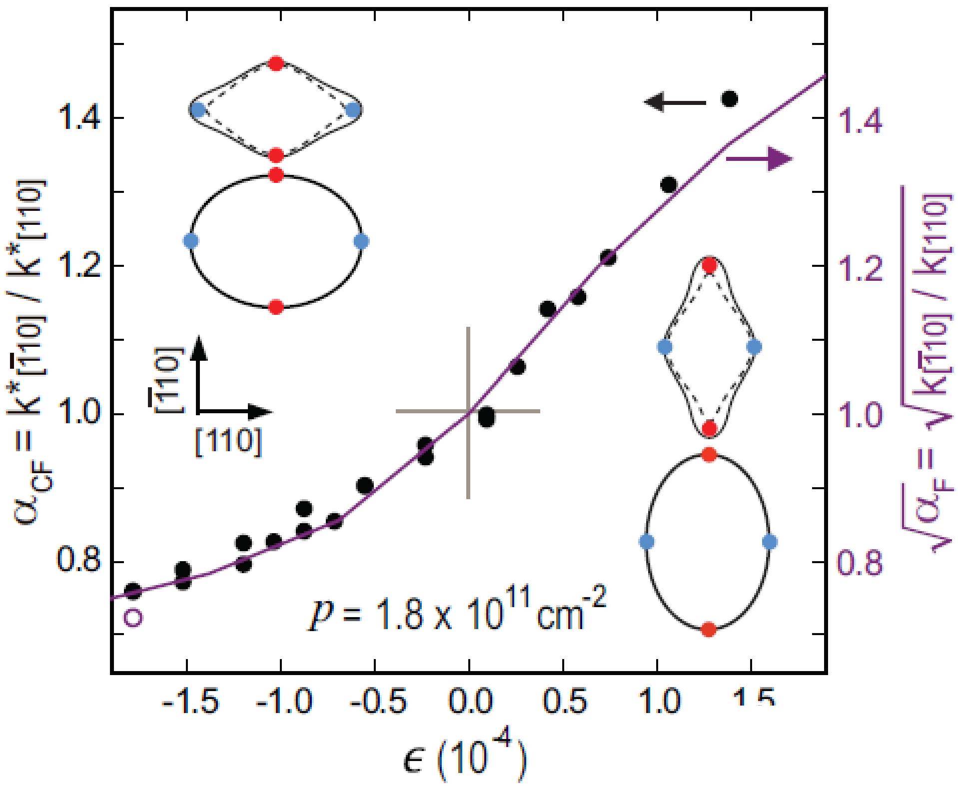
$$v_q = v_0 e^{-q^2 s^2/2}, \qquad 1 < g < a.$$

g is geometry parameter of FQH/CF Fermi surface.

Qualitative agreement with experiments.

Numerically confirmed (Ippoliti, Geraedts, Bhatt 17)

Quantitative Comparison with Experiment (Jo et al. 17)



Empirical finding:

$$\alpha_{CF} = \sqrt{\alpha_F}$$

Exact agreement if interaction range chosen to be magnetic length!

Compare with KY13: $\alpha_{CF} = \sqrt{\alpha_F}\sqrt{\dfrac{\alpha_F \ell_B^2 + s^2}{\alpha_F s^2 + \ell_B^2}}$

Numerics finds square root relation for Coulomb interaction. (Ippoliti, Geraedts, Bhatt 17)

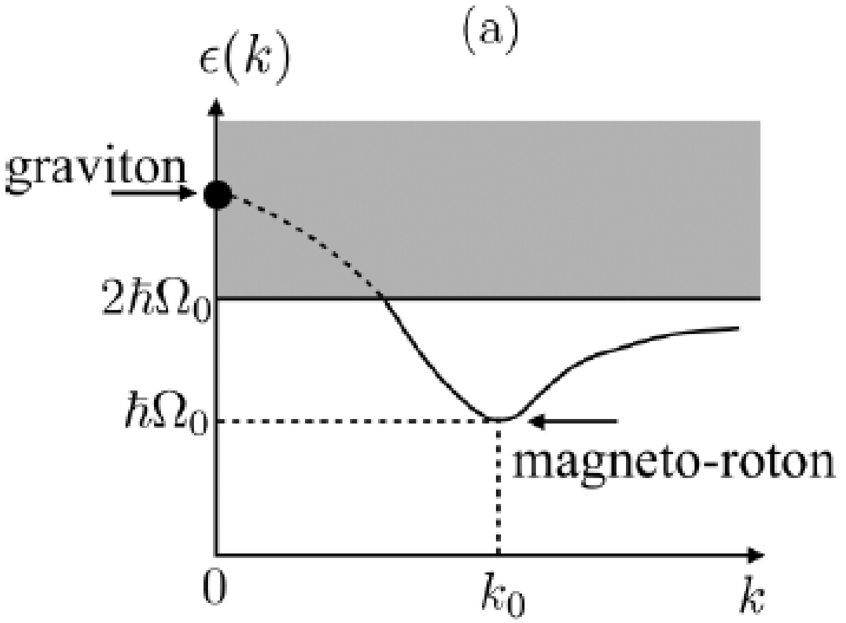
Balram and Jain 16: $\alpha_{CF} = \alpha_F$

Compare with KY13: Agreement for $s=0$!
(zero interaction-range)

$$\alpha_{CF} = \sqrt{\alpha_F}\sqrt{\frac{\alpha_F \ell_B^2 + s^2}{\alpha_F s^2 + \ell_B^2}}$$

Origin: BJ insist on n-fold zero on each electron, corresponding to ultra-local interaction.

Collective geometrical degree of freedom revealed; determines shape of correlation hole, and can be parameterized by an internal metric (Haldane 11). Its quantum dynamics gives rise to a quadruple mode that carries spin-2, or graviton (Haldane, Son; also Lee and Zhang, 1990).



How to excite the graviton?

Use “gravitational wave”.

Acoustic Wave Absorption as a Probe of Dynamical Gravitational Response of Fractional Quantum Hall Liquids

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(Dated: August 7, 2015)

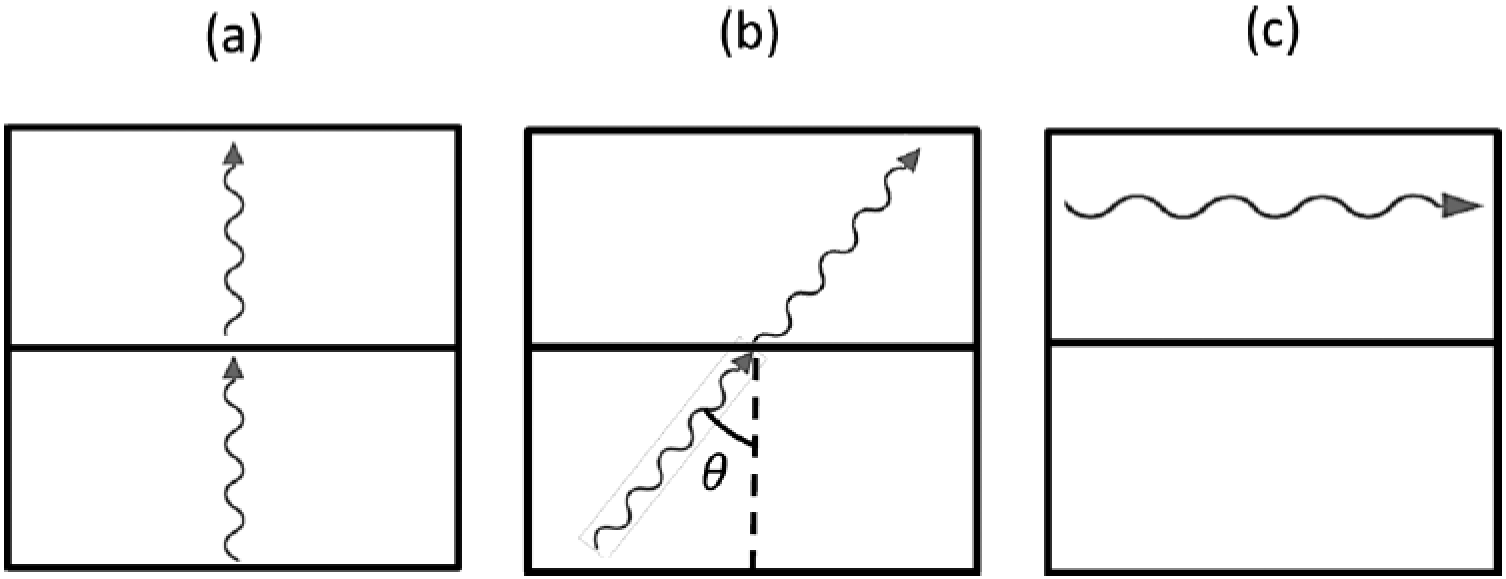
We show that acoustic crystalline wave gives rise to an effect similar to that of a gravitational wave to an electron gas. Applying this idea to a two-dimensional electron gas in the fractional quantum Hall regime, this allows for experimental study of its dynamical gravitational response. To study such response we generalize Haldane's geometrical description of fractional quantum Hall states to situations where the external metric is time-dependent. We show that such time-dependent metric (generated by acoustic or effective gravitational wave) couples to collective modes of the system, including a quadrapolar mode similar to graviton at long wave length, and magneto-roton at finite wave length. Energies of these modes can be revealed in spectroscopic measurements. We argue that such gravitational probe provides a potentially highly useful alternative probe of quantum Hall liquids, in addition to the usual electromagnetic response.

PACS numbers: 73.43.Nq, 73.43.-f

6 Aug 2015

Acoustic wave induces strain that modifies effective mass metric; behaves like gravitational wave! (KY16)

$$T = \frac{1}{2}(m^{-1})^{\mu\nu}\Pi_\mu\Pi_\nu = \frac{g^{\mu\nu}\Pi_\mu\Pi_\nu}{2m_0}$$



$$\tilde{V} = \frac{1}{2}\sum_{\mathbf{q}} V_{\mathbf{q}} e^{-(1/2)(aq_x^2+q_y^2/a)\ell^2}\overline{\rho}_{\mathbf{q}}\overline{\rho}_{-\mathbf{q}}, \quad \overline{\rho}_{\mathbf{q}} = \sum_i e^{i\mathbf{q}\cdot\mathbf{R}_i}$$

$$a = 1 + \xi(t)$$

$$\delta\tilde{V}(t) = \frac{\xi(t)}{4}\sum_{\mathbf{q}} \left(q_y^2 - q_x^2\right) V_{\mathbf{q}} e^{-(1/2)q^2\ell^2}\overline{\rho}_{\mathbf{q}}\overline{\rho}_{-\mathbf{q}}$$

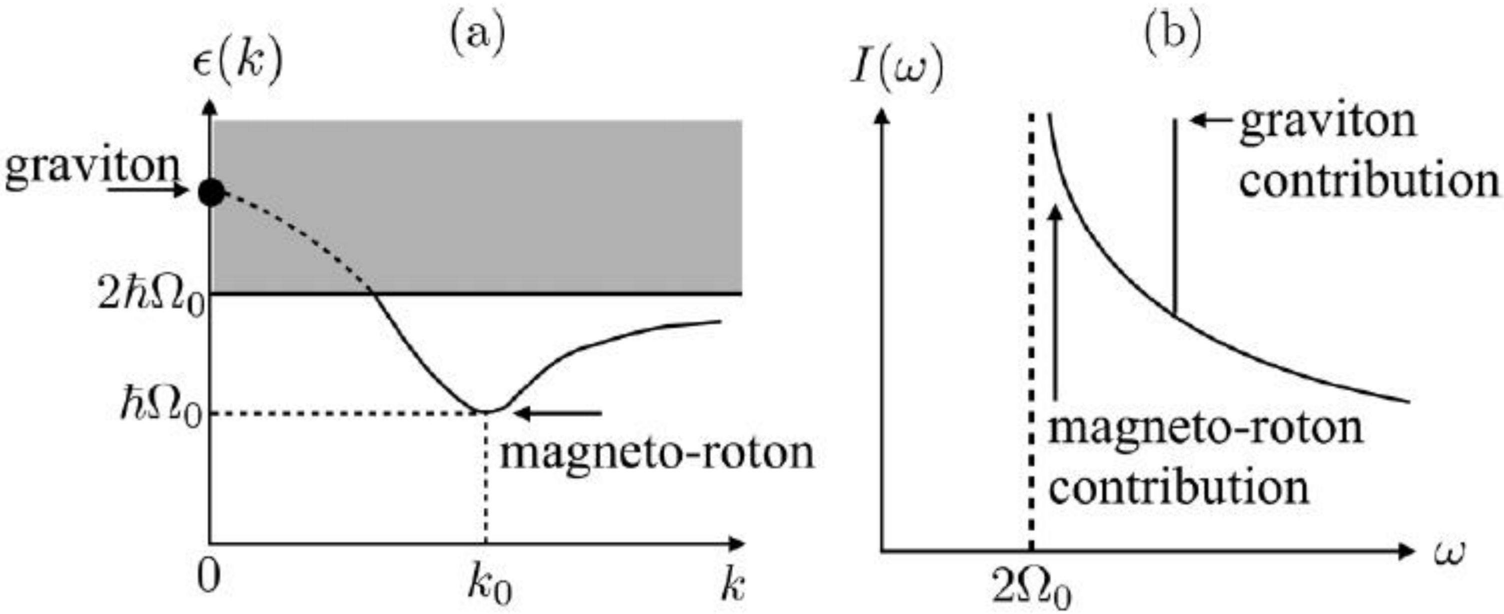
$$\hat{O} = \sum_{\mathbf{q}} \left(q_y^2 - q_x^2\right) V_{\mathbf{q}} e^{-(1/2)q^2\ell^2}\overline{\rho}_{\mathbf{q}}\overline{\rho}_{-\mathbf{q}}$$ D-wave!

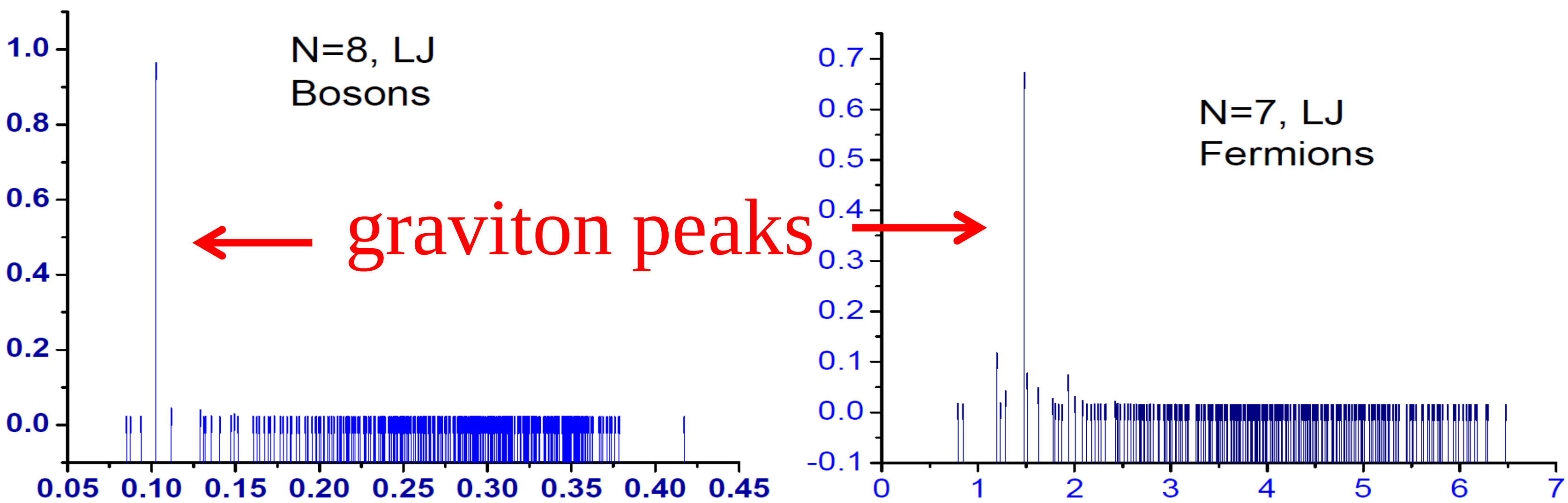
$$I(\omega) = \sum_n |\langle n|\hat{O}|0\rangle|^2 \delta(\omega - \omega_n)$$

Measurable via acoustic wave absorption rate

(gravitational wave analog of cyclotron resonance).

Prediction of Spectral Function (KY16):





E. Rezayi, F. D. M. Haldane and KY, in progress

Closing Remarks

- There is life beyond topology in fractional quantum Hall liquids; geometry a crucial ingredient!
- Geometry of fractional quantum Hall liquids can be directly measured experimentally (KY 13).
- Elementary neutral excitations of fractional quantum Hall liquids are spin-2 **gravitons**, which can be excited and probed by acoustic waves, acting like ***gravitational waves*** (KY16).
- Time to go beyond electromagnetic response, and explore (spectroscopic) gravitational response experimentally?
- **FQHE a platform to study quantum gravity?**