Transforming Bell's Inequalities into State Classifiers with Machine Learning

Author: Yue-Chi Ma, Man-Hong Yung

Speaker: Yue-Chi Ma

Contribution of our work

Build a bridge between ***Bell Inequality*** (physics) and ***Artificial Neuron Network*** (computer science)

Train ANN to detect entanglement with all or partial information and without ***prior*** knowledge

What is Entanglement

However,

E nglement verified experimentally (Bell Inequality),
a becomes kernel resource of quantum information!

It's impossible!

Quantum Mechanics is incomplete.

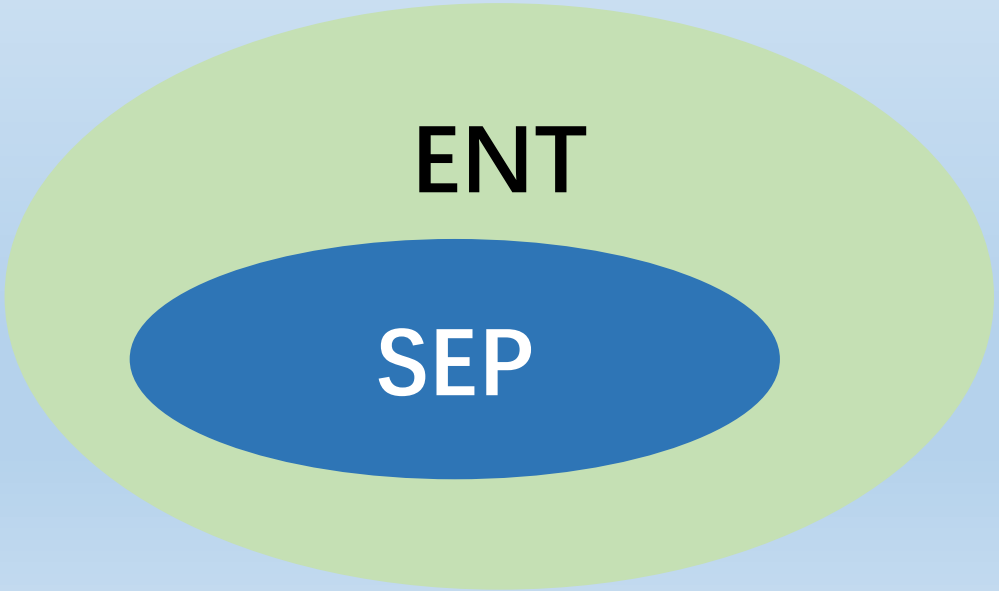
lication of entanglement:
Quantum computation, Teleportation,
Quantum key distribution…

1935. EPR paradox

Definition of Entanglement

$$\underset{2^n \times 2^n}{\rho_{\text{sep}}} = \sum_i p_i \underset{2\times 2}{\rho_i^{(1)}} \otimes \rho_i^{(2)} \cdots \rho_i^{(n)}$$

n : qubit number
p_i : probability



separable
or
entangled

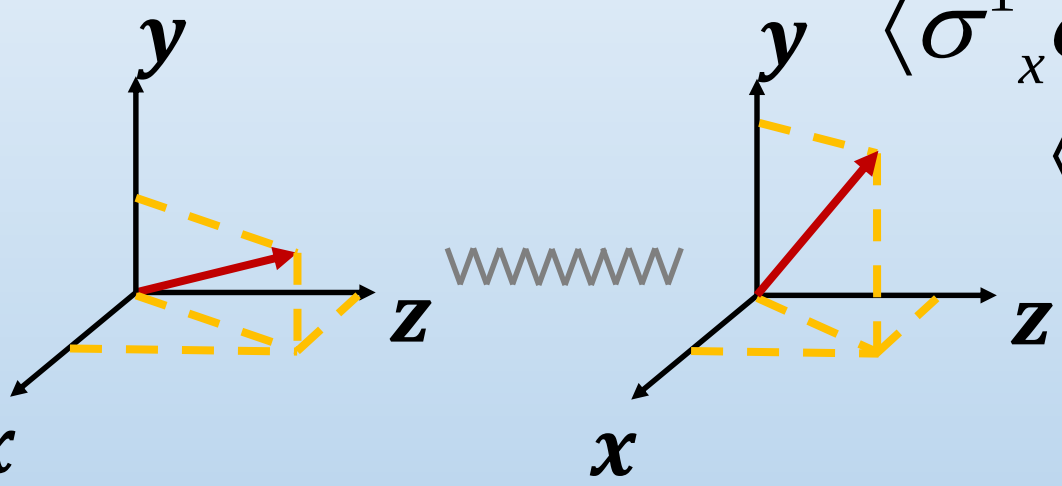
Positive Partial Transpose (PPT criterion)

$$\lambda_{\min} = \min(\mathrm{eig}(\rho^{T_A}))$$

$\lambda_{\min} < 0$	Any system	entangled
$\lambda_{\min} \geq 0$	2-qubit	separable
	n-qubit (n=3,4,…)	???

Problem

reconstruct ρ is resource-consuming



Alice Bob

$\langle\sigma^1{}_x\sigma^2{}_x\rangle\langle\sigma^1{}_y\sigma^2{}_y\rangle\langle\sigma^1{}_z\sigma^2{}_z\rangle$

$\langle\sigma^1{}_x\sigma^2{}_y\rangle\ \langle\sigma^1{}_x\sigma^2{}_z\rangle\ldots$

3 angles

$\{I,\sigma_x,\sigma_y,\sigma_z\}\times\{I,\sigma_x,\sigma_y,\sigma_z\}$ **Tomography**

$\{I,I\sigma_x,\cdots\sigma_y\sigma_z,\cdots\sigma_z\sigma_z\}$ $4^n-1=15$ **features**

CHSH (Bell) inequality

Alice's measurement:

Bob's measurement:

$$\{a, a'\} \times \{b, b'\}$$ **2 angles**

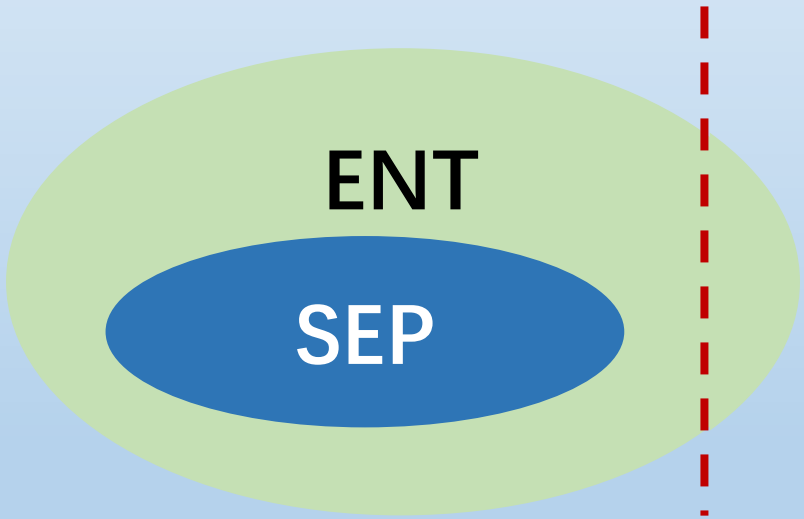
$$\{ab, ab', a'b, a'b'\}$$ **4 features**

CHSH (Bell) inequality

$$ab - ab' + a'b + a'b' + 2 \geq 0$$

not violate

???



violate

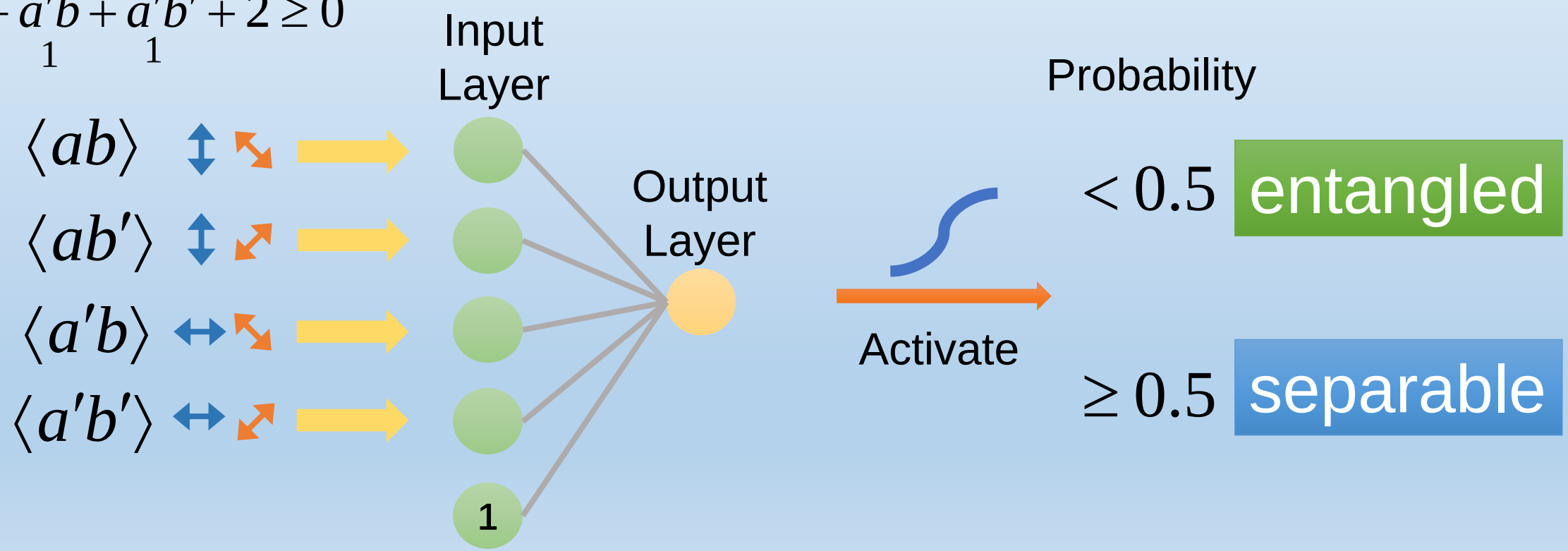
entangled

Artificial Neuron Network (ANN)

Bell to ANN!

CHSH(Bell) Inequality :

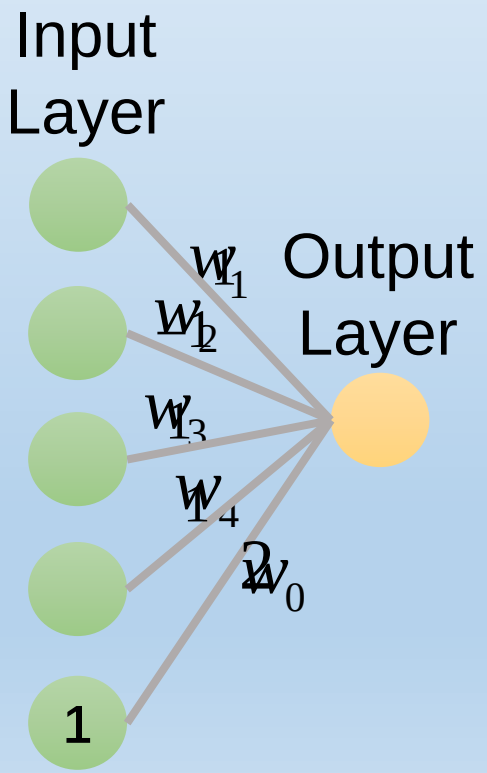
$$\underset{1}{ab} - \underset{-1}{ab'} + \underset{1}{a'b} + \underset{1}{a'b'} + 2 \geq 0$$



Supervised learning

Optimize

w_0 w_1 w_2 w_3 w_4

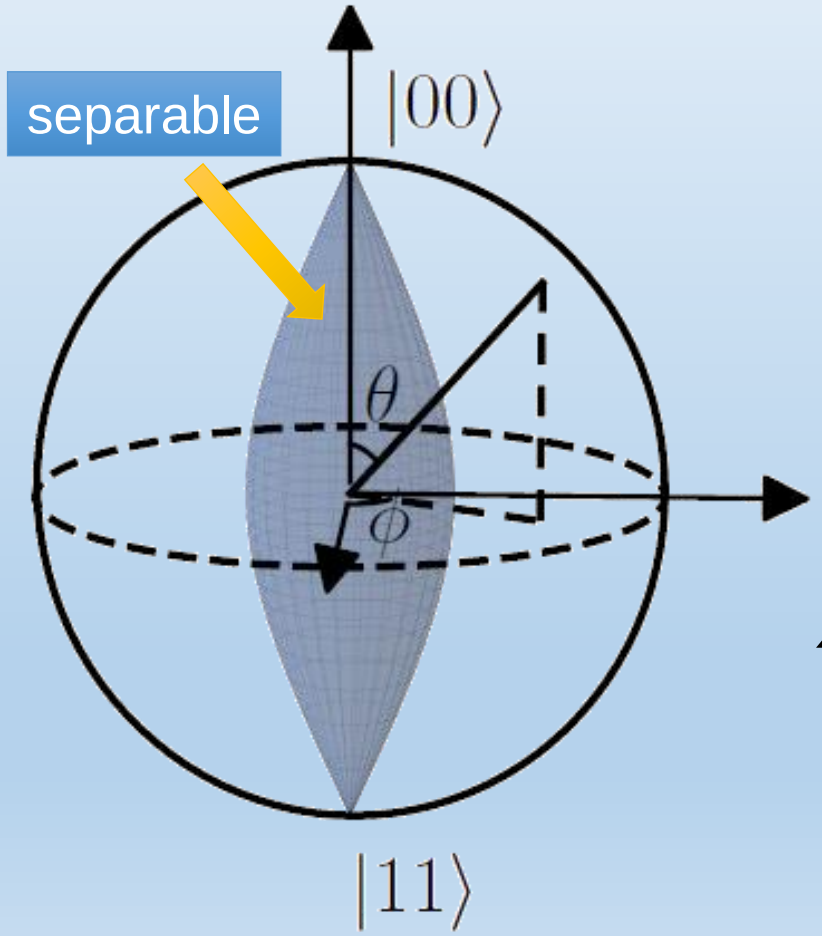


Probability

< 0.5 entangled

≥ 0.5 separable

“Test Run” state



$$\rho_{\theta,\phi} = p\,|\psi_{\theta,\phi}\rangle\langle\psi_{\theta,\phi}| + (1-p)I/4$$

$$|\psi_{\theta,\phi}\rangle = \cos(\theta/2)\,|00\rangle + e^{i\phi}\sin(\theta/2)\,|11\rangle$$

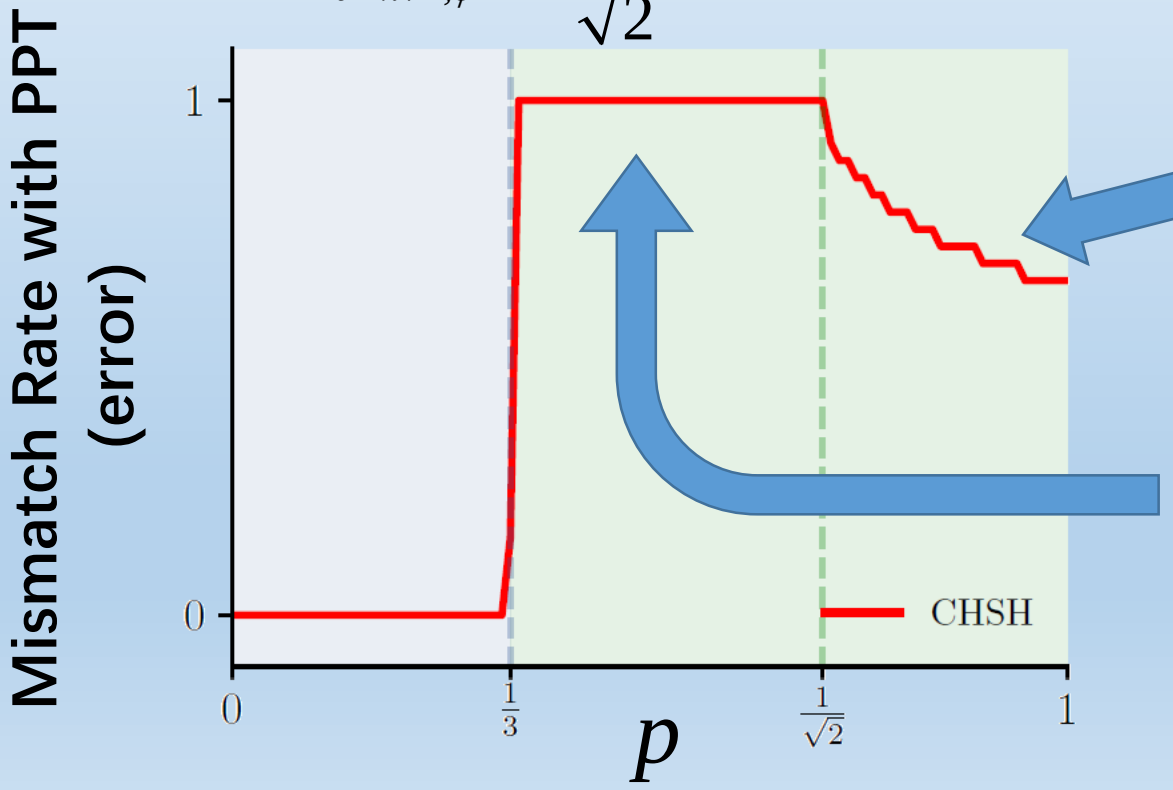
$$\lambda_{\min}(\rho_{\theta,\phi}^{T_B}) = (1-p)/4 - p\cos(\theta/2)\sin(\theta/2)$$

Independent of ϕ

Defect of original CHSH

$$a_0 = \sigma_z, a_0' = \sigma_x, b_0 = (\sigma_x - \sigma_z)/\sqrt{2}, b_0' = (\sigma_x + \sigma_z)/\sqrt{2}$$

$$|\psi_{\theta=\pi/2,\phi}\rangle = \frac{1}{\sqrt{2}}(|00\rangle + e^{i\phi}|11\rangle) \qquad \rho_{\theta,\phi} = p|\psi_{\theta,\phi}\rangle\langle\psi_{\theta,\phi}| + (1-p)I/4$$



$$\frac{1}{\sqrt{2}} \sim 1$$

No knowledge of ϕ

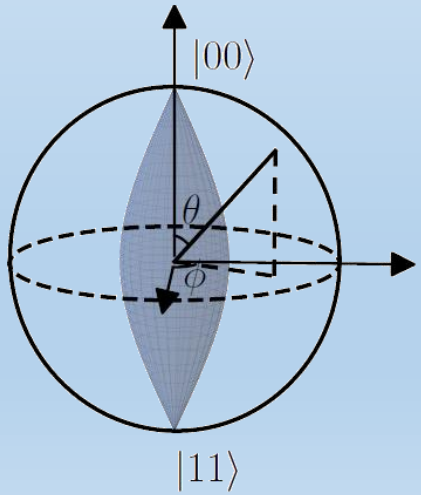
$$\frac{1}{3} \sim \frac{1}{\sqrt{2}}$$

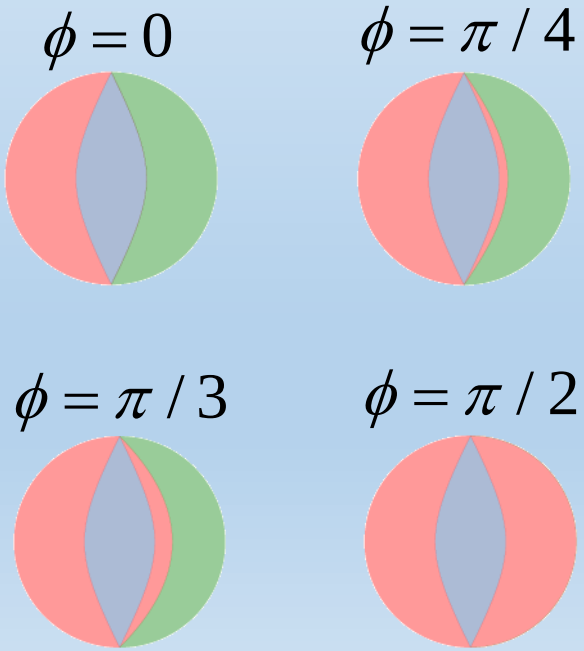
Total failure

Defect of Witness

W is Witness iff

$$\mathrm{Tr}(\rho W) \geq 0 \quad \text{for all } \rho_{sep}$$
$$\mathrm{Tr}(\rho W) < 0 \quad \text{for some } \rho_{ent}$$





Separable (success)

Entangled (success)

Entangled (fail)

3 angles

$$\sigma_x, \sigma_y, \sigma_z$$

5 features

$$\langle \sigma^1{}_x \sigma^2{}_x \rangle \; \langle \sigma^1{}_y \sigma^2{}_y \rangle \; \langle \sigma^1{}_z \sigma^2{}_z \rangle$$
$$\langle \sigma^1{}_z \rangle \quad \langle \sigma^2{}_z \rangle$$

Experimental detection of entanglement via witness operators and local measurements, Journal of Modern Optics, 2003, Gühne et al.

Linear optimization by ANN

$$-\langle a_0 b_0 \rangle = \langle a_0 b_0' \rangle = p/\sqrt{2},$$

$$\langle a_0' b_0 \rangle = \langle a_0' b_0' \rangle = p \sin\theta \cos\phi / \sqrt{2}$$

If trained with only $\phi = 0$

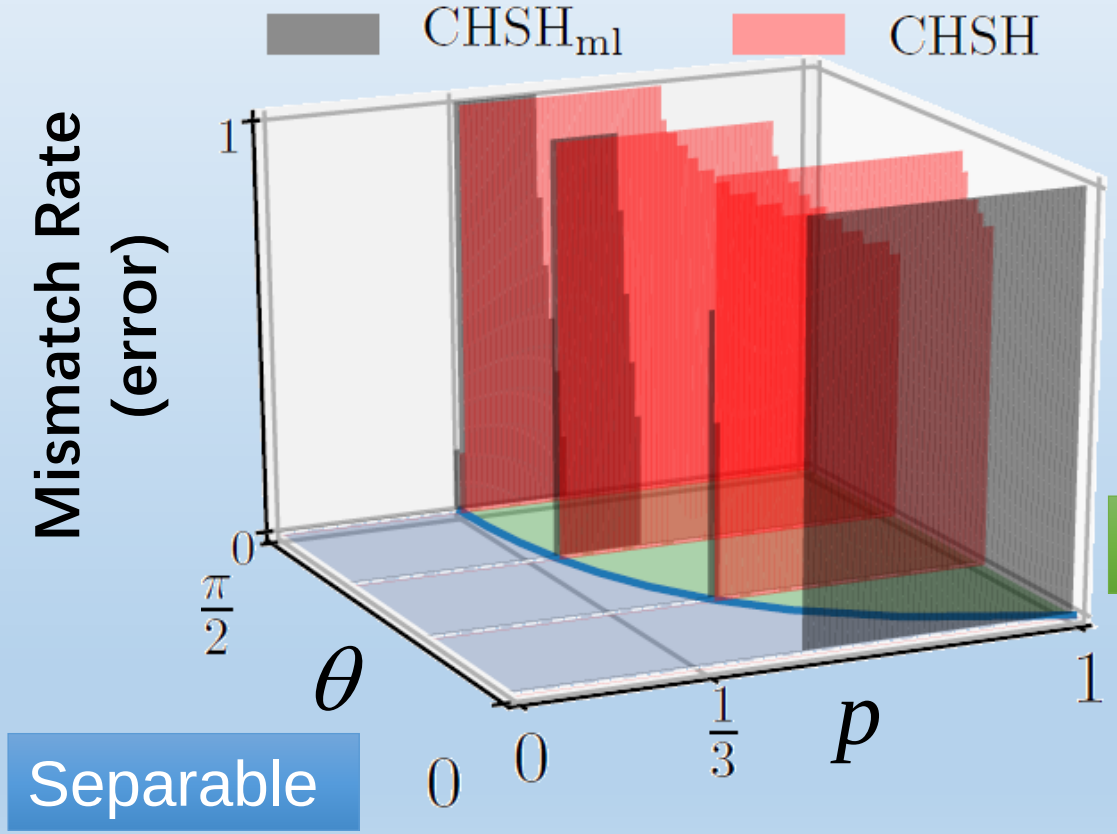
Our ANN result $-14\langle a_0 b_0' \rangle - 28\langle a_0' b_0' \rangle + 10$

Analytical result $-10\sqrt{2}\langle a_0 b_0' \rangle - 20\sqrt{2}\langle a_0' b_0' \rangle + 10$ $\sqrt{2} \approx 1.4$

Reduced to $p(1 + 2\sin\theta) - 1 < 0$

If (linear) solution exists, ANN can find it out.

Trained with Randomized ϕ



Separable

Entangled

No solution for lack of information

Our ANN result

$$\mathrm{CHSH}_{ml}$$

$$p < 1/2$$

How to improve $CHSH_{ml}$?

$$a_0 = \sigma_z, a_0' = \sigma_x, b_0 = (\sigma_x - \sigma_z)/\sqrt{2}, b_0' = (\sigma_x + \sigma_z)/\sqrt{2}$$

No information about σ_y

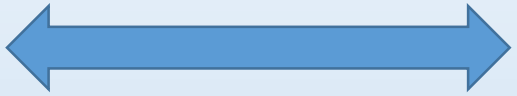
So we…

Randomize angles

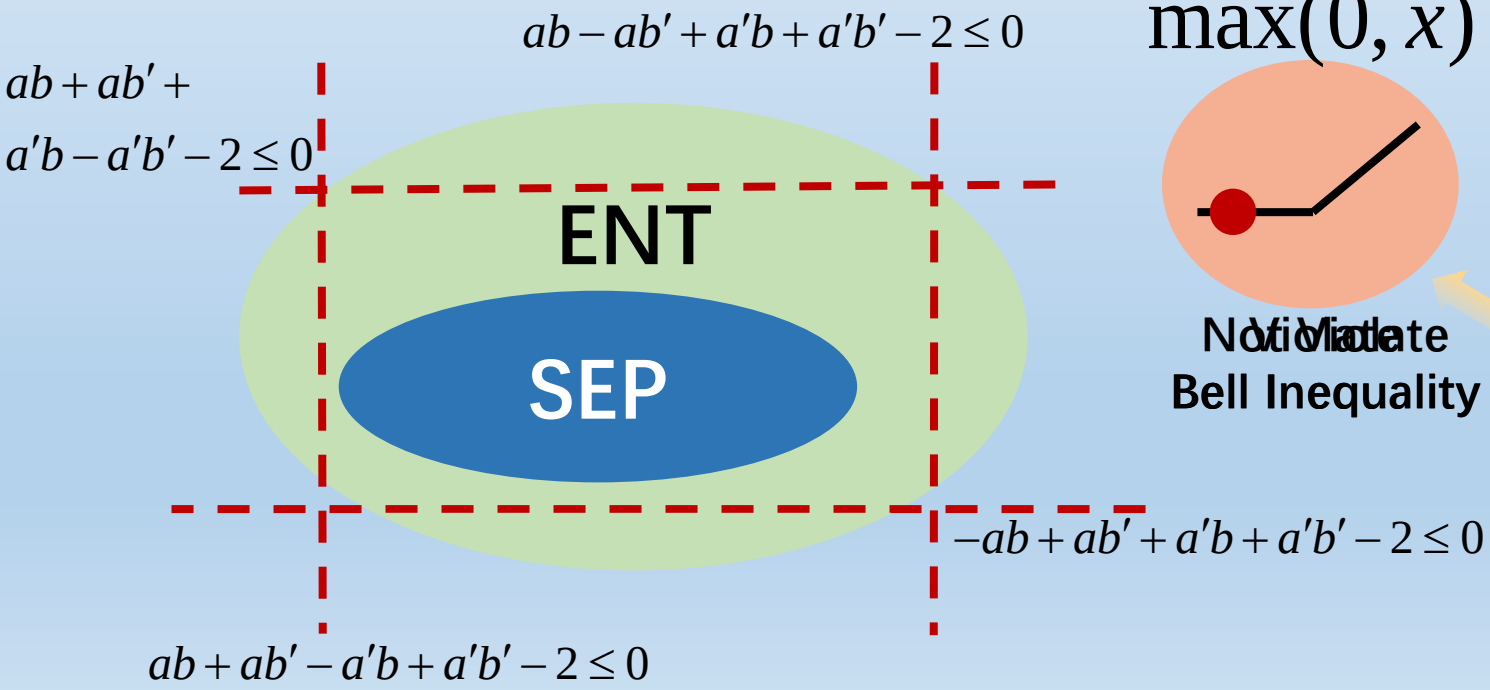
$$a, a', b, b'$$

How to Multi Bell to 1 CNN! Bell_{ml} ?

More inequalities, more entanglement detected



hidden layer



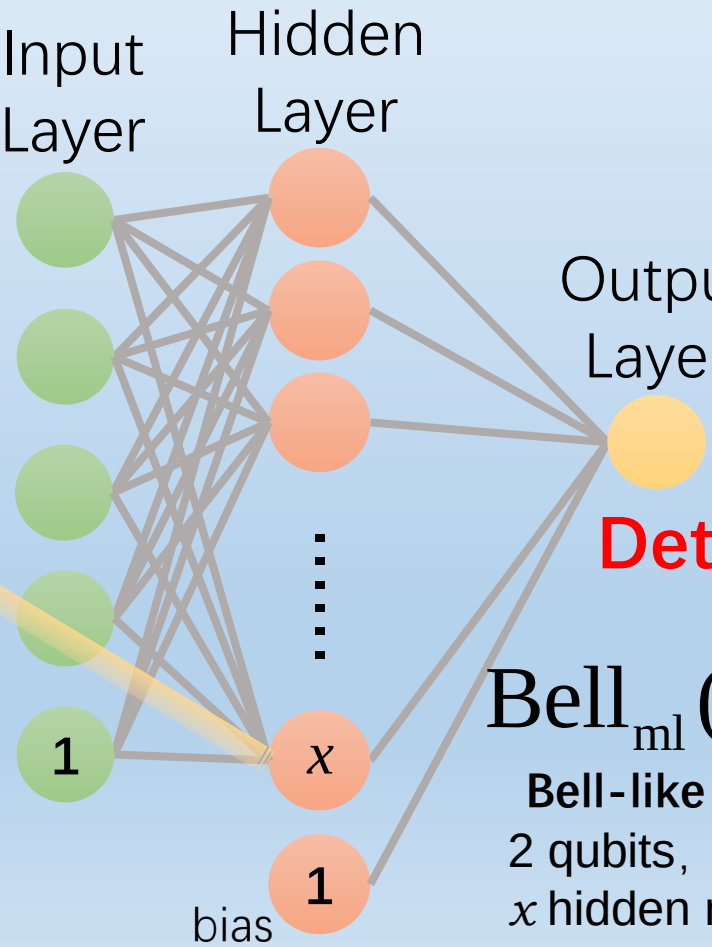
Perfect activation:

ReLu

$\max(0, x)$

Violate Bell Inequality

Not violate



Detect !

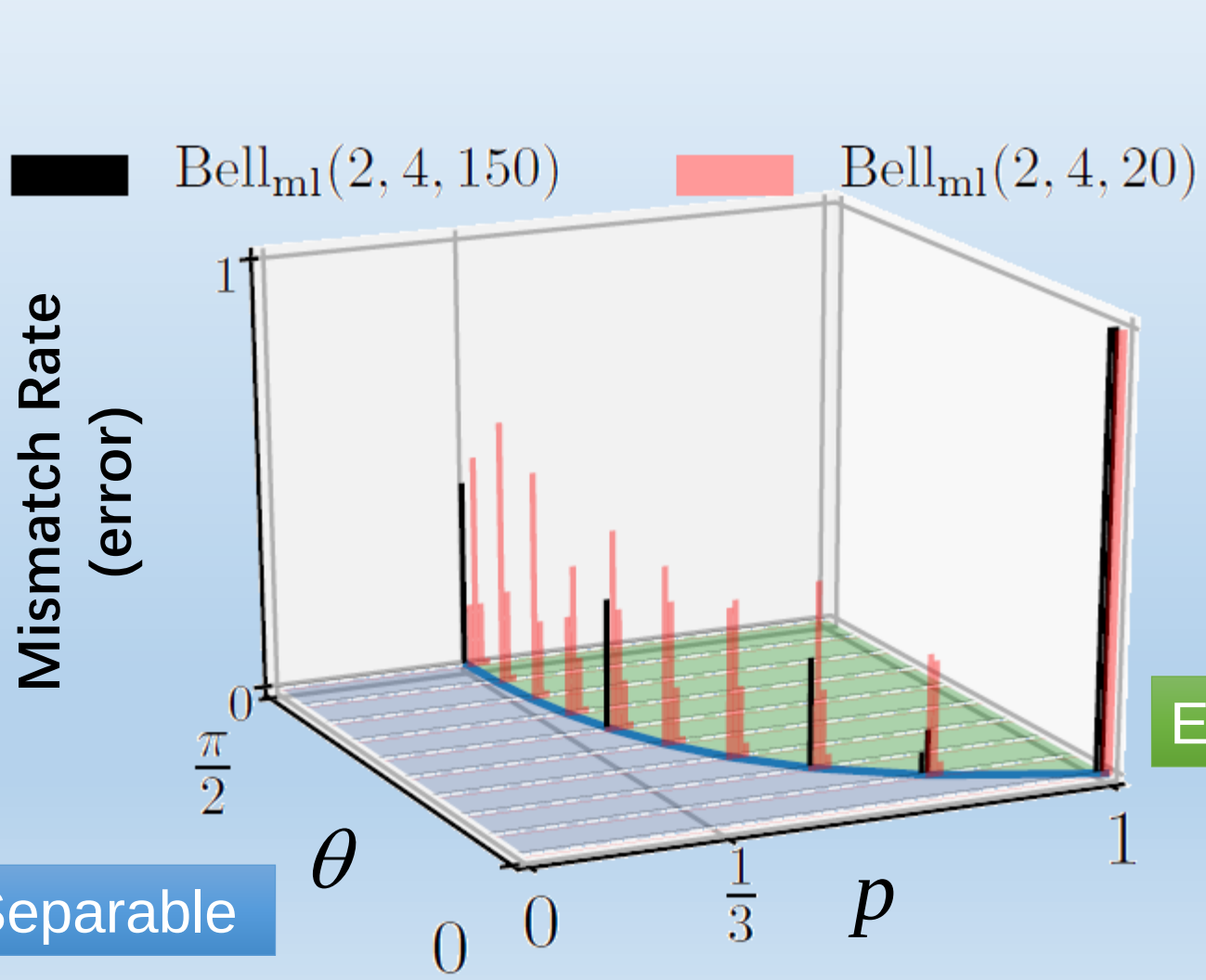
$\text{Bell}_{\text{ml}}(2, 4, x)$

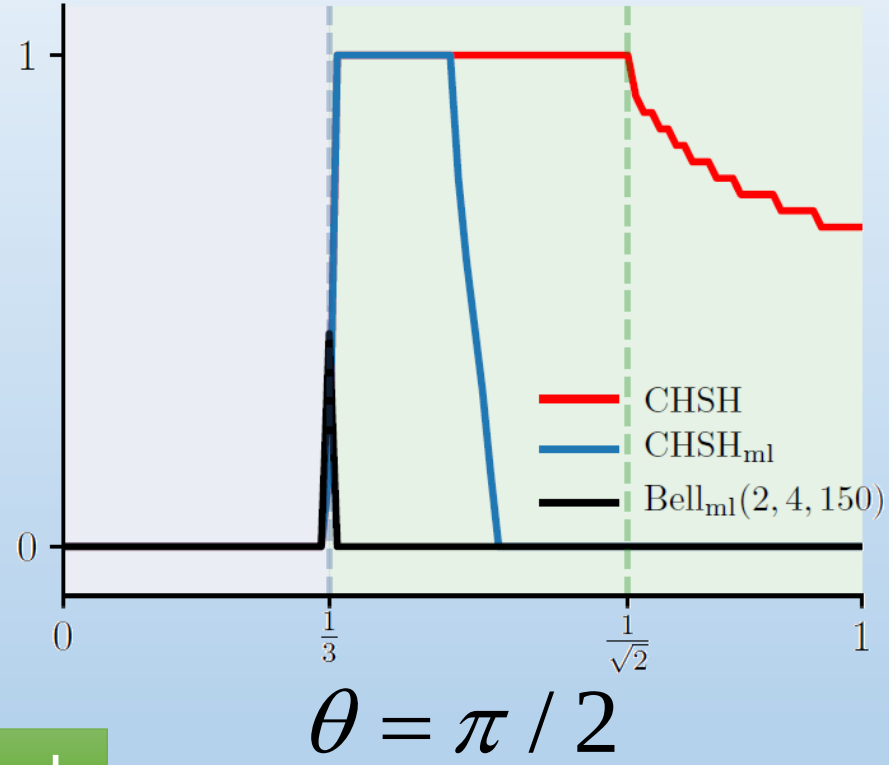
Bell-like Predictor

2 qubits, 4 features

x hidden neurons

Results of Bell-like Predictor





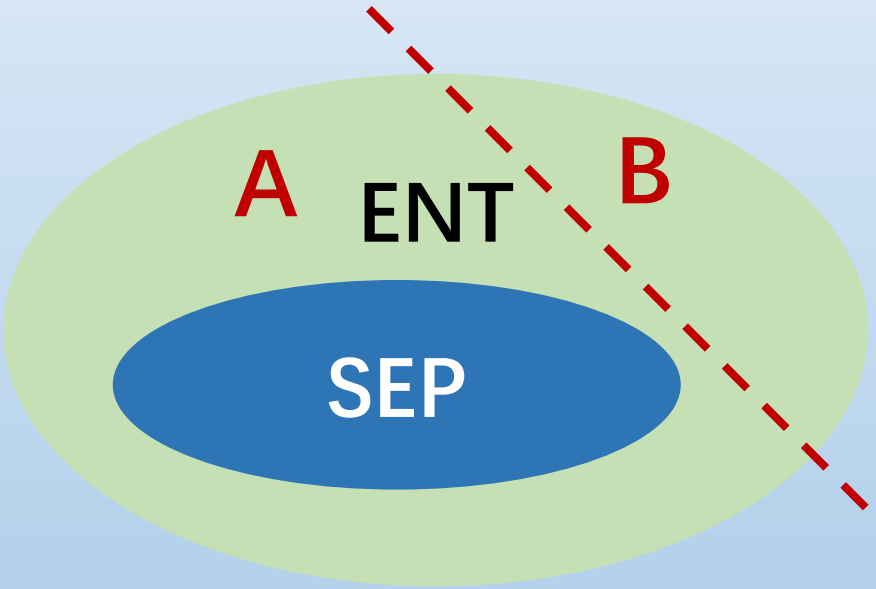
Entangled

Separable

Error occurs on the boundary

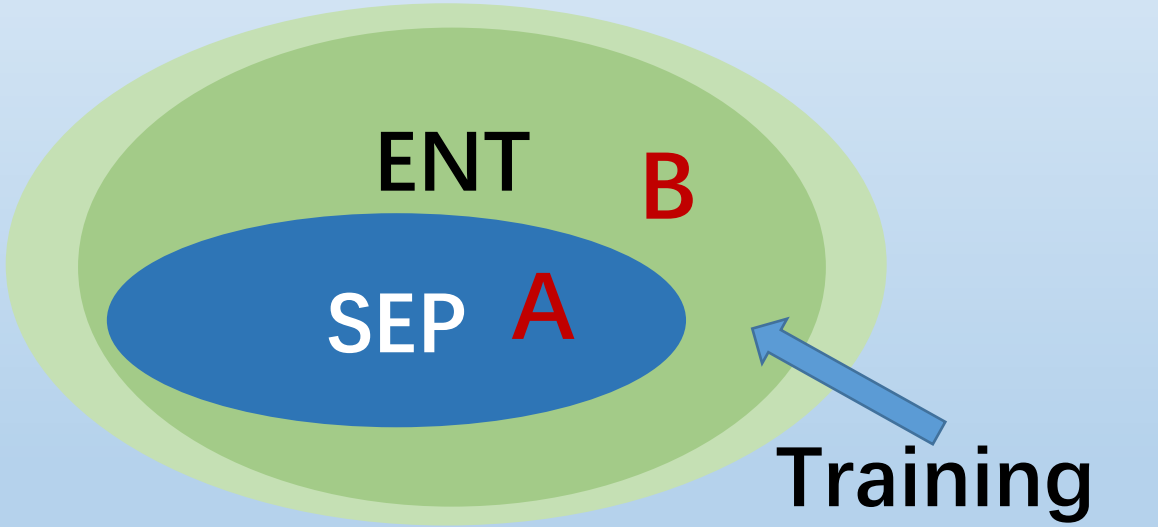
Contrast

PPT \ Bell Inq. \ Witness



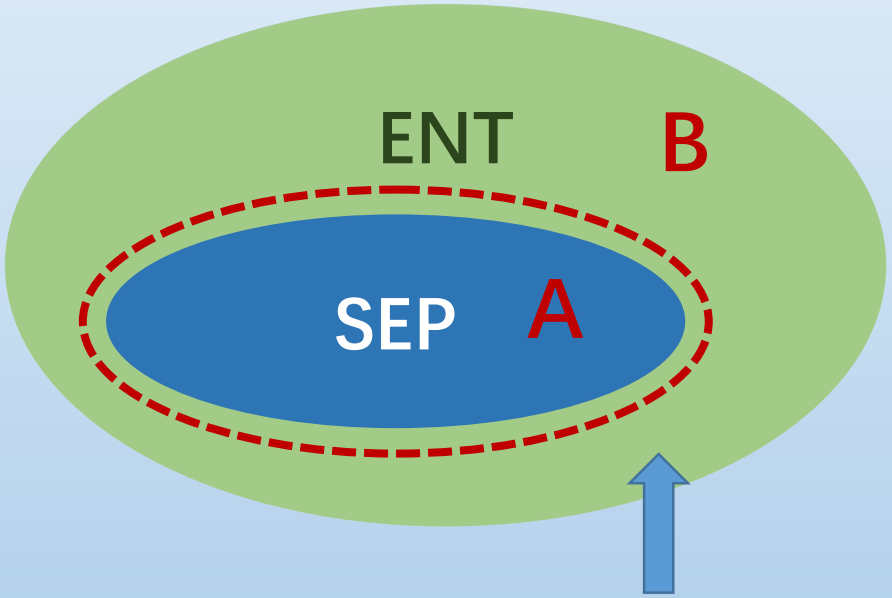
Entanglement detection

Our Bell-like Predictor (ANN)



Entanglement detection

General 2-qubit system



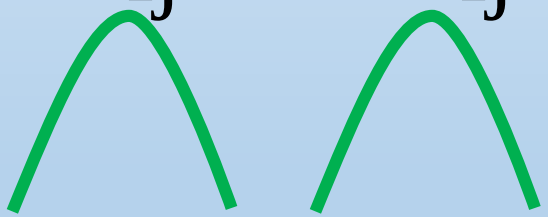
Harr Measure (mix)

$$\rho_{\text{rand}} = \frac{\sigma\sigma^{\dagger}}{\text{tr}(\sigma\sigma^{\dagger})}$$

Hermitian

Normalize

$$\sigma_{ij} = a_{ij} + ib_{ij}$$



Detect all entangled states

General 2-qubit system

$\{I, \sigma_x, \sigma_y, \sigma_z\} \times \{I, \sigma_x, \sigma_y, \sigma_z\}$

Tomographic Predictor (15 features)

2-4% error

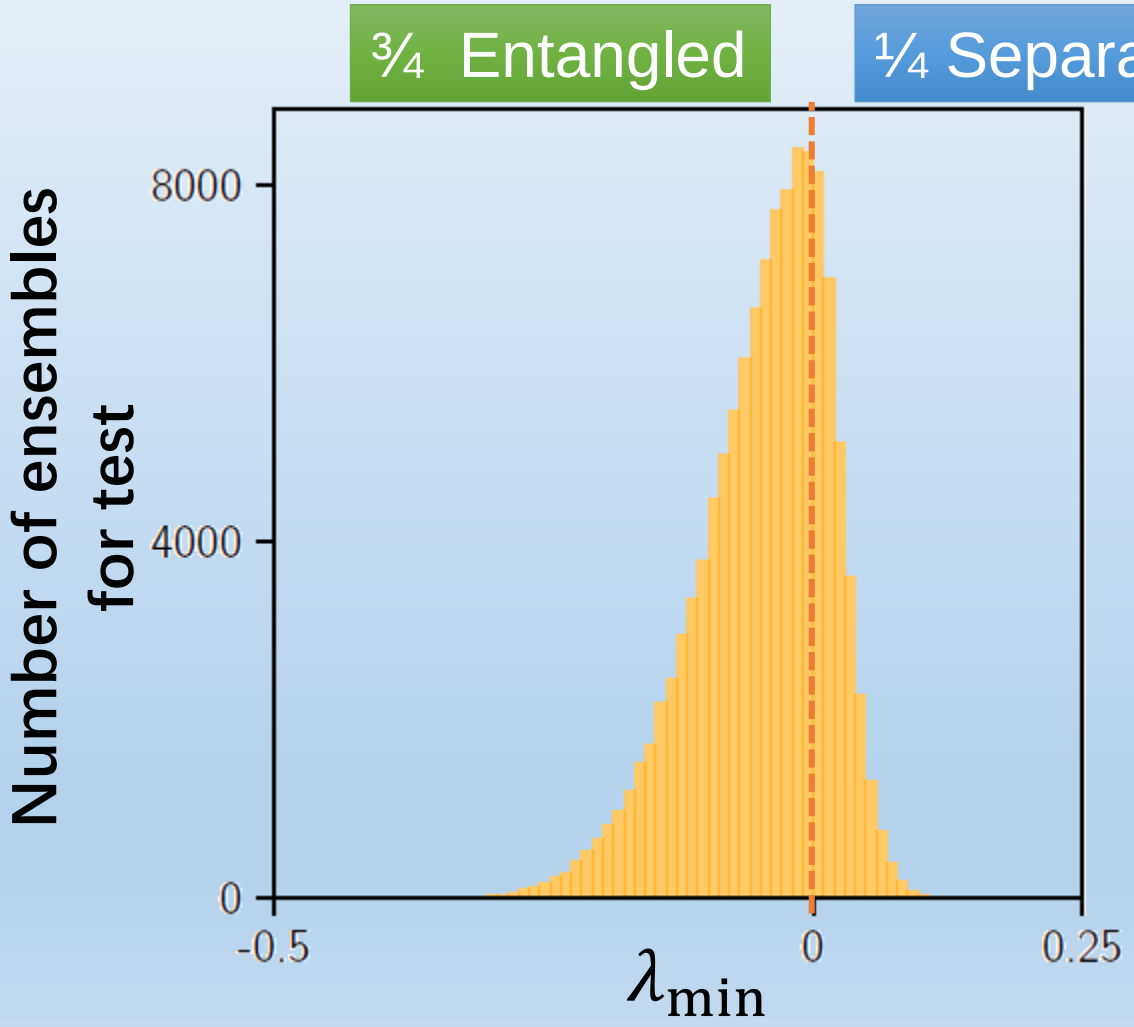
$\{I, a, a'\} \times \{I, b, b'\}$

Bell-like Predictor (8 features)

25% error (lack of information)

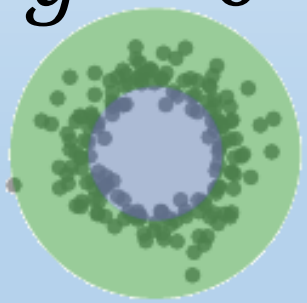
Where the error occurs?

Add Gap

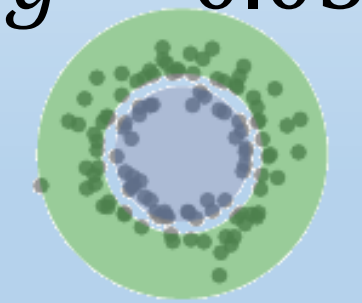


remove $-g < \lambda_{\min} < 0$

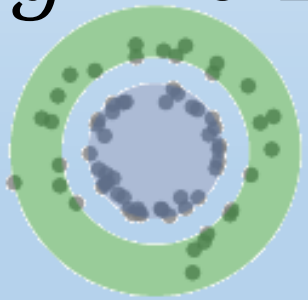
$g = 0$



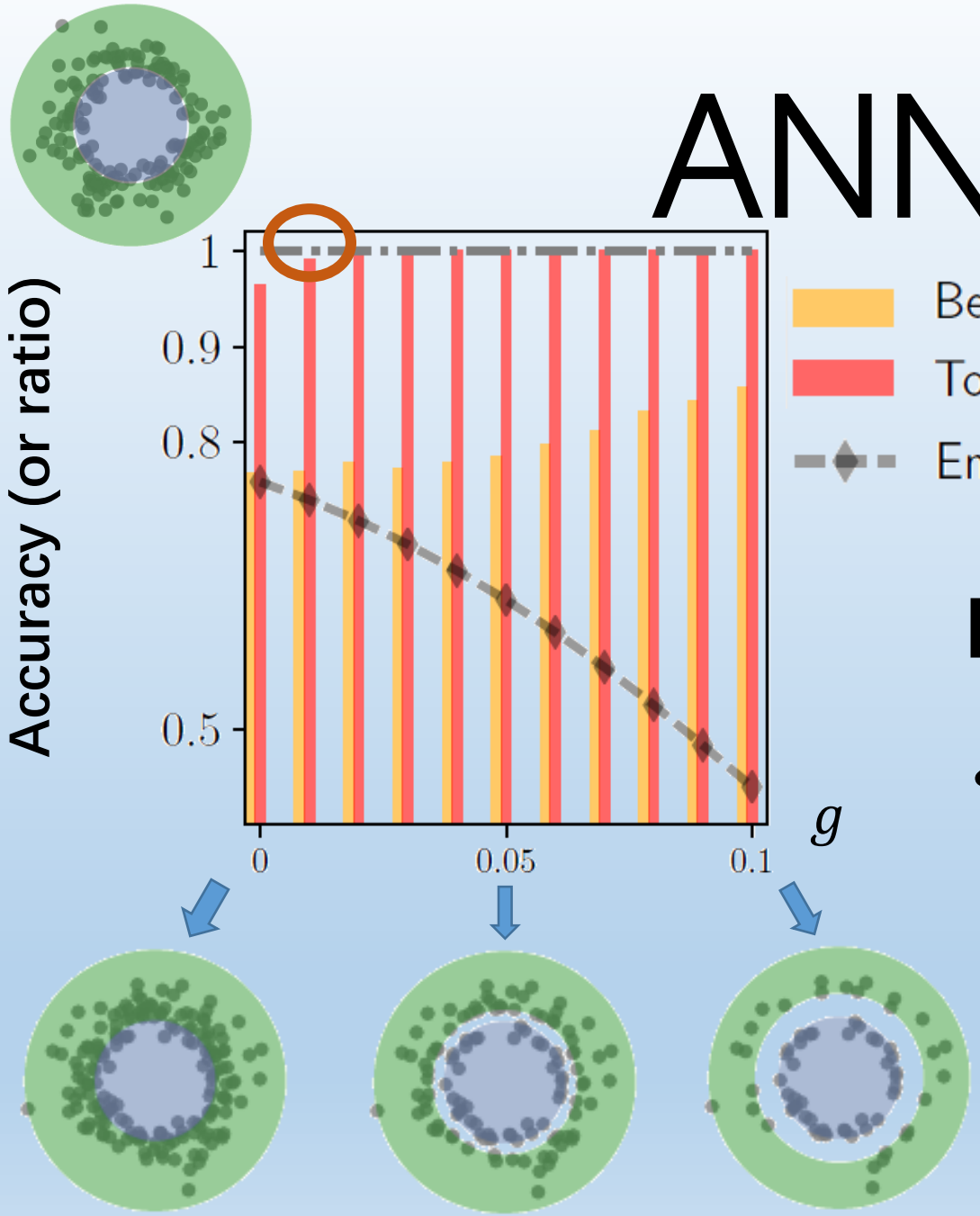
$g = 0.05$



$g = 0.1$



ANN results



$g_{\max} = 0.5$

For Tomographic Predictor:

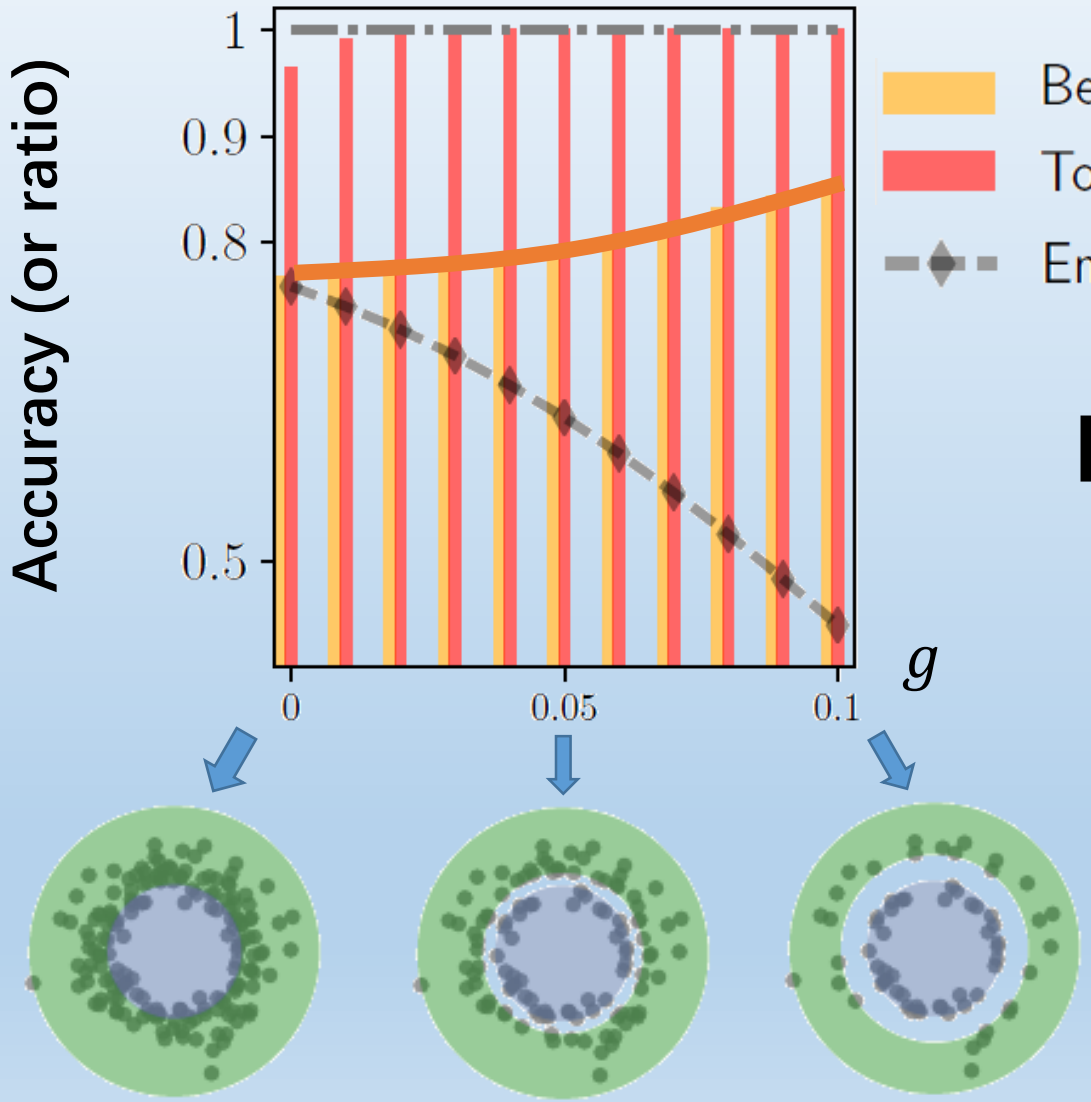
$g = 0.01$

Depends on data, hidden layer and training time

Accuracy larger than 99%

If solution exists,
ANN fit it,
even if it is non-linear.

ANN results



$g_{\max} = 0.5$

For Bell-like Predictor:

g ↑ Accuracy ↑

More difference,
Better performance

Multi-qubit system

General 3-qubit detection?

We cannot label

All data are entangled

The only constraint is Data!!!

Fortunately, we can cover (fully) separable states as much as posiible

$$\rho_{\text{sep}} = \sum_i p_i \rho_i^{(1)} \otimes \rho_i^{(2)} \cdots \rho_i^{(n)}$$

$$\rho_i = \frac{\sigma\sigma^\dagger}{\text{tr}(\sigma\sigma^\dagger)}$$

$$\sum_i p_i = 1$$

Uniform distribution

Classical noise

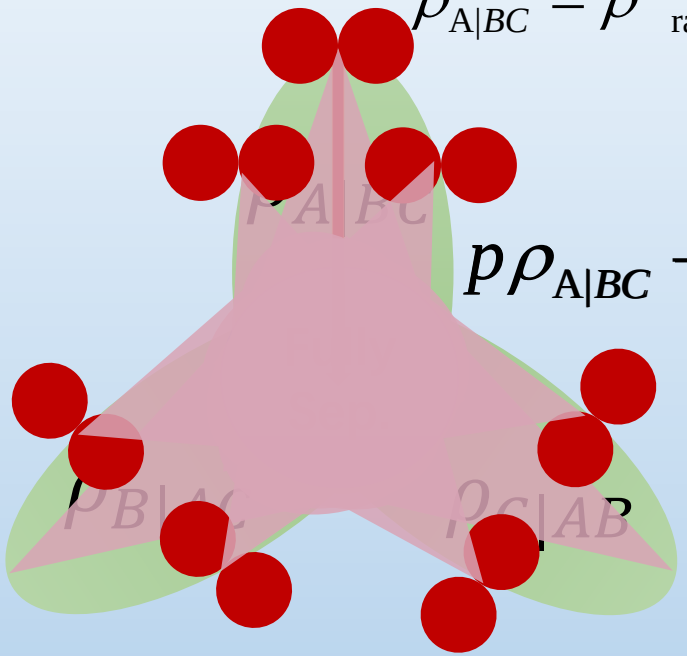
Bipartite entanglement

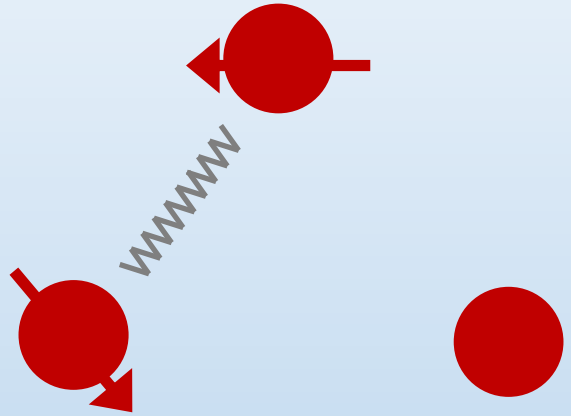
$$\rho_{A|BC} = \rho^{A}_{rand} \otimes |\psi^{BC}\rangle\langle\psi^{BC}|$$

$$|\psi^{BC}\rangle = |v_1, v_2, v_3, v_4\rangle_{rand}$$

Harr Measure (pure)

$$p\rho_{A|BC} + (1-p)I\rho/8_{sep}$$

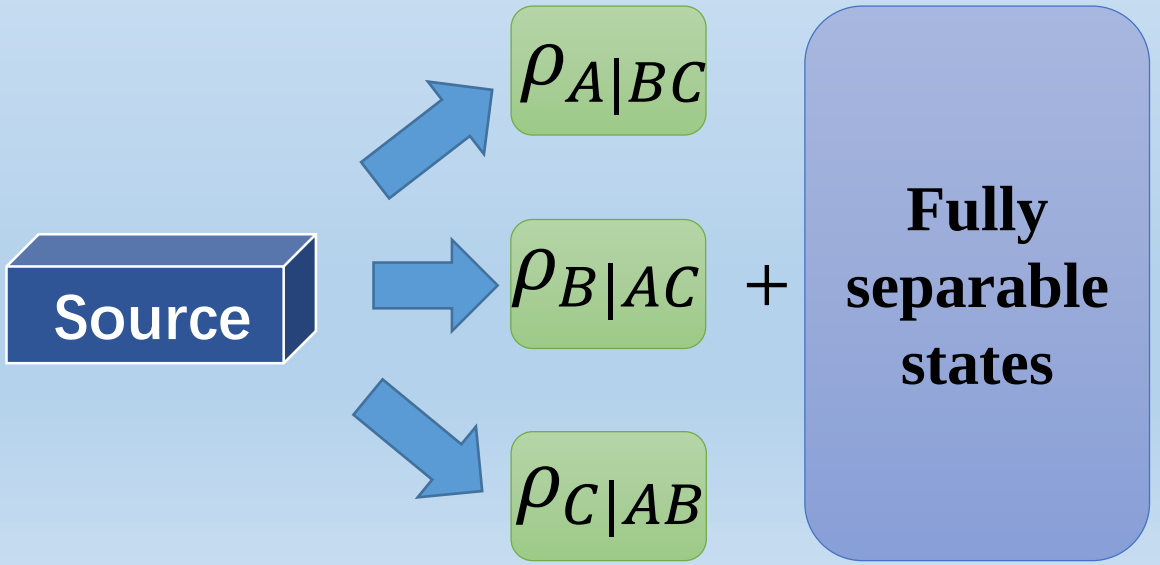




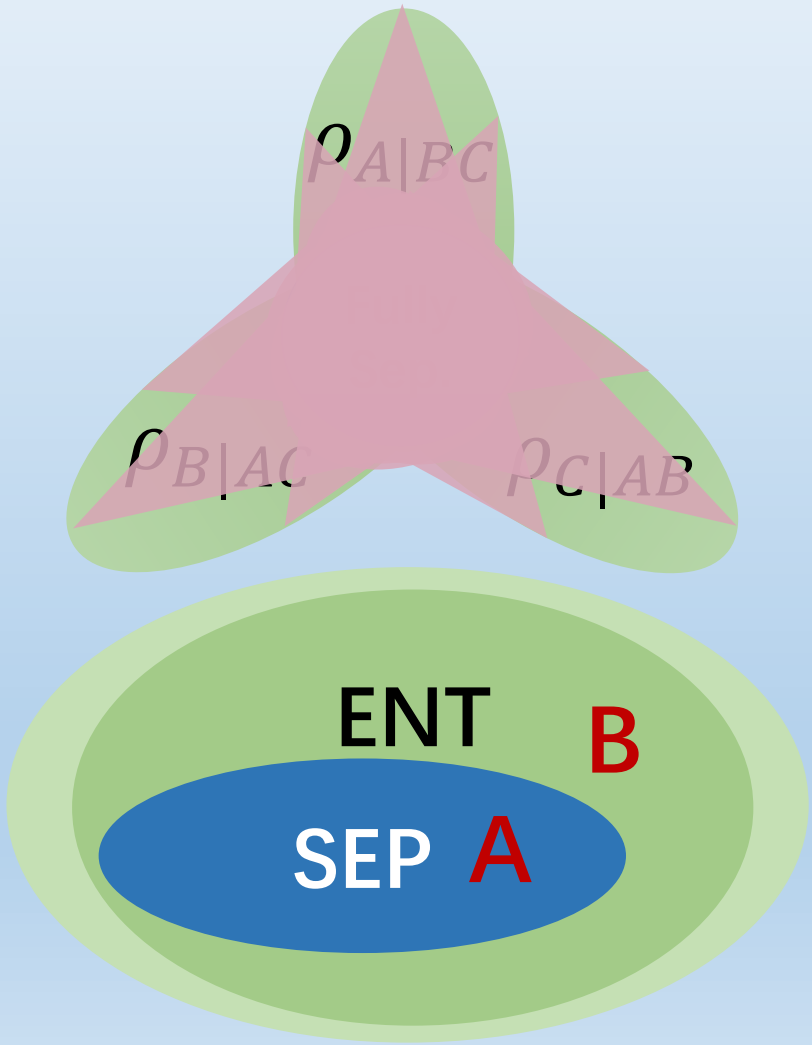
Accuracy (or ratio)

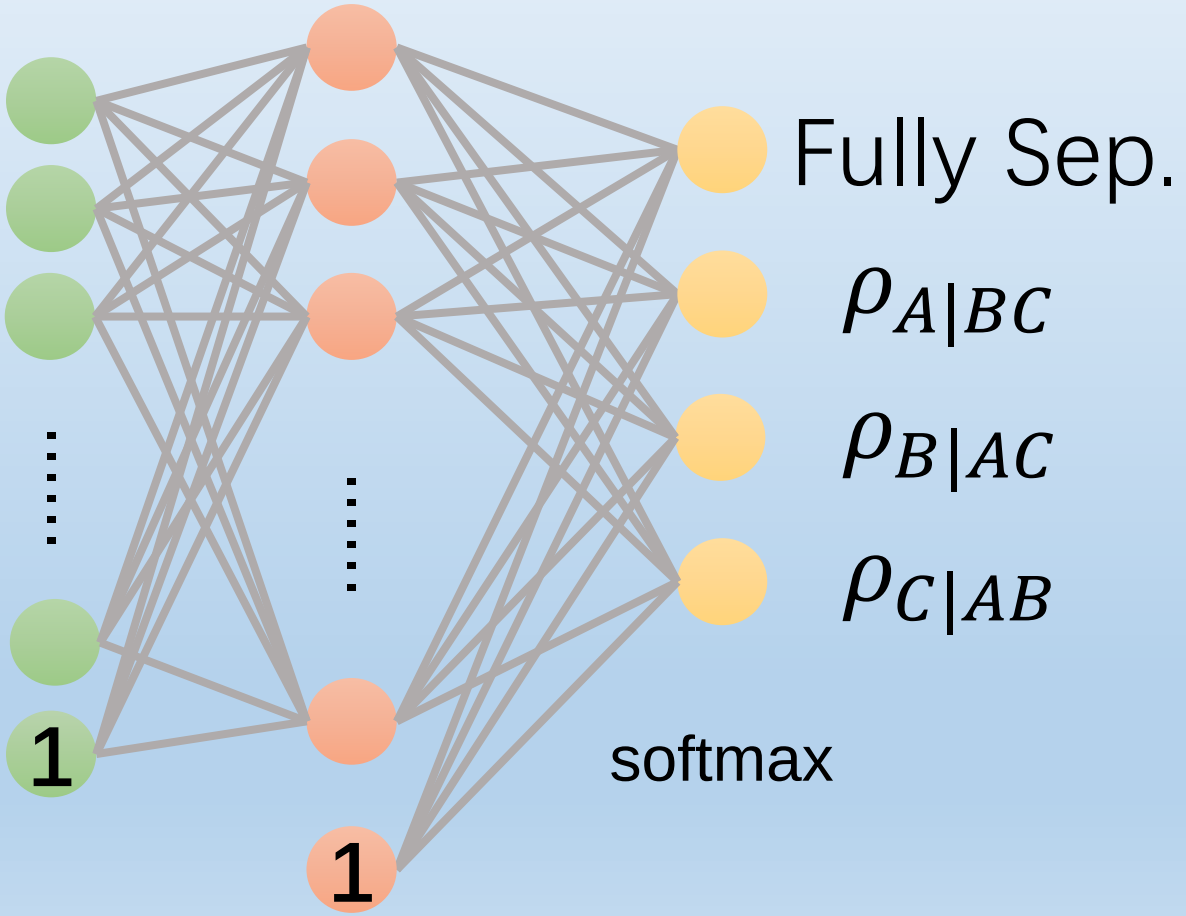
Biseparable states

Fully separable states

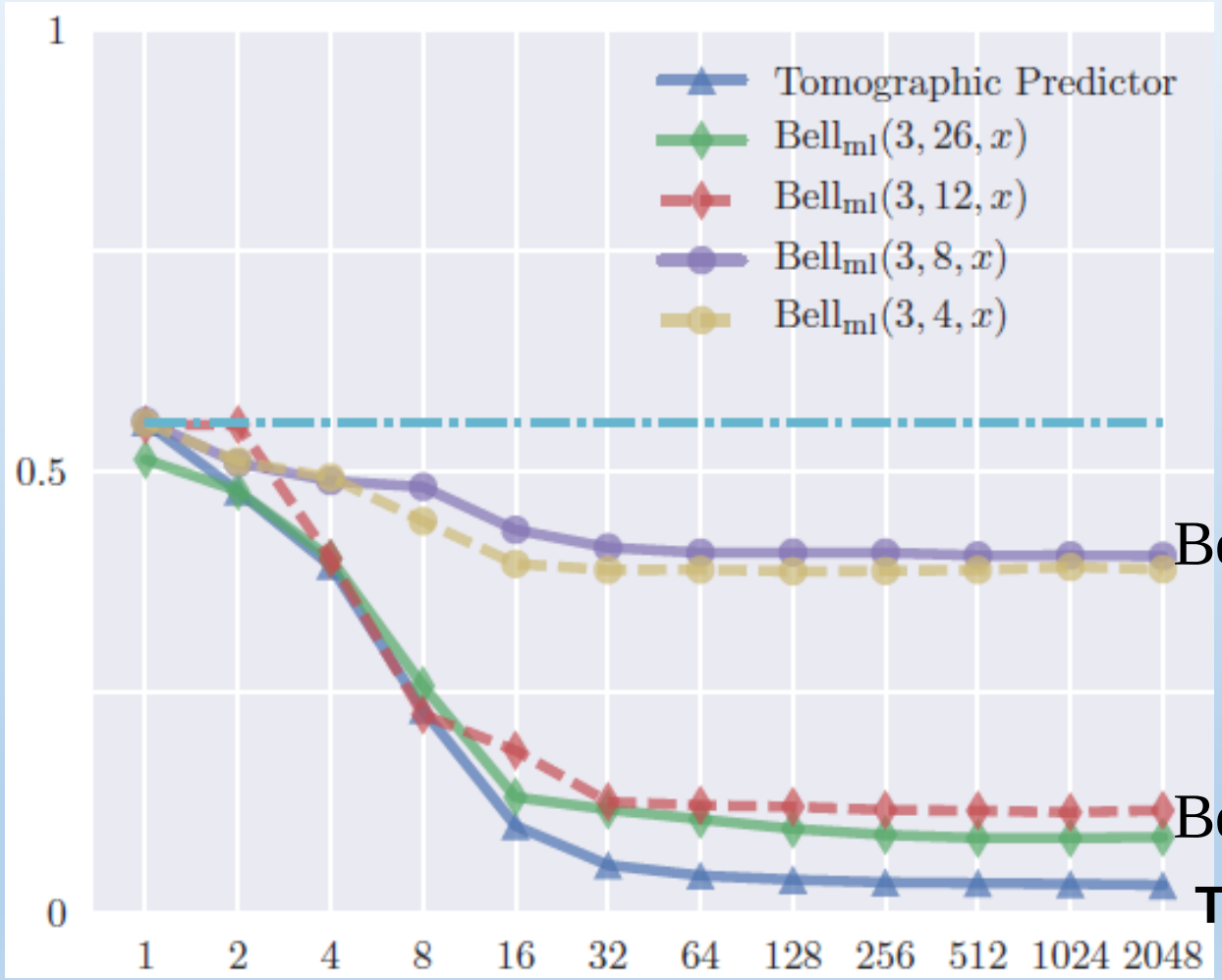


3-qubit system





Error of biseparable states



Random guess

$\mathrm{Bell}_{\mathrm{ml}}(3,4,x)$ $\mathrm{Bell}_{\mathrm{ml}}(3,8,x)$

$\mathrm{Bell}_{\mathrm{ml}}(3,12,x)$ $\mathrm{Bell}_{\mathrm{ml}}(3,26,x)$

Tomographic Predictor (63 features)

The features of Bell-like predictors are composed of
2 random angles

$\mathrm{Bell}_{\mathrm{ml}}(3,26,x)$

$\{I,a,a'\}\times\{I,b,b'\}\times\{I,c,c'\}$

$\mathrm{Bell}_{\mathrm{ml}}(3,12,x)$

3 CHSH Inequalities

$\mathrm{Bell}_{\mathrm{ml}}(3,4,x)$ $\left|\langle abc\rangle-\langle a'b'c\rangle-\langle ab'c'\rangle-\langle a'bc'\rangle\right|\le 2$ **Mermin inequality**

$\mathrm{Bell}_{\mathrm{ml}}(3,8,x)$

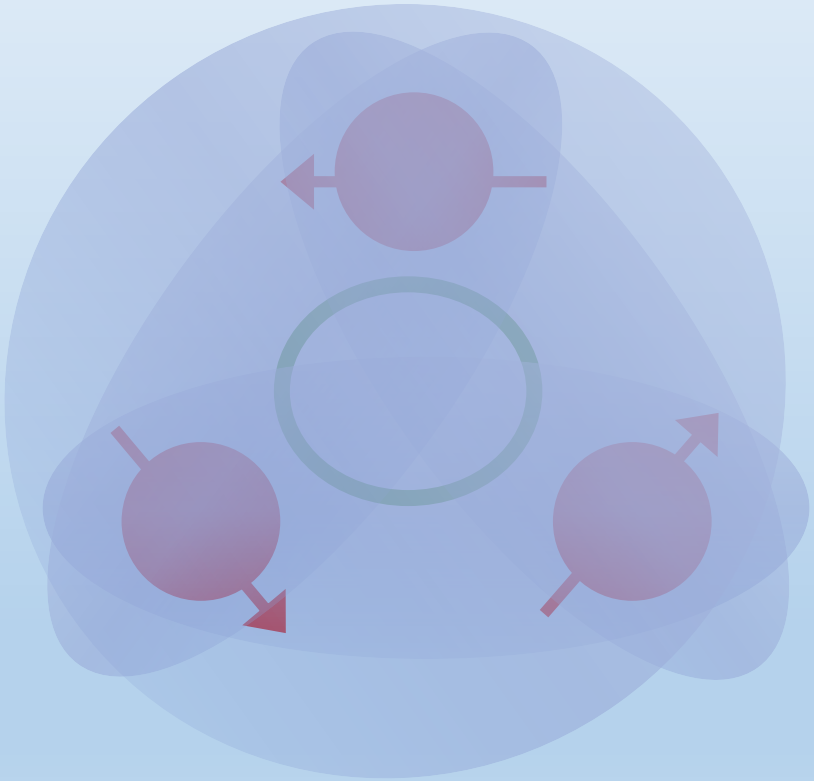
$$\left|\langle abc'\rangle+\langle ab'c\rangle+\langle a'bc\rangle-\langle a'b'c'\rangle+\langle a'b'c\rangle+\langle a'bc'\rangle+\langle ab'c'\rangle-\langle abc\rangle\right|\le 4$$

Svetlichny inequality

Detect **tripartite**, but not **bipartite**
entangled states

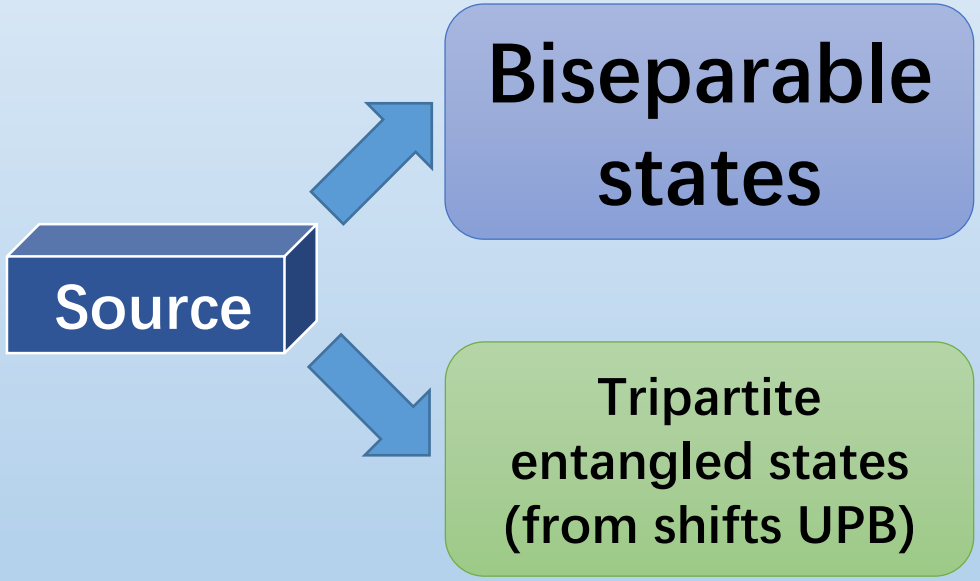
Do PPT can not do (shifts UPB)

Tripartite entangled states



$$\rho_{\text{bisep}} = \lambda_1 \rho_{\text{A|BC}} + \lambda_2 \rho_{\text{B|AC}} + \lambda_3 \rho_{\text{C|AB}}$$

$$\rho_{\text{A|BC}} = \sum_i p_i \rho^{\text{A}}{}_i \otimes \rho^{\text{BC}}{}_i \qquad \rho_i \text{ Harr measure (mix)}$$



ρ_{bisep}

ρ_{tile} SLOCC

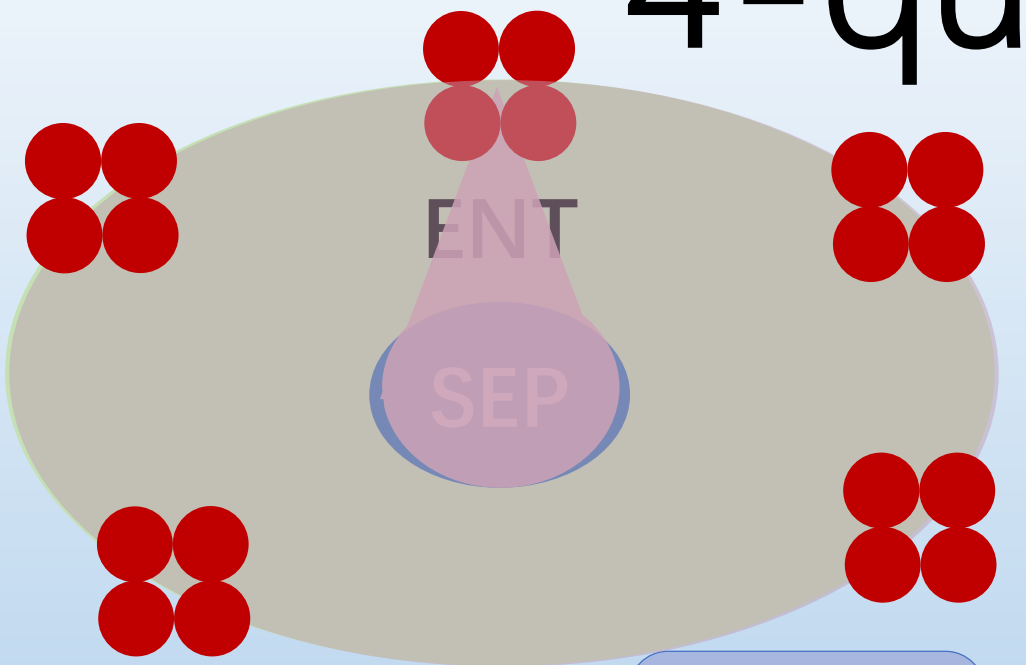
PPT can not detected!

Unextendible Product Bases and Bound Entanglement
1998 Physical Review Letters, Bennette et al.

Error of UPB

Unpublished data

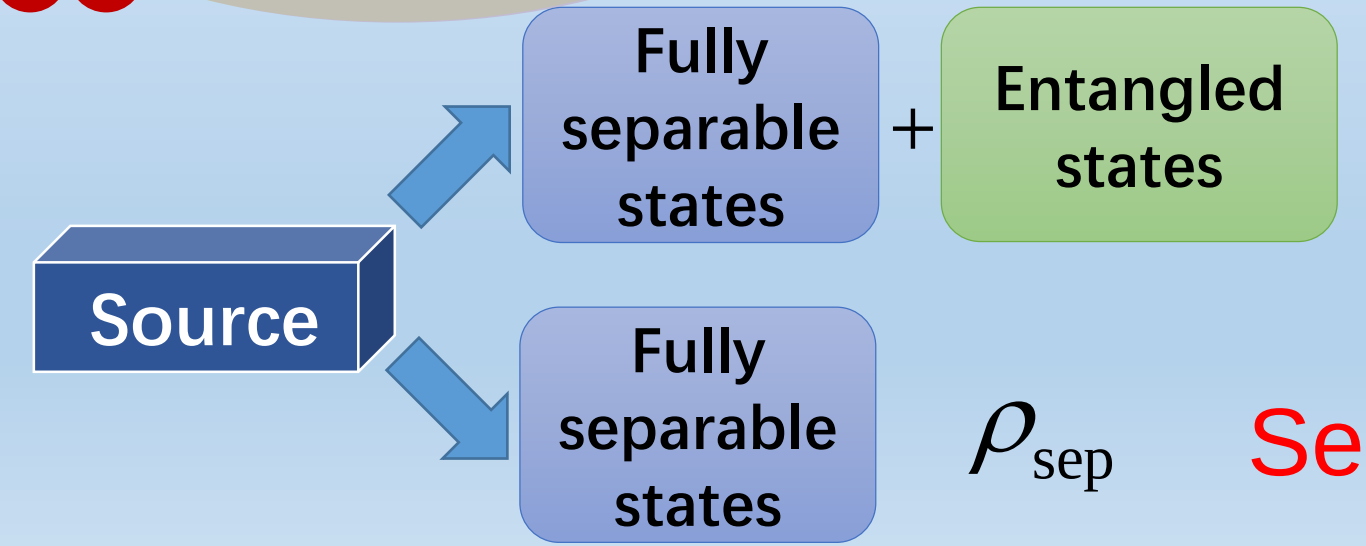
4-qubit system



Entangled or not sure

$$\rho = p\,|\psi\rangle\langle\psi\,| + (1-p)\rho_{\text{sep}}$$

$|\psi\rangle$ **Harr measure (pure)**



ρ_{sep} Separable

4-qubit system

	Tomographic (255 features)	Bell-like (80 features)
Entangled	99%	98%
Separable	near 100%	99%

Separable

Accuracy

In the future

Supervised learning

Cross entropy y_{pred} y_{label} (PPT)

Unsupervised learning

??? y_{pred}

Application & Summary

New tool to detect entanglement with *partial* information

Verify other quantum entanglement detection methods


THANKS