Topological Hybrid-Materials towards Robust Quantum Computation

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Outline

- Introduction
- Universal gate for Majorana qubit in nanowires: 1D
- Zero-energy Majorana bound states in vortex: 2D
- All-dielectric topological photonics
- Summary

Topological superconductivity in 1D

- Model system: Lutchyn, Sau and Das Sarma, PRL 105, 077001 (2010)

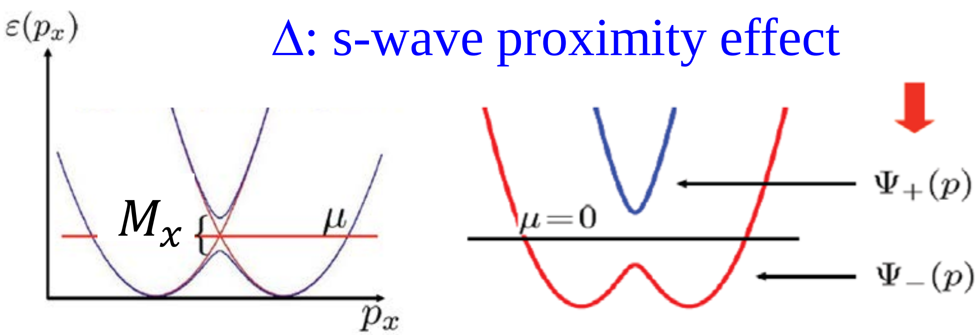
$$H = t \sum_{\langle i,j\rangle,\sigma} c^\dagger_{i\sigma} c_{j\sigma} + (\mu + eV) \sum_{i,\sigma} c^\dagger_{i\sigma} c_{i\sigma} + \frac{\eta}{2} \sum_{i,\sigma,\sigma'} c^\dagger_{i+1,\sigma} (i\sigma_y)_{\sigma\sigma'} c_{i,\sigma'}$$

$$+ \sum_{i,\sigma} c^\dagger_{i,\sigma} (M_x \sigma_x)_{\sigma\sigma'} c_{i,\sigma'} + \sum_i \Delta c^\dagger_{i,\uparrow} c^\dagger_{i,\downarrow} + h.c.$$

c.f. A. Kitaev (2001)

- effect of SOC & Zeeman field

Δ: s-wave proximity effect

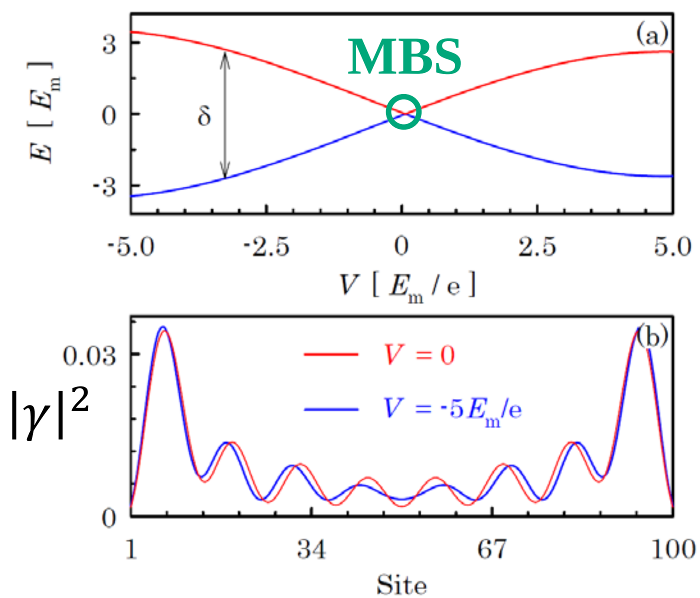


- condition for topological state

$$M_x^2 > (\mu + eV)^2 + \Delta^2$$

$$\gamma^\dagger = \int d\vec{r} \sum_\sigma \left[u_\sigma(\vec{r}) c^\dagger_\sigma(\vec{r}) + v_\sigma(\vec{r}) c_\sigma(\vec{r}) \right]$$

- spectrum of quasiparticle

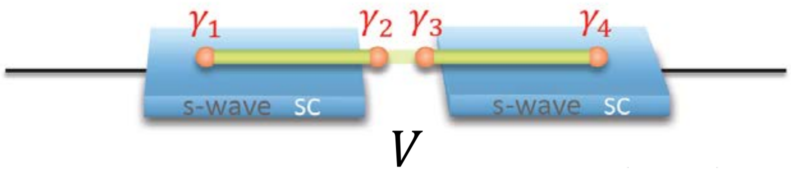


only two states inside gap

Majorana parity qubit

Z. Wang & X. Hu et al.: Sci. Rep. (in press, arXiv.1607.08491)

- Josephson-Majorana junction:



$\theta = \phi_1 - \phi_2$

quantum tunneling of MFs

$$\mathcal{H} = iE_m \cos\frac{\theta}{2}\gamma_2\gamma_3 + i\delta_L\gamma_1\gamma_2 + i\delta_R\gamma_3\gamma_4$$

- voltage bias: ① ac Josephson effect → $\theta(t) \propto \omega t$

  ② induce MQP interaction $\delta_{L,R}$: $\delta_{L,R} \ll E_m < V < \Delta$

Conservation of whole parity: one qubit from two nanowires

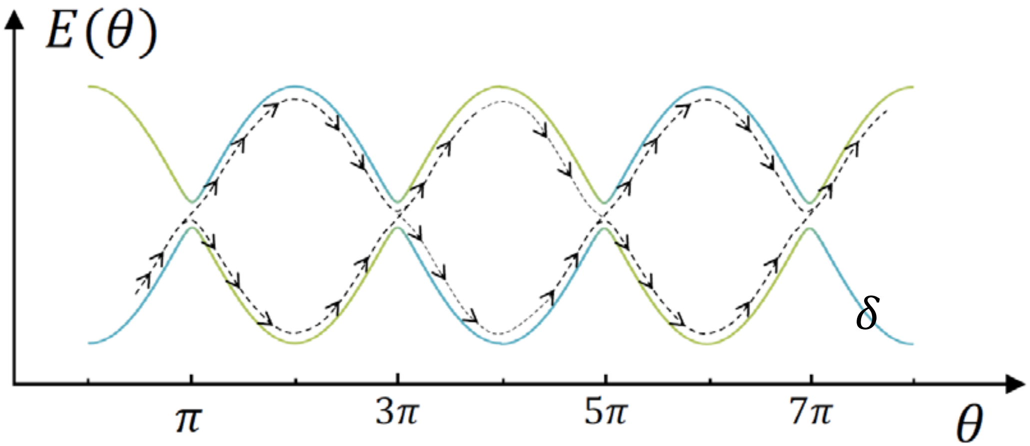
- basis: $i\gamma_2\gamma_3|0,1\rangle = \pm|0,1\rangle$
- qubit state: $\psi_0|0\rangle + \psi_1|1\rangle$

- Time-dependent Schrodinger equation: $\delta = \delta_L + \delta_R$

$$i\hbar\frac{d}{dt}\begin{bmatrix}\psi_0\\ \psi_1\end{bmatrix} = \begin{bmatrix} E_\mathrm{m}\cos\frac{\theta(t)}{2} & \delta \\ \delta & -E_\mathrm{m}\cos\frac{\theta(t)}{2}\end{bmatrix}\begin{bmatrix}\psi_0\\ \psi_1\end{bmatrix}$$

Landau-Zener-Stückelberg interference

- Accumulation of quantum phase at LZ transitions: Stückelberg (1932)



E. C. G. Stückelberg

- Two frequencies: ○ fast process: superconducting phase evolution ⇔ ω

  ○ slow process: variation of Majorana qubit state

Floquet theory:

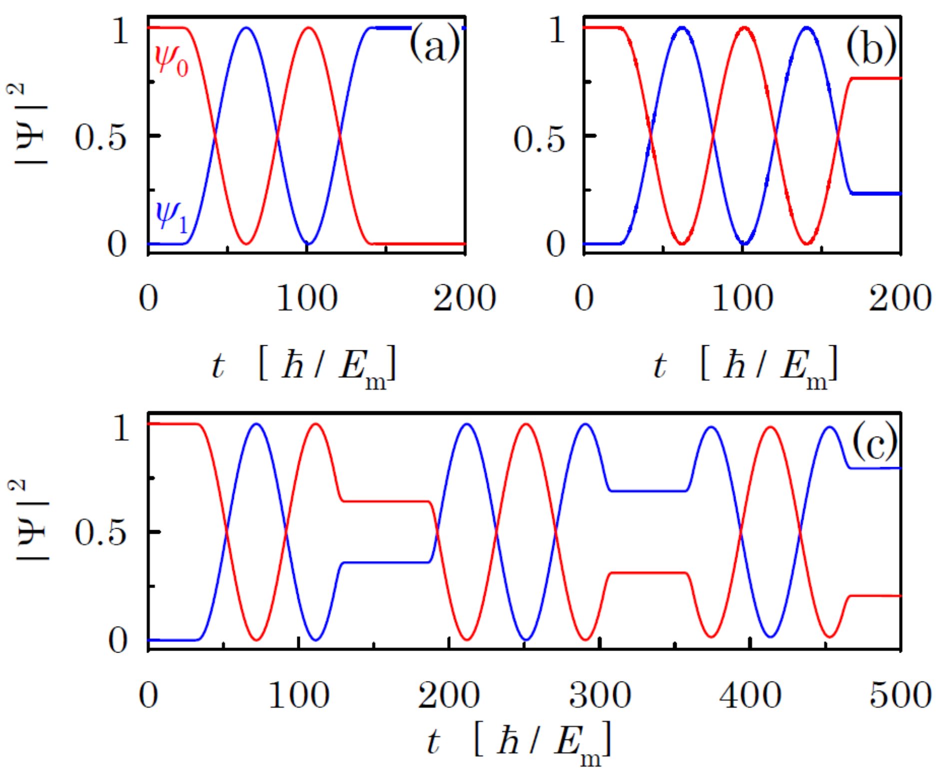
$$\omega_{\mathrm{m}} = \delta J_0(4E_{\mathrm{m}}/\hbar\omega)/\hbar$$

$$|\psi_0(t)|^2 = \cos^2(\omega_{\mathrm{m}}t/2)$$

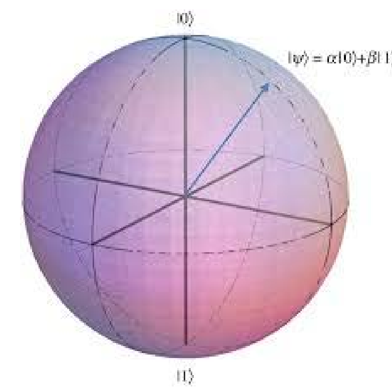
$$|\psi_1(t)|^2 = \sin^2(\omega_{\mathrm{m}}t/2)$$

LZS interferometry for Majorana qubit

- Weights of Majorana parity states: $\delta / E_{\mathrm{m}} = 0.04$



Bloch sphere



LZS interferometry → universal gate for Majorana quibt

- Control on phases of Majorana parity states:

zero voltage →

θ: constant; $\delta = 0$

$$i\hbar \frac{d}{dt}\begin{bmatrix}\psi_0\\ \psi_1\end{bmatrix} = \begin{bmatrix} E_{\mathrm{m}}\cos\frac{\theta(t)}{2} & \delta \\ \delta & -E_{\mathrm{m}}\cos\frac{\theta(t)}{2}\end{bmatrix}\begin{bmatrix}\psi_0\\ \psi_1\end{bmatrix}$$

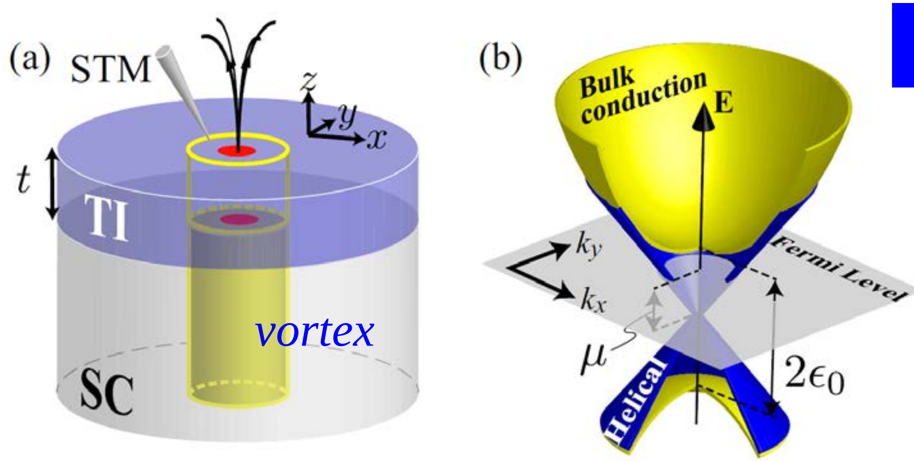
Hybrid structure of s-SC and 3D topological insulator

T. Kawakami and XH: Phys. Rev. Lett. 115, 177001 (2015)

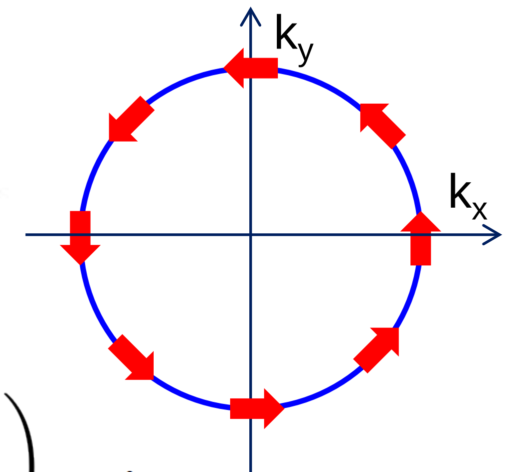
- Geometry

Fu and Kane, PRL 100 (2008) 96407

J. F. Jia Group: PRL 114 (2015) 017001



surface state of TI



- Hamiltonian

$$\mathcal{H}_{\rm BdG} = \begin{pmatrix} \hat{\mathcal{H}}_{\rm TI}(\boldsymbol{k}) & \hat{\Delta} \\ \hat{\Delta}^{\dagger} & -\hat{\mathcal{H}}^{*}_{\rm TI}(-\boldsymbol{k}) \end{pmatrix} \quad \hat{\Delta} = e^{i\phi}\Delta_0 i\hat{s}_y$$

$$\hat{\mathcal{H}}_{\rm TI} = \epsilon\hat{\sigma}_z - iv_{\rm F}\left[e^{-i\phi\hat{s}_z}\left(\partial_r\hat{s}_x + \frac{1}{r}\partial_\phi\hat{s}_y\right) + \partial_z\hat{s}_z\right]\hat{\sigma}_x - \mu$$

σ: orbital
s: spin

$$\epsilon = -\epsilon_0 - \frac{1}{2m}\left(\partial_r^2 + \frac{1}{r}\partial_r + \frac{1}{r^2}\partial_\phi^2 + \partial_z^2\right)$$

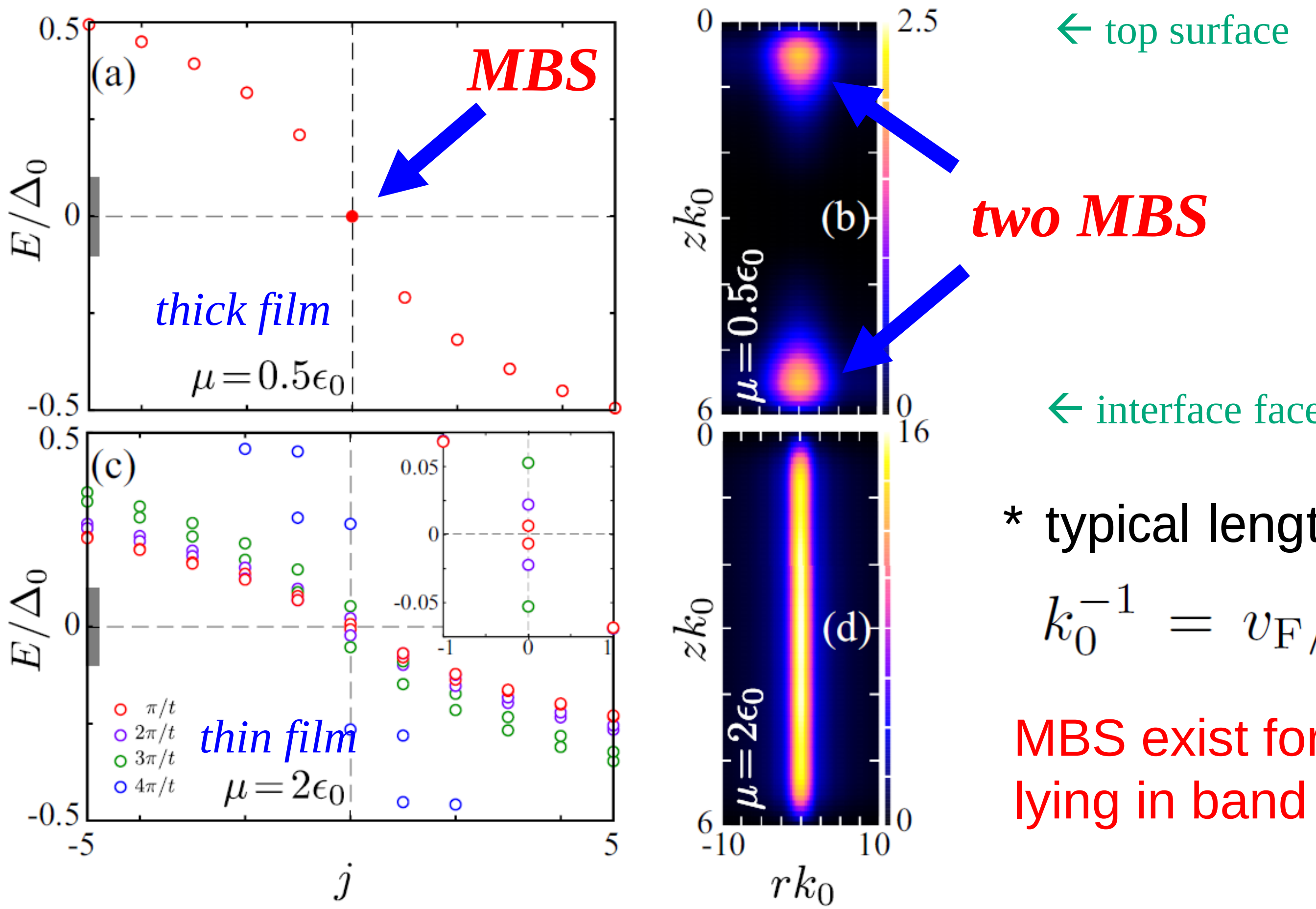
band inversion → topological insulating state

- Parameters:

* Band gap of Bi_2Te_3: $2\varepsilon_0$~0.2eV
* $v_{\rm F} \simeq 0.2\ {\rm nm\cdot eV}$
* SC gap of $NbSe_2$: $\Delta=0.02\varepsilon_0$~2meV
* chemical potential: $\mu=0.5\varepsilon_0$~$2\varepsilon_0$

MBS in hybrid structure

- Energy dispersion and distribution of DOS of quasiparticles



* typical length

$$k_0^{-1} = v_F/\epsilon_0 \simeq 2\text{nm}$$

MBS exist for Fermi level lying in band gap of TI.

Fu and Kane, PRL **100**, 96407 (2008) Hosur *et al.*, PRL **107**, 097001 (2011)

Z.-Z. Li, F.-C. Zhang, and Q.-H. Wang, Sci. Rep. **4**, 6363 (2014).

Capturing MBS as a single quantum state

- Total angular momentum of quasiparticle: good quantum number

$$j = l + \frac{s}{2} - \frac{1}{2}$$

spin moment: $s = \pm 1$

phase winding of vortex

orbital moment: l: integer → spatial distribution

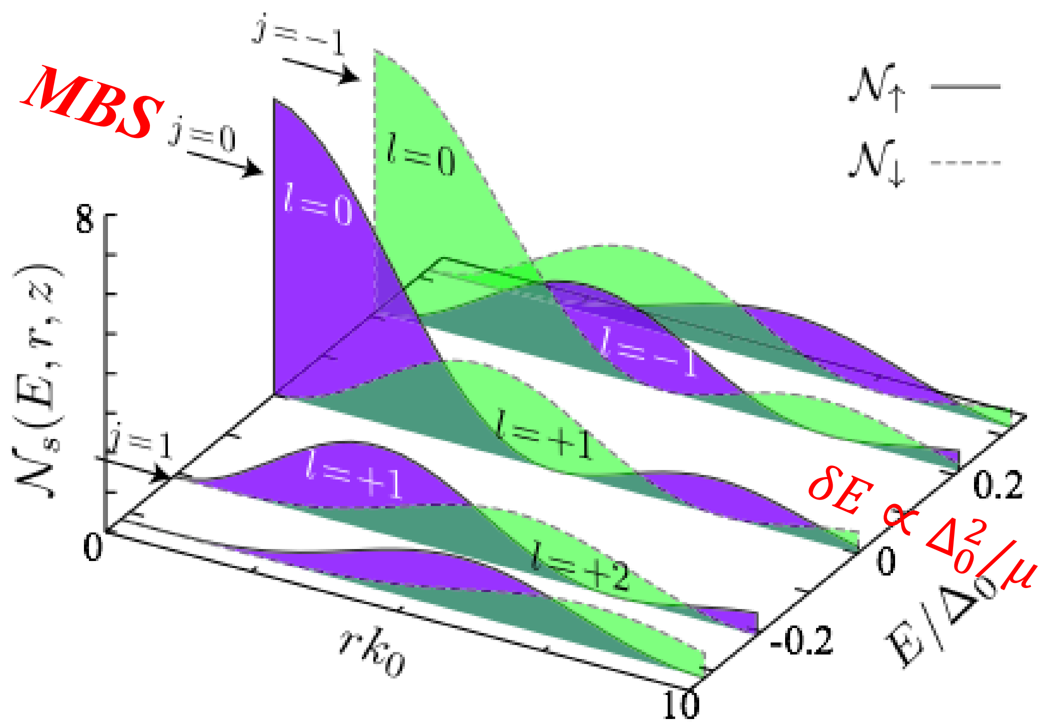
MBS: zero energy ➔ $j = 0$

$$E \propto j$$

* two components in each state: spin-up and –down & two orbital angular momenta

* out-of-phase oscillations

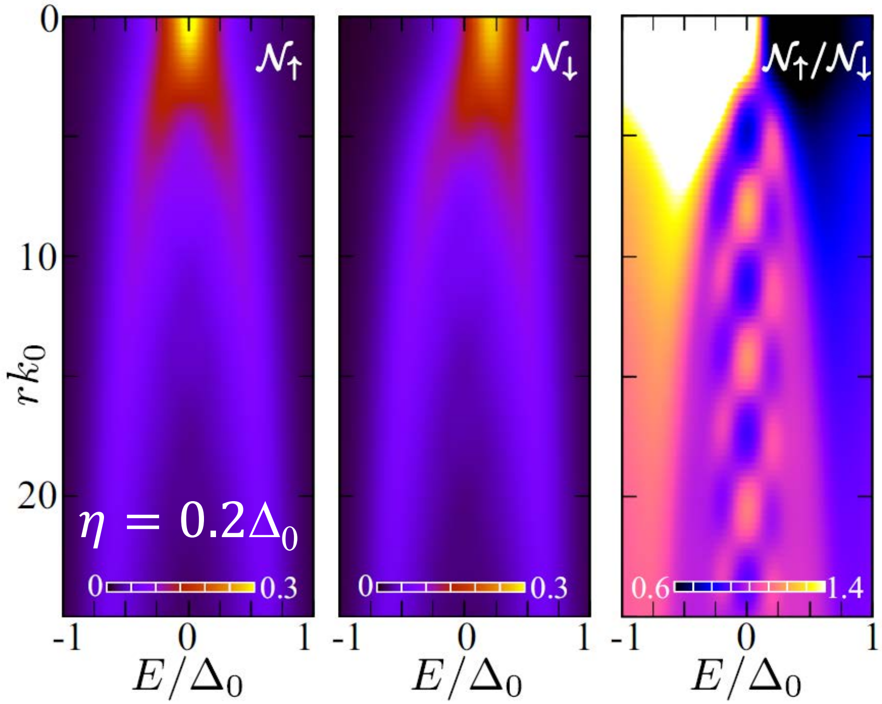
* out-of-phase between MBS & first-excited state, which carry same total LDOS



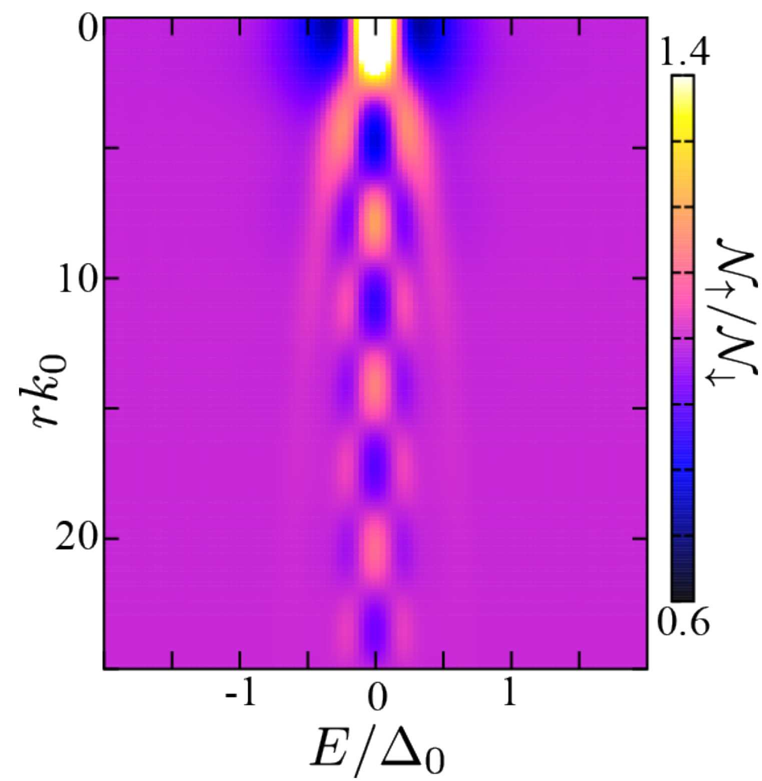
$$|\vec{u}^{\uparrow}_{0,0}| = e^{-r/\xi} J_0(rk_F)$$

Relative spin-resolved LDOS

T. Kawakami and XH: Phys. Rev. Lett. 115, 177001 (2015): cover story



hole tunneling to tip considered



power of taking relative LDOS !

Period of checkerboard pattern:
10nm & 0.4meV

Spin-polarized STM to capture MBS

Metamaterial and photonic crystal

- Maxwell's equations of electromagnetic wave

$$\nabla \cdot \boldsymbol{E} = 0 \qquad \nabla \cdot B = 0$$

$$\nabla \times \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t} \qquad \nabla \times \boldsymbol{B} = \mu\epsilon \frac{\partial \boldsymbol{E}}{\partial t}$$

$$\text{for vacuum } c = 1/\sqrt{\mu_0 \epsilon_0} = 3 \times 10^8 m/s$$

metamaterials & photonic crystal ⇔ real-space arrangements of $\overleftrightarrow{\epsilon}$ & $\overleftrightarrow{\mu}$

Chance : easy to fabricate, already yielding negative refractive index

Challenge: no Lorentz force, no spin, no spin-orbit coupling

A milestone: gyrotropic materials with Faraday effect under H_{ext}

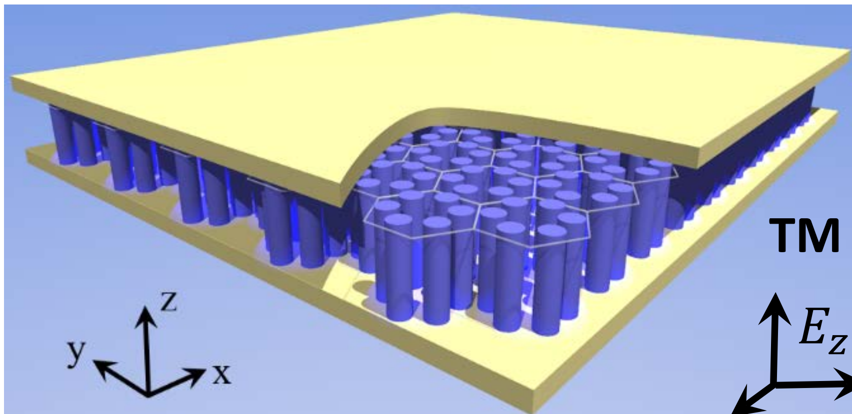
Theory: Haldane and Raghu (2008) Experiment: Wang & Cheong et al. (2009)

QAHE of photon, but limited below near infrared in frequency

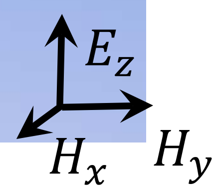
Topological photonic crystal of dielectrics

L.-H. Wu and X. Hu: PRL **114**, 223901 (2015); 胡:「応用物理」**85**, 474（2016）

- Honeycomb array of dielectric cylinders: Si, Al_2O_3, GaAs, GaN ...



TM mode



- Maxwell's equations:

$$\begin{cases} \partial_x H_y - \partial_y H_x = -\dfrac{i\omega}{c}\varepsilon(x,y)E_z \\ \partial_x E_z = -\dfrac{i\omega}{c}H_y \\ \partial_y E_z = \dfrac{i\omega}{c}H_x \end{cases}$$

harmonic mode: $E(\vec{r},t) = E(\vec{r})e^{i\omega t}$

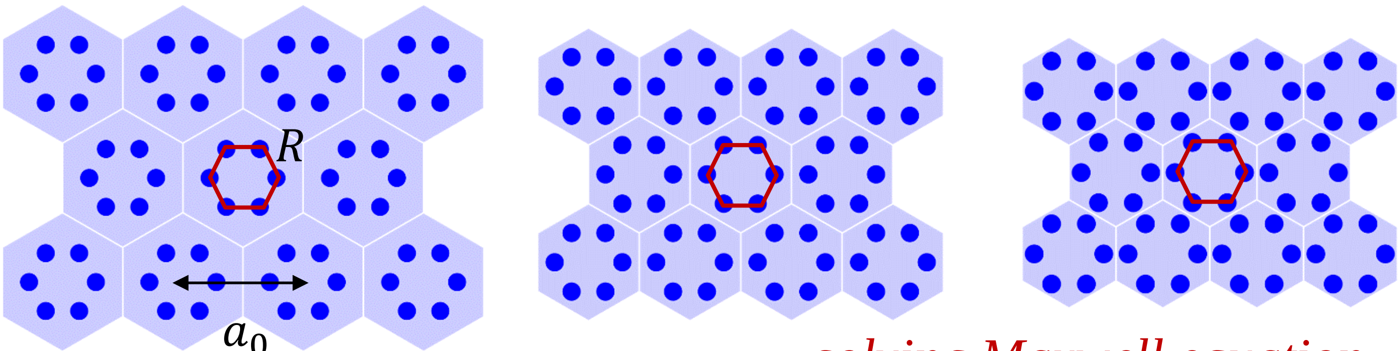
$$H(\vec{r},t) = H(\vec{r})e^{i\omega t}$$

$$-\frac{1}{\varepsilon(x,y)}\left[\partial_x^2 + \partial_y^2\right]E_z = \frac{\omega^2}{c^2}E_z$$

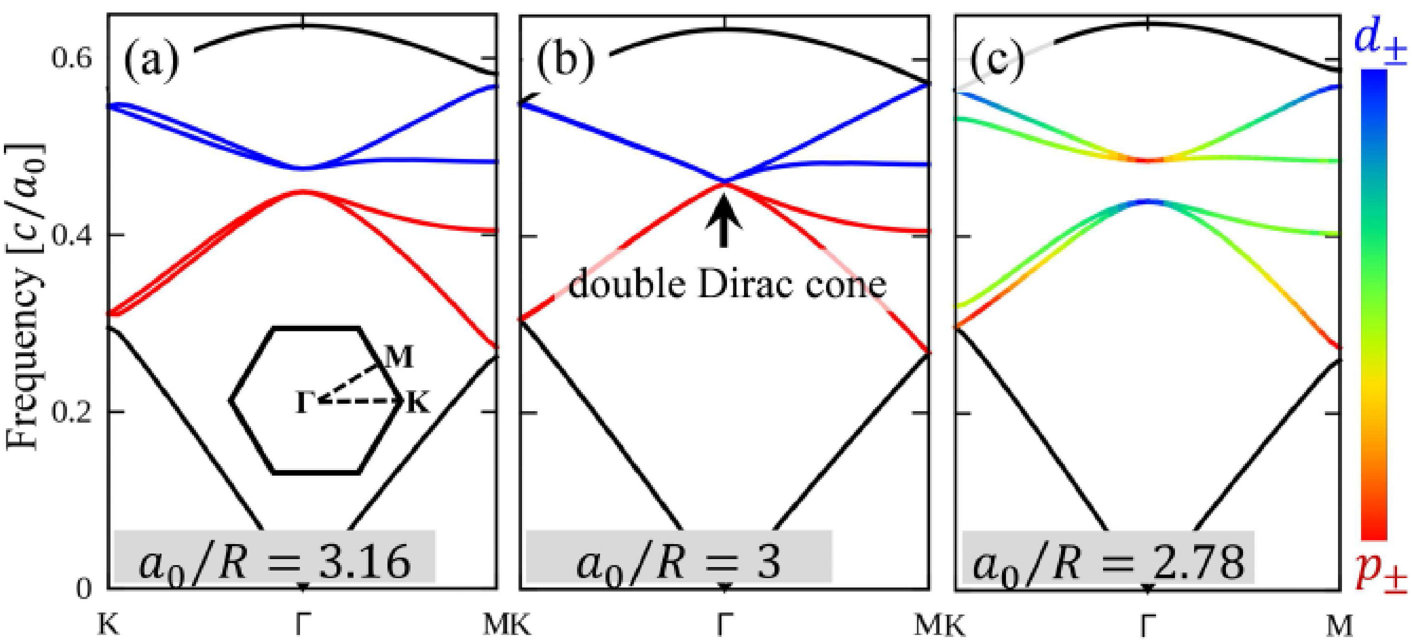
- ε periodic ➔ Bloch theory ➔ bands

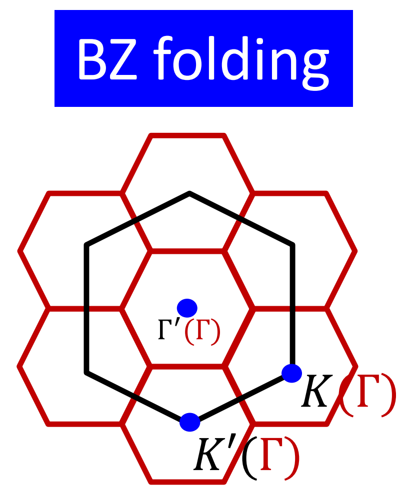
All-dielectric topological photonic crystal

- Photonic crystals derived from honeycomb structure



solving Maxwell equation → photonic band



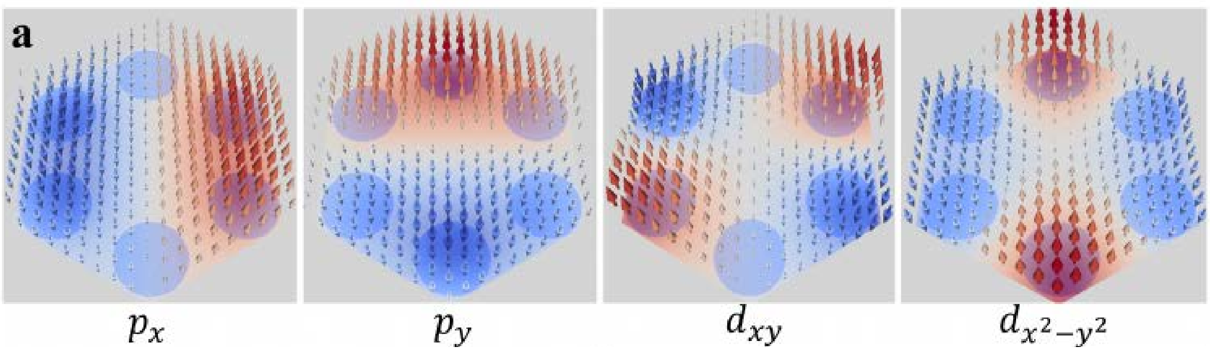


triangular lattice

honeycomb lattice

Artificial atom and emergent Kramers pair

- Distribution of E_z field on single hexagon: "atomic" orbitals



2D representations of C_{6v} point group: E' E''

$x \;\&\; y$ $\quad$ $xy \;\&\; x^2 - y^2$

- Emergent Kramers pair:

pseudo spin

$$p_\pm = (p_x \pm i p_y)/\sqrt{2}$$

sign of orbital angular momentum

$$d_\pm = (d_{x^2-y^2} \pm i d_{xy})/\sqrt{2}$$

clockwise vs counter-clockwise current

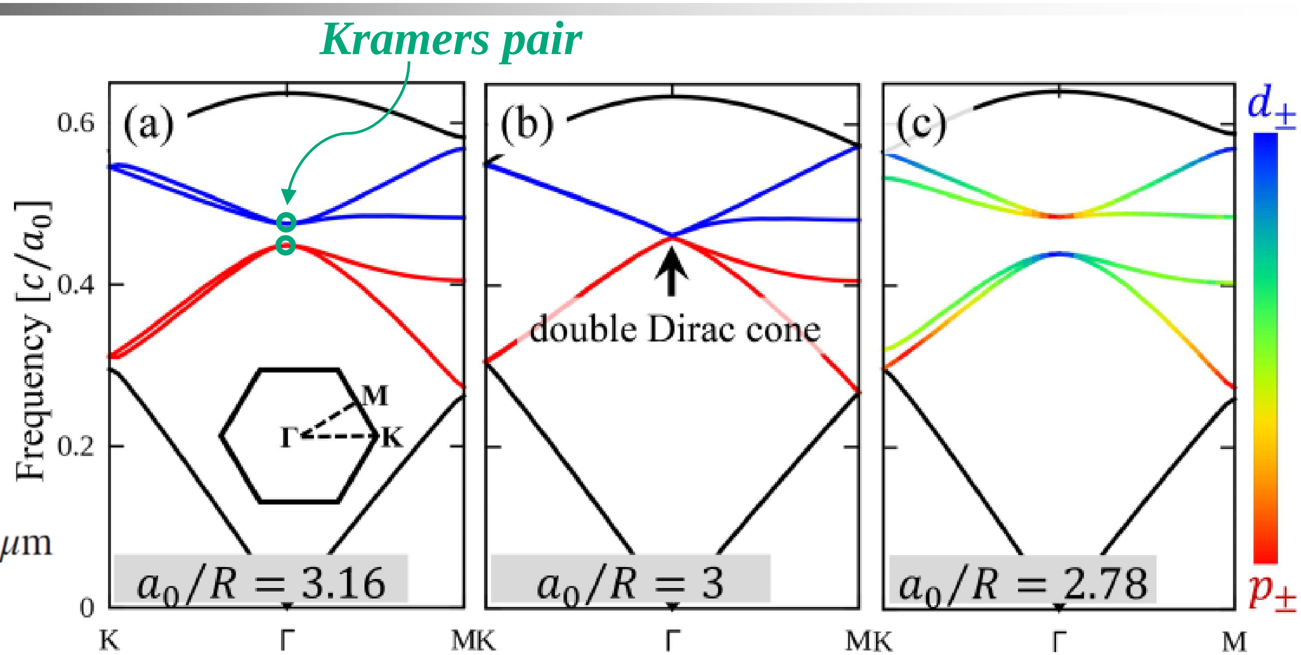
- Inversion symmetry: p-orbital: odd parity d-orbital: even parity

Topological EM state

- Band inversion

honeycomb = critical point

$\omega = 138.77$ THz with $a_0 = 1\ \mu$m



- $k \cdot p$ model: $k_\pm = k_x \pm i k_y$ $\quad \boldsymbol{k}^2 = k_x^2 + k_y^2$ $\quad$ basis $[p_+, d_+, p_-, d_-]^T$

$$H_\Gamma(\boldsymbol{k}) = \begin{pmatrix} H_+(\boldsymbol{k}) & 0 \\ 0 & H_-(\boldsymbol{k}) \end{pmatrix} \qquad H_\pm(\boldsymbol{k}) = \begin{pmatrix} M + B\boldsymbol{k}^2 & A k_\pm \\ A^* k_\mp & -M - B\boldsymbol{k}^2 \end{pmatrix}$$

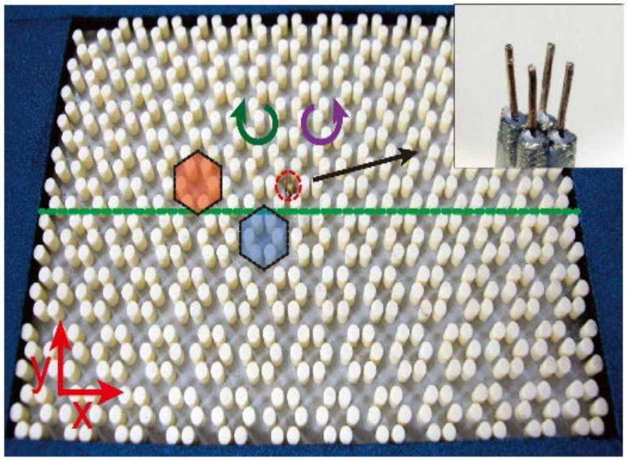
BHZ model for QSHE (2006)

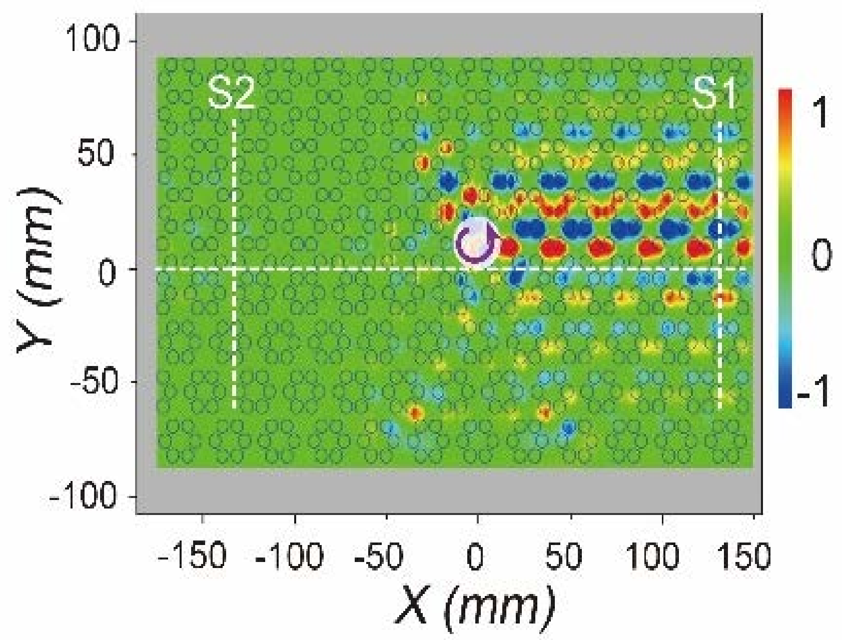
* topological crystalline insulator: L. Fu (2010) * quantum orbital Hall effect

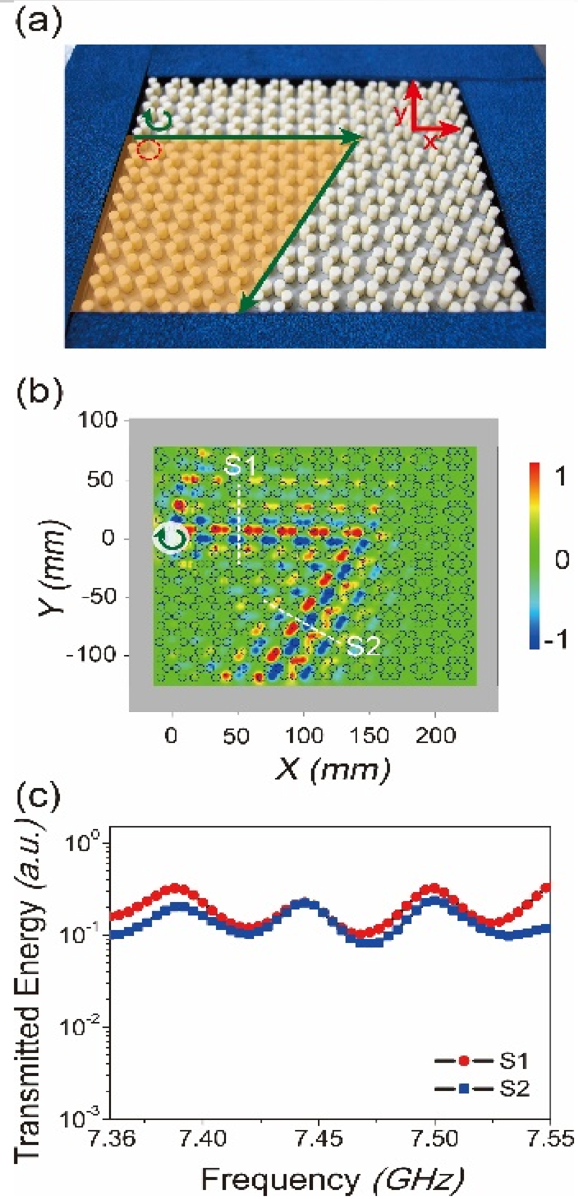
Proof-of-principle microwave experiment

Y. T. Yang, XH, Z. H. Hang et al.: arXiv.1610.07780 (PRL in press)

Al_2O_3



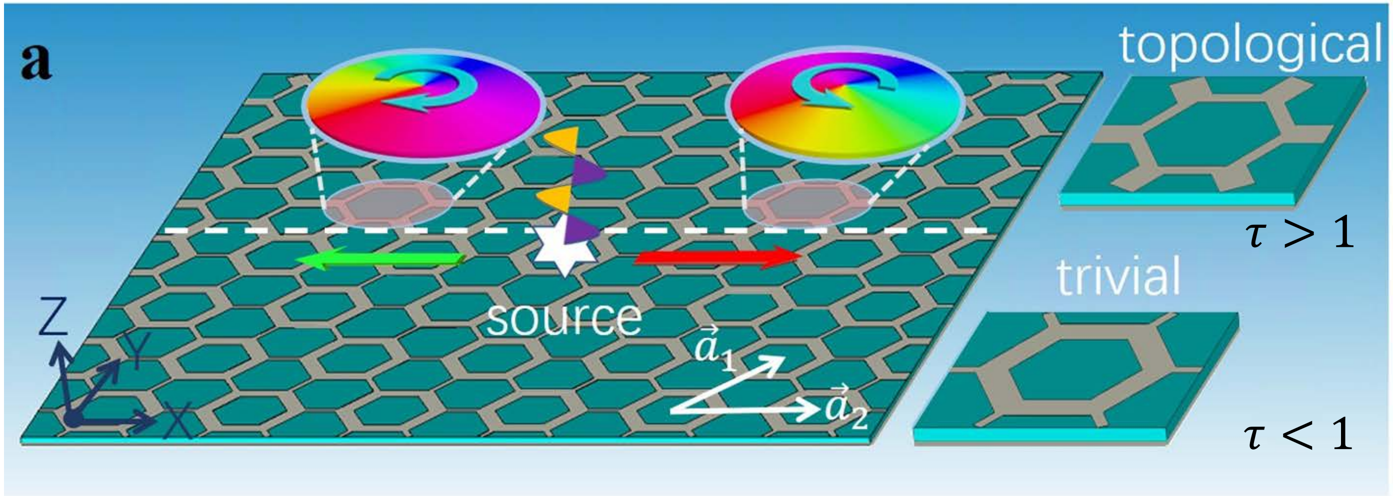




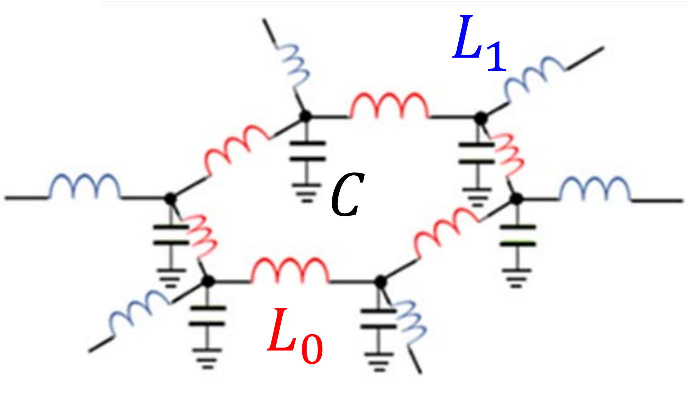
Topological LC circuit and transmission line

Y. Li, Y. Sun, T. Kariyado, H. Chen and XH, arXiv:1801.04395 (under review)

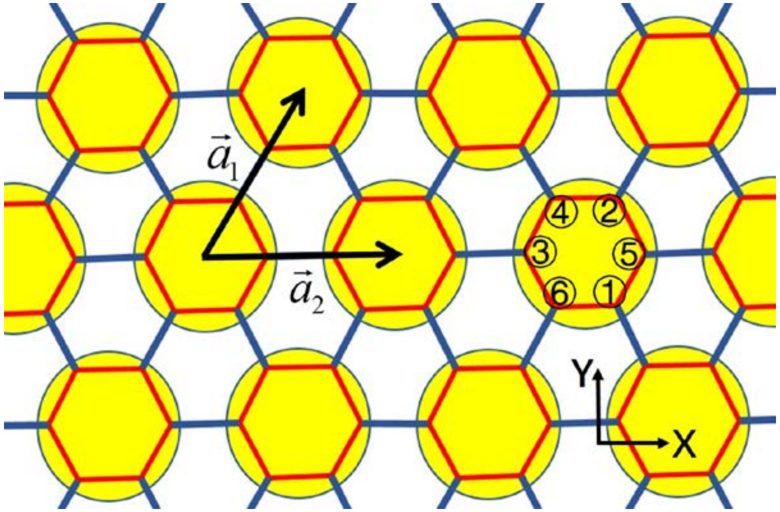
microstrip



- Lumped-element circuit of honeycomb structure

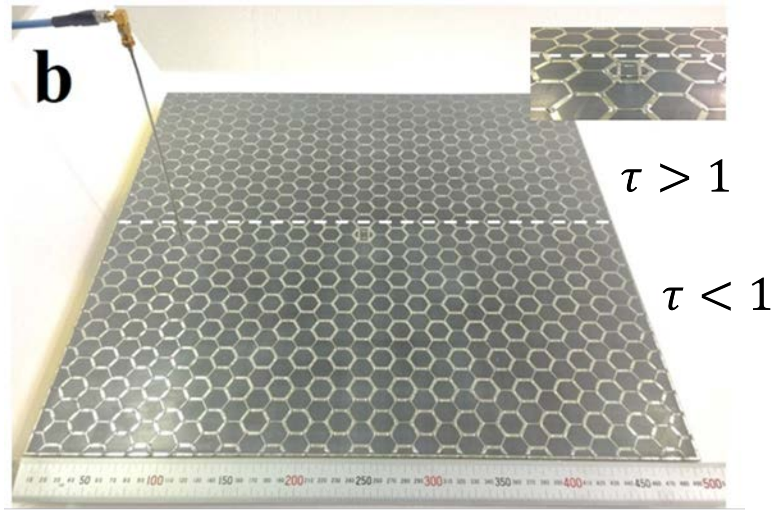


$$\tau = L_0/L_1 = w_{out}/w_{in}$$



Generating chiral EM wave by microstrip

- Experimental setup

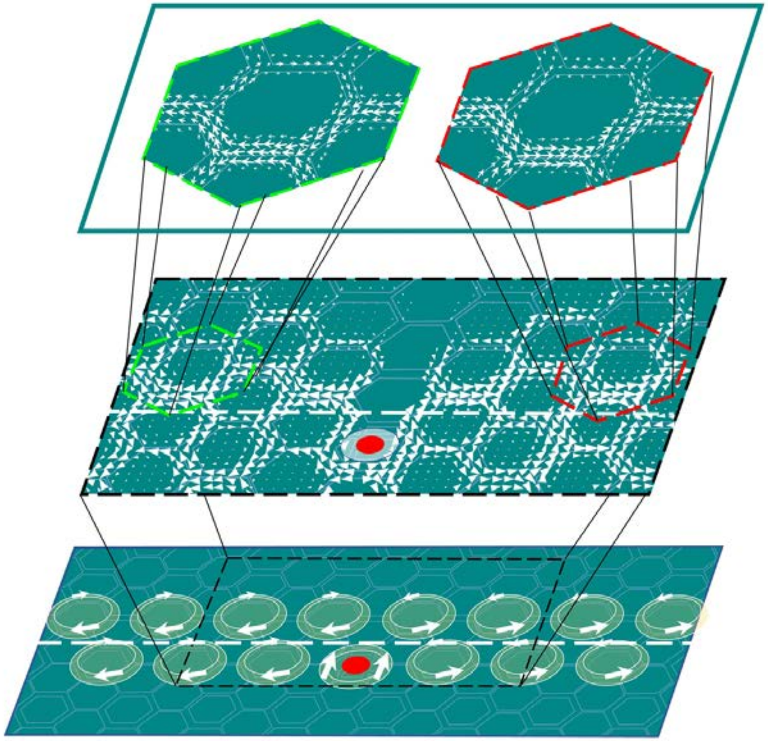


- precise measurements on amplitude & phase of E_z

➔ ***Poynting vector***

$$\boldsymbol{S} = \frac{|E_z|^2}{2\mu\omega}\left(\frac{\partial\varphi}{\partial x}\hat{\boldsymbol{x}} + \frac{\partial\varphi}{\partial y}\hat{\boldsymbol{y}}\right)$$

- Topological interface modes



- well confined along the interface
- circular vs. net energy flows

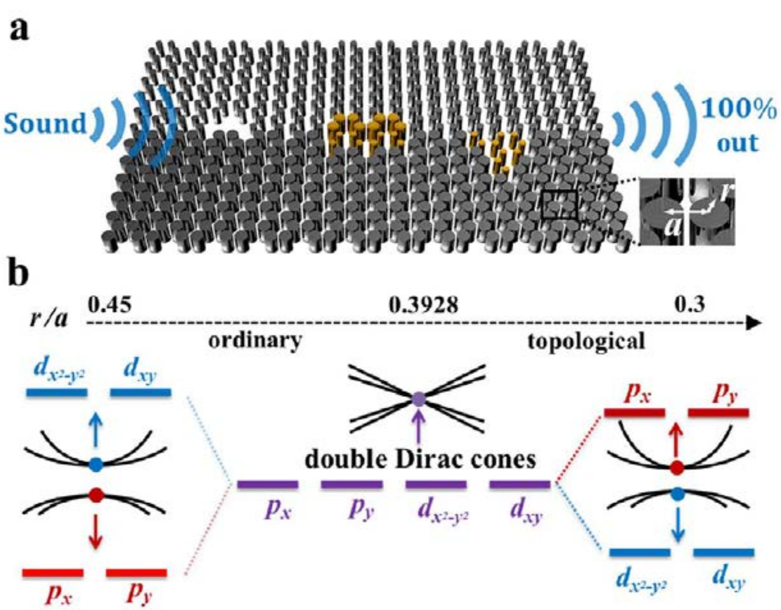
Chiral light ⇔ EM waves w OAM

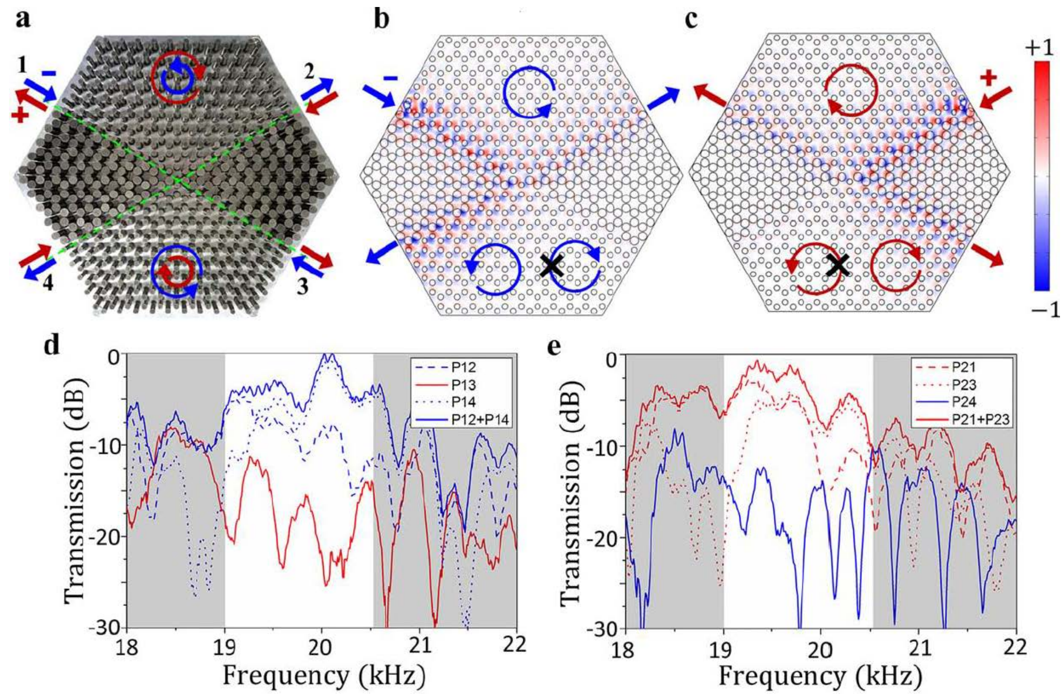
Realization in acoustic waves

- Topological phononic crystal

H. Chen, **X.-C. Sun**, M.-H. Lu and Y.-F Chen et al.: Nat. Phys. vol. 12, 1124 (2016)

- steel rods in air



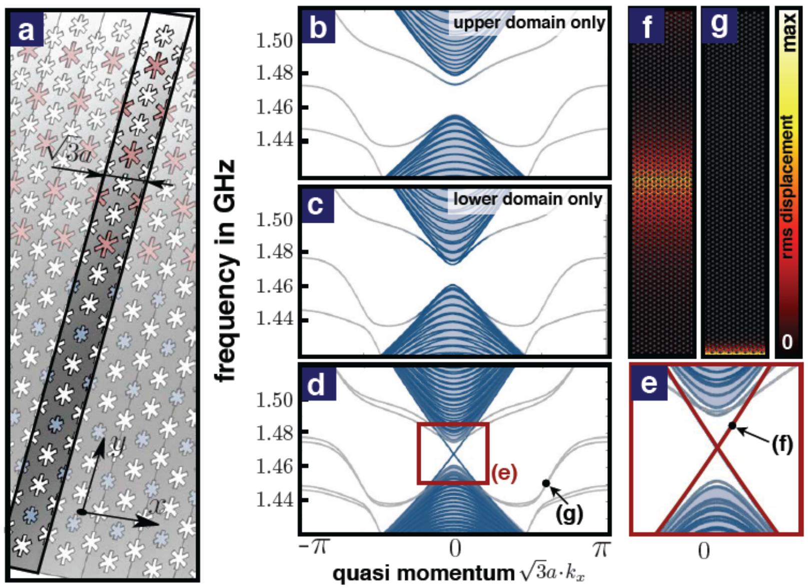


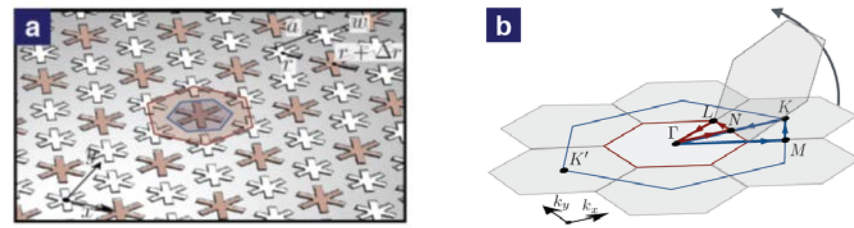
- changing r/a ratio
- “dielectric constant” smaller in rods

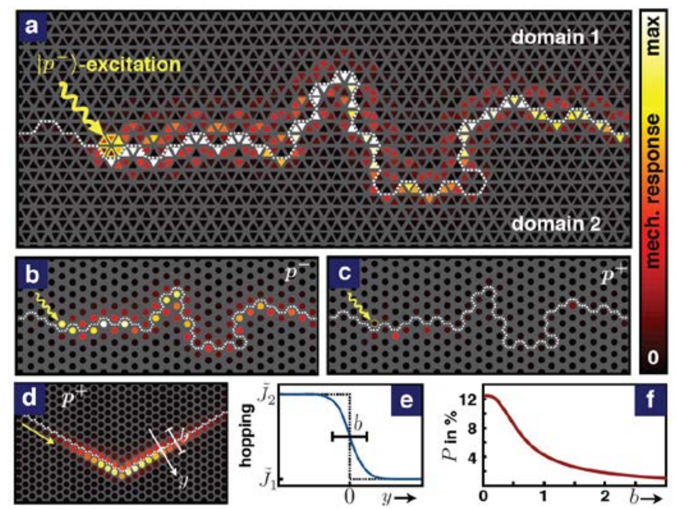
Topological optomechanical crystal

Brendel, Peano, Painter and Marquardt, PRB 97, 020102(R) (2018)

- Snow flakes for acoustic device







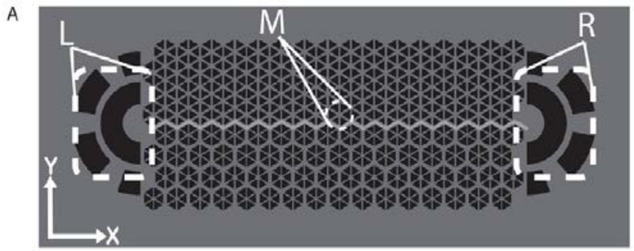
Snowflake phononic topological insulator at the nanoscale – featured in ***Physics***

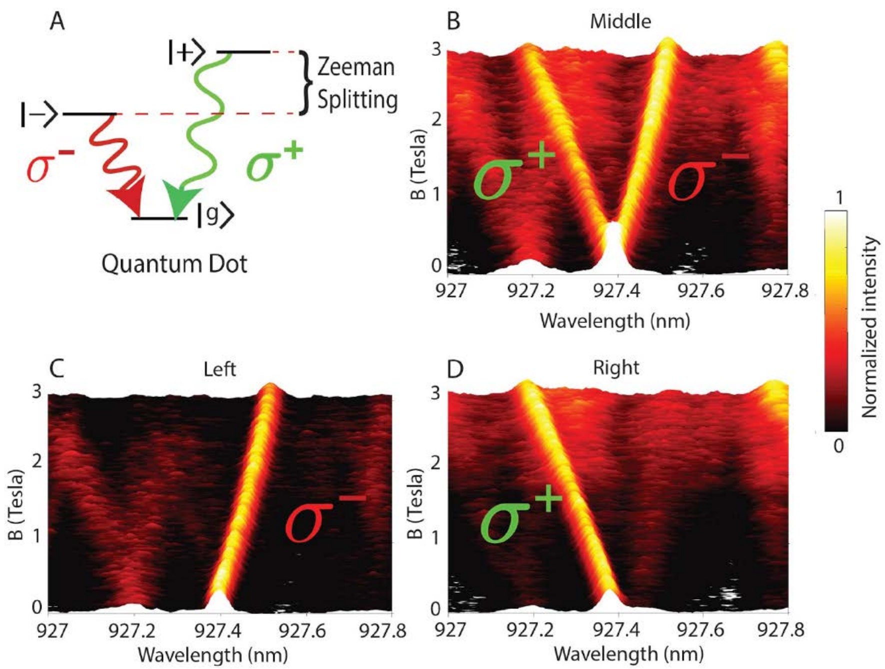
a versatile platform for generating ***arbitrary phononic circuits and networks on the chip****, which may* ***interact with hybrid quantum systems*** *including various kinds of* ***qubits coupled via surface-acoustic waves****.*

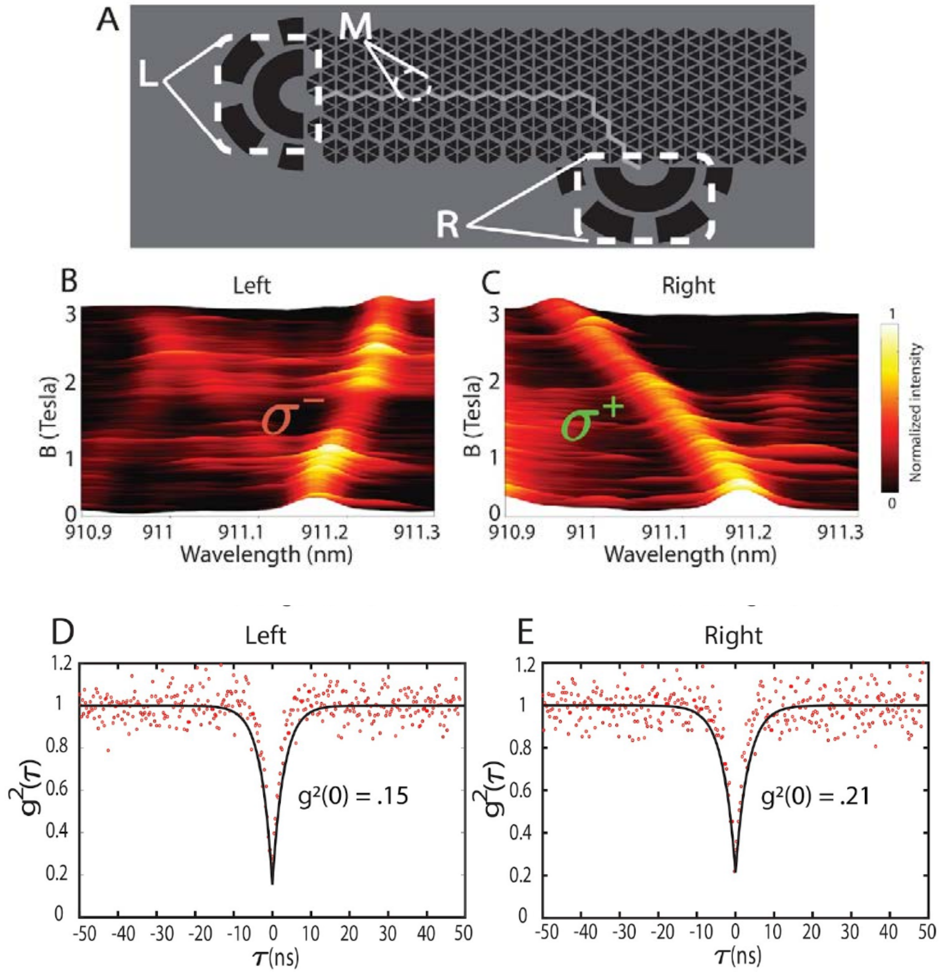
Topological quantum optics interface

S. Barik et al. : Science **359**, 666 (2018)

- GaAs





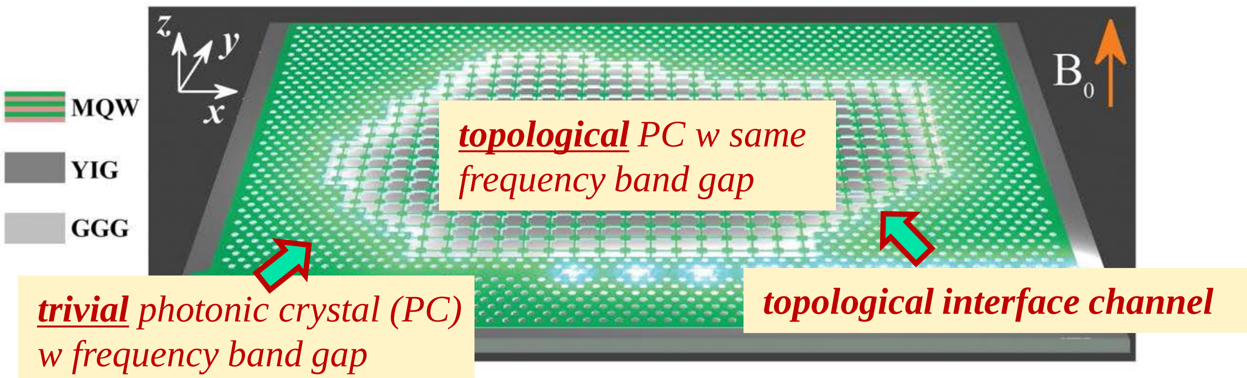


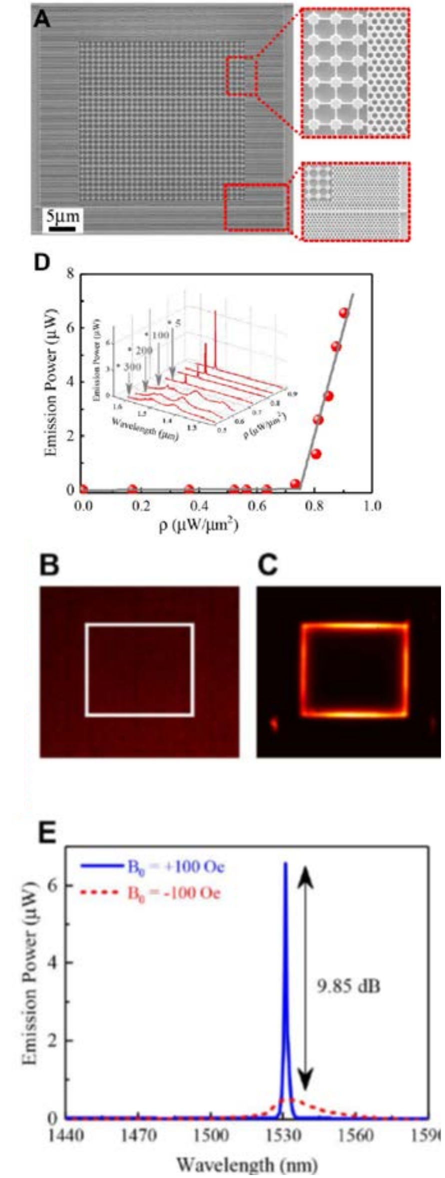
- momentum-pseudospin locking
- topology of individual photons

Towards topological lasing

- Laser based on topological edge states

B. Bahari et al., Science 10.1126/science.aao4551(2017)





Use the topological interface channel for lasing

merits: ** cavity with arbitrary shape*
** single mode → stable lasing*
** robust against defects and noises*

demerits: ** requires external magnetic field*
** only for infrared frequency*

We are now trying to vercome the demerits using our theory !

Topological states of electrons on honeycomb

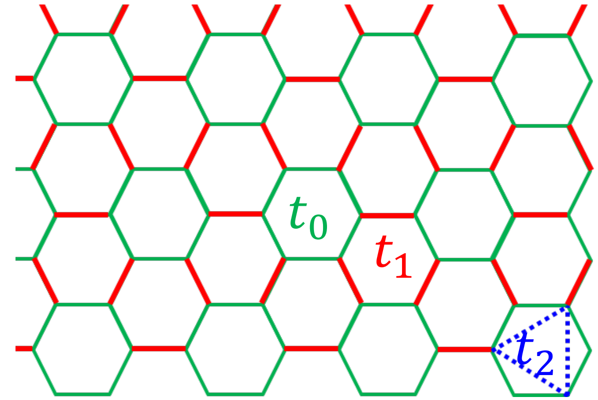
L.-H. Wu and XH: Sci. Rep. ***6****, 24347 (2016); T. Kariyado and XH, ibid* ***7****, 16515 (2017)*

- Tight-binding model: spinless electron

$$H = t_0 \sum_{\langle i,j \rangle} c_i^\dagger c_j + t_1 \sum_{\langle i',j' \rangle} c_{i'}^\dagger c_{j'}$$

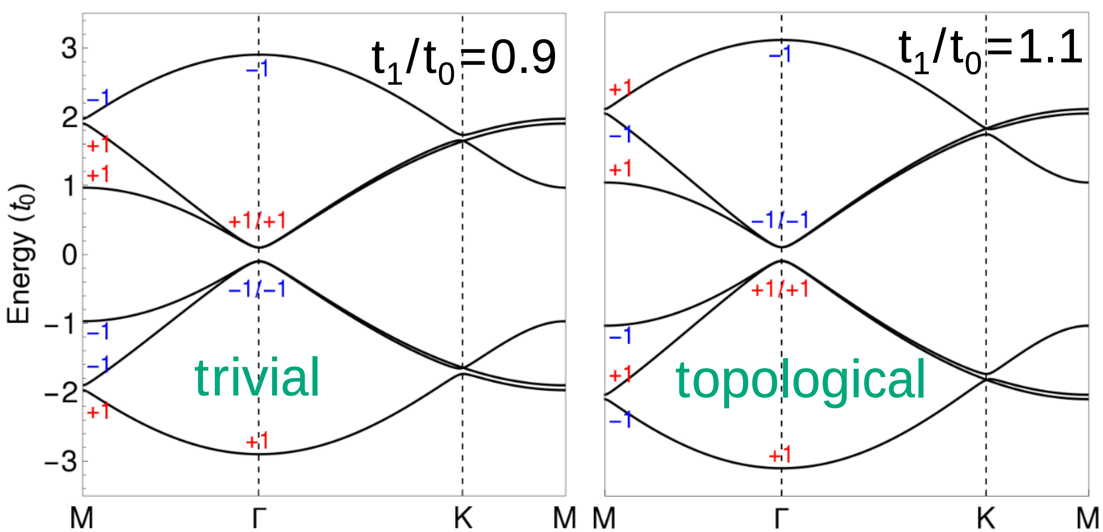
texture in nearest-neighbor (NN) hopping energy

- topological energy gap: $\delta = t_1 - t_0$



t_0 & t_1: NN t_2: Next NN

- band structure: *p-d band inversion*



- topological index: ***mirror winding number***

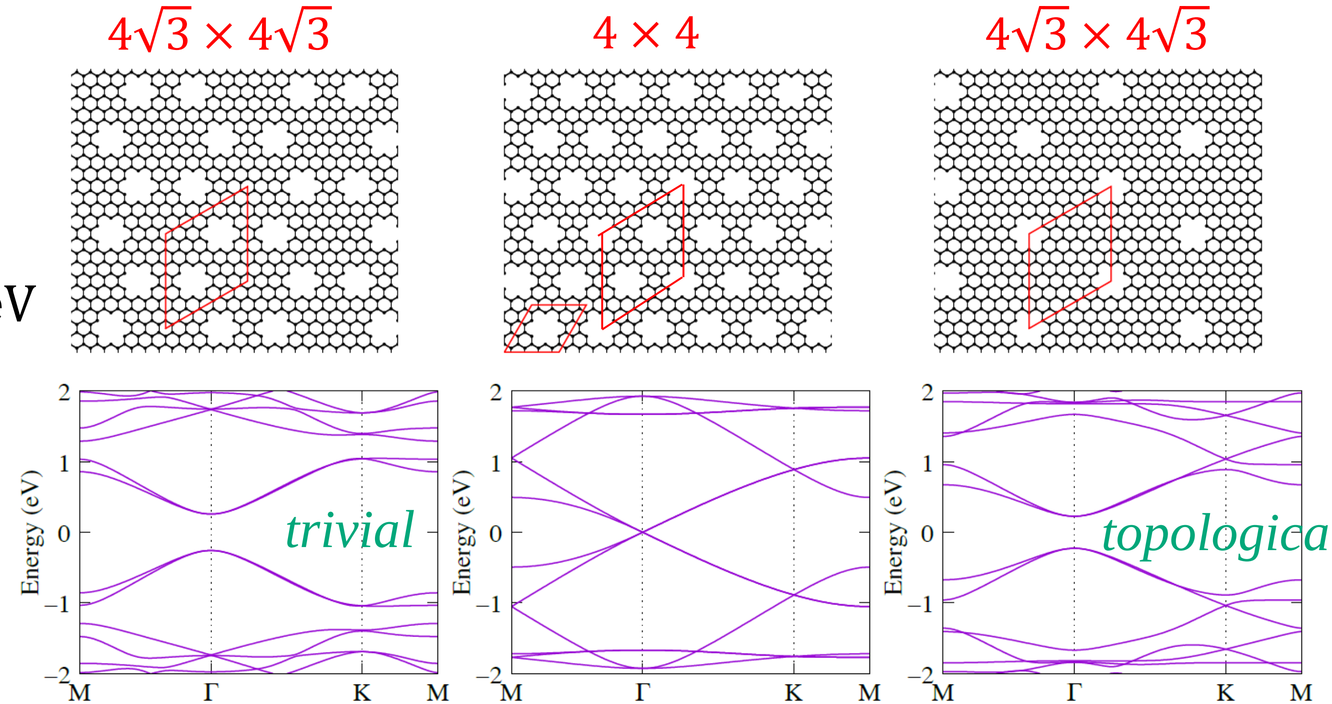
- Honeycomb lattice
  - * Haldane model: Z
  - * Kane-Mele model: Z_2
  - *next-nearest neighbor*
- One-dimensional chain
  - * SSH model
  - * Rice-Mele model
  - winding number

Topological graphene patchwork

T. Kariyado, Y.-C. Jiang, H.-X. Yang and XH: arXiv:1801.03115

- Hole arrays and band structures

with $t = 2.7$ eV



C_2 index at Γ & M

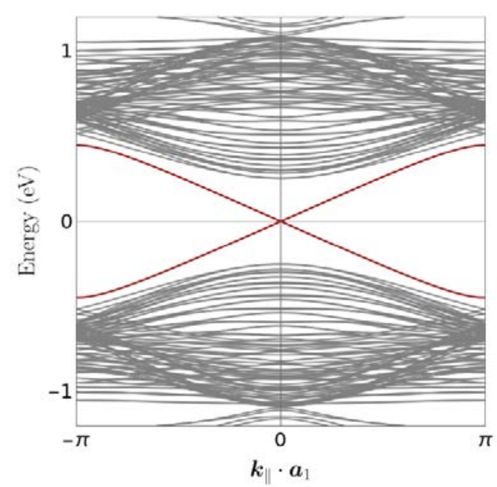
*Benalcazar, Teo & Hughes: PRB **89**, 224503 (2014)*

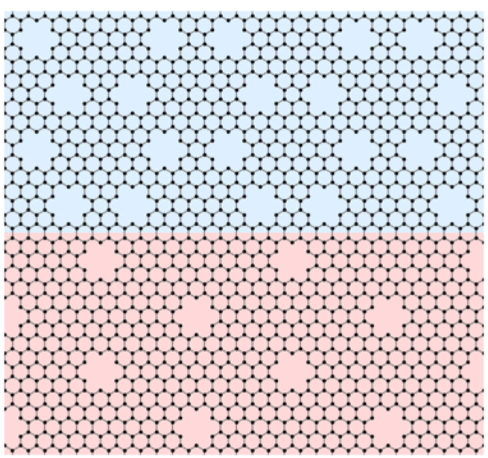
- Patchworking

topological gap: $\Delta = 0.5$ eV

Δ_{max}= 1.88 eV ~ 18000 Kelvin

hopefully overcome strongly correlated effect in 1D edge states





Summary

- A possible universal gate for Majorana qubits in nanowires

  * LZS interferometry based on quantum tunneling

  * quite stable but not topologically protected

- MBS inside vortex of topological superconductor

  * checkerboard-type pattern in spin-resolved LDOS

  * used to detect MBS by spin-polarized STM/STS

- Top-down approach for topological state in honeycomb structure

  * synthetic topological functionalities

  * nano-fabricated artificial graphenes


Thank you very much!