A Web of Dualities in Condensed Matter: from Quantum Hall to Quantum Criticality

Chong Wang
Harvard University

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Field theory duality

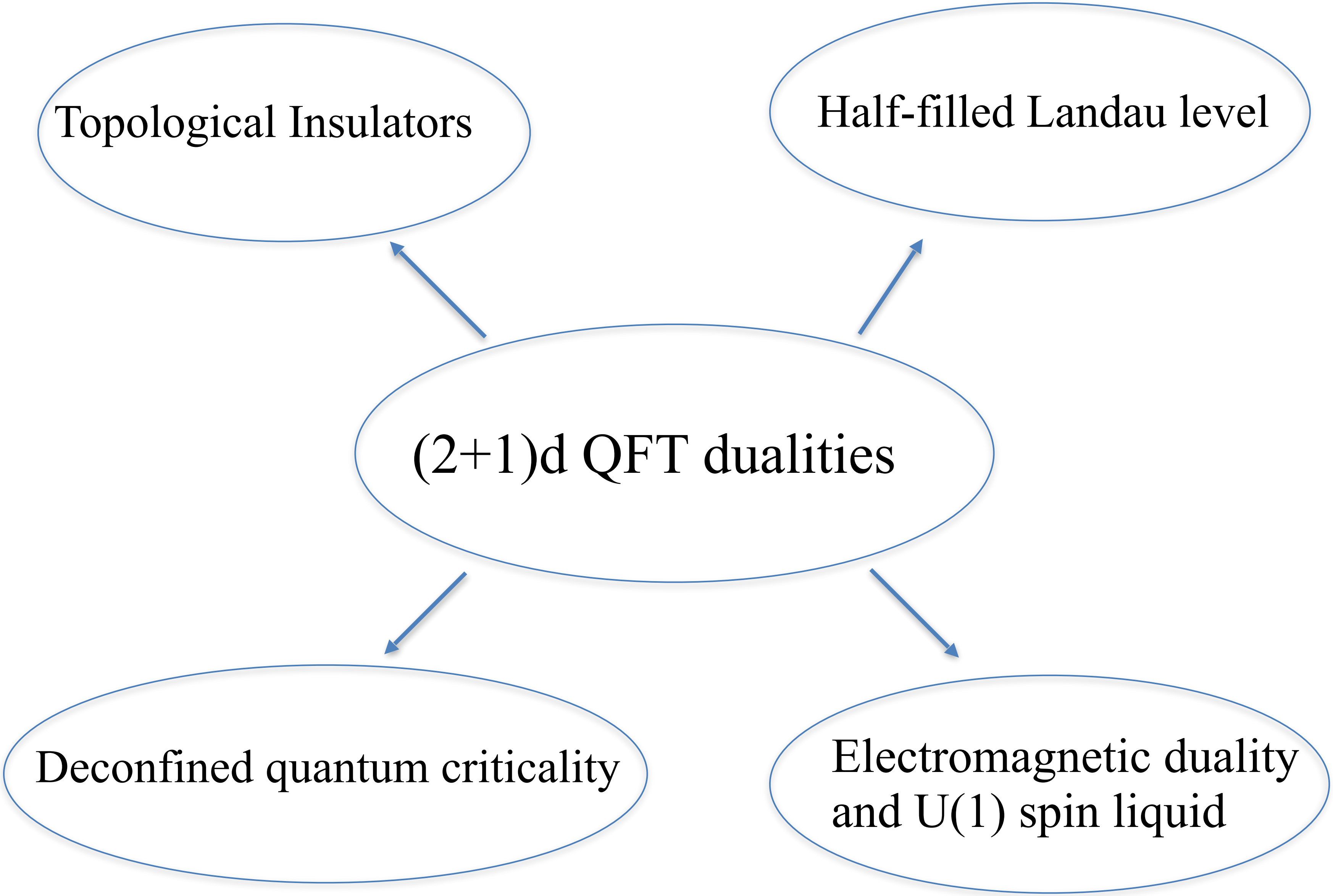
Quantum field theory 1 "=" Quantum field theory 2

"Non-local change of variables"

Familiar examples in (1+1)d: Bosonization, Jordan-Wigner…

This talk: QFT dualities in (2+1)d

Recent developments



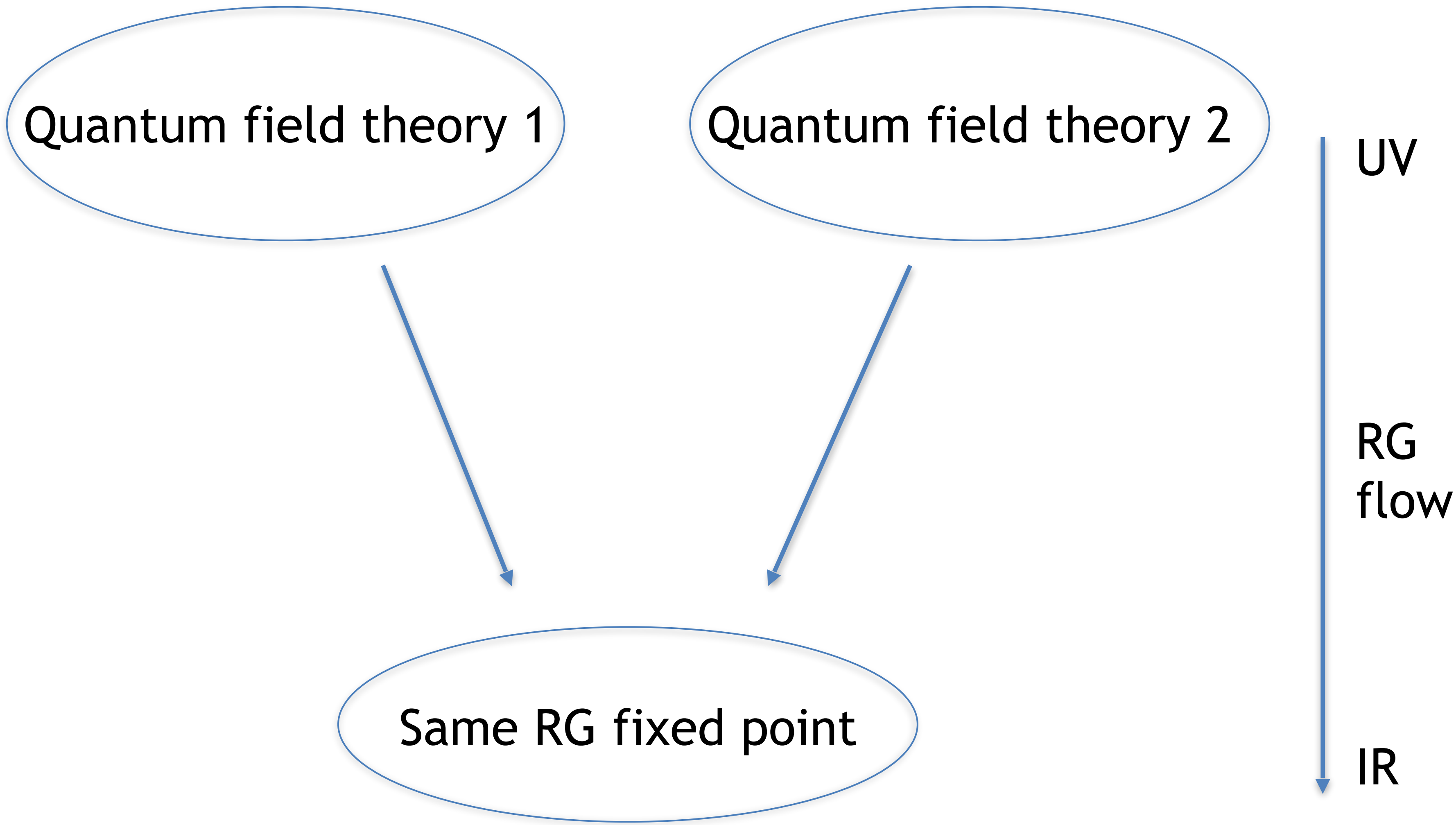
Outline

- A web of QFT dualities
- Half-filled Landau level and particle-hole symmetry
- Deconfined quantum criticality and QED_3

Part I: A Web of Dualities

- Seiberg, Senthil, **CW**, Witten, 1606.01989
- Karch, Tong, 1606.01893

Duality: what do we mean?



Simplest example in (2+1)d: Boson-Vortex Duality

$$|D_B\phi|^2 - |\phi|^4 \qquad \longleftrightarrow \qquad |D_b\widehat{\phi}|^2 - |\widehat{\phi}|^4 + \frac{1}{2\pi}bdB$$

- Left: O(2) Wilson-Fisher (particle picture)
- Right: gauged O(2) Wilson-Fisher (vortex picture)
- Postulate: same IR fix point
- Consistency checks:
  - same local operators and phase diagram
  - lattice "derivation" at strong coupling

Peskin; Dasgupta, Halperin; Fisher, Lee

Consistency checks

$$|D_B\phi|^2 - |\phi|^4 \longleftrightarrow |D_b\widehat{\phi}|^2 - |\widehat{\phi}|^4 + \frac{1}{2\pi} bdB$$

$$U(1)_B : j_\phi \longleftrightarrow \frac{1}{2\pi}\epsilon_{\mu\nu\lambda}\partial^\nu b^\lambda$$

$$\text{Operator} : \phi \longleftrightarrow \mathcal{M}_b$$

$$-r|\phi|^2 \longleftrightarrow +r|\widehat{\phi}|^2$$

$r < 0$

LHS: $\langle\phi\rangle \neq 0$, Goldstone boson

RHS: $\widehat{\phi}$ massive, free photon

$r > 0$

LHS: ϕ massive

RHS: $\langle\widehat{\phi}\rangle \neq 0$, Higgs phase

A Fermion-Boson Duality

$$i\overline{\Psi}\not{D}_A\Psi - \frac{1}{8\pi}AdA \quad \longleftrightarrow \quad |D_b\phi|^2 - |\phi|^4 + \frac{1}{4\pi}bdb + \frac{1}{2\pi}bdA$$

$$U(1)_A : j_\Psi \quad \longleftrightarrow \quad \frac{1}{2\pi}\epsilon_{\mu\nu\lambda}\partial^\nu b^\lambda$$

$$\text{Operator} : \Psi \quad \longleftrightarrow \quad \phi^\dagger \mathcal{M}_b$$

Gapless version of "flux-attachment" or "composite boson"

(Aharony; Wilczek; Polyakov; Schaposnik, Fradkin……)

Can be "derived" on lattice or coupled wires

(Mross, Alicea, Motrunich; Chen, Son, Wang, Raghu)

Related to "Mirror dualities" in SUSY QFT

(Kachru, Mulligan, Torroba, Wang)

Phase diagram

$$i\bar{\Psi}\not{D}_A\Psi - \frac{1}{8\pi}AdA \quad \longleftrightarrow \quad |D_b\phi|^2 - |\phi|^4 + \frac{1}{4\pi}bdb + \frac{1}{2\pi}bdA$$

$$m\bar{\Psi}\Psi \quad \longleftrightarrow \quad m|\phi|^2$$

$m < 0 : -\frac{1}{4\pi}AdA$ $\qquad$ $m > 0$: trivial

LHS: integrating out massive $\Psi \rightarrow \frac{m}{|m|}\frac{1}{8\pi}AdA$

RHS: $\langle\phi\rangle \neq 0 \rightarrow$ trivial Higgs phase

$$\text{massive } \phi \rightarrow \quad \frac{1}{4\pi}bdb + \frac{1}{2\pi}bdA \quad \longleftrightarrow \quad -\frac{1}{4\pi}AdA$$

Time-reversal Symmetry?

$$i\bar{\Psi}\not{D}_A\Psi \qquad \longleftrightarrow \qquad |D_b\phi|^2 - |\phi|^4 + \frac{1}{4\pi}bdb + \frac{1}{2\pi}bdA + \frac{1}{8\pi}AdA$$

- Left: invariant
- Right: Chern-Simons term odd — not invariant?
- Invariant up to a particle-vortex duality on ϕ — composite boson goes to its vortex!

“Deriving” more dualities

Starting from

$$\mathcal{L}_1[A] \qquad \longleftrightarrow \qquad \mathcal{L}_2[A]$$

Operation S:

$$\mathcal{L}_1[a] + \frac{1}{2\pi} adA \qquad \longleftrightarrow \qquad \mathcal{L}_2[a] + \frac{1}{2\pi} adA$$

Operation T:

$$\mathcal{L}_1[A] + \frac{1}{4\pi} AdA \qquad \longleftrightarrow \qquad \mathcal{L}_2[A] + \frac{1}{4\pi} AdA$$

- Infinitely many dualities are generated!
- How many are interesting for condensed matter physics?

Part II: Half-filled Landau level and particle-hole symmetry

- Son, 1502.03446
- **CW**, Senthil, 1505.05141; 1507.08290
- Metlitski, Vishwanath, 1505.05142
- Mross, Alicea, Motrunich, 1510.08455
- Cheung, Raghu, Mulligan, 1611.08910
- Levin, Son, 1612.06402
- **CW**, Cooper, Halperin, Stern, 1701.00007

A fermion-fermion duality

$$i\bar{\chi} \not{D}_a \chi + \frac{1}{4\pi} adA \qquad \longleftrightarrow \qquad i\bar{\Psi} \not{D}_A \Psi$$

"Fermionic particle-vortex duality"

(Son; **CW**, Senthil; Metlitski, Vishwanath; Mross, Alicea, Motrunich)

Related to previous dualities using S&T operations

(Seiberg, Senthil, **CW**, Witten; Karch, Tong; Murugan, Nastase)

Free Dirac ↔ QED$_3$

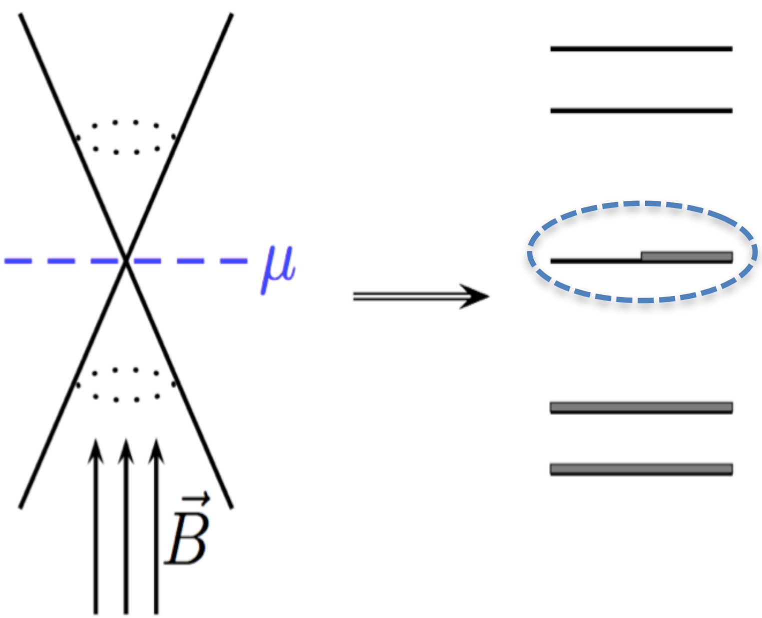
$$i\bar{\Psi}\not{D}_A\Psi \quad \leftrightarrow \quad i\bar{\chi}\not{D}_a\chi + \frac{1}{4\pi}adA$$

$$U(1)_A : j_\Psi \quad \leftrightarrow \quad \frac{1}{4\pi}\epsilon_{\mu\nu\lambda}\partial^\nu a^\lambda$$

$$\mathcal{CT} : \Psi \to i\sigma_2\Psi^\dagger \quad \leftrightarrow \quad \mathcal{T} : \chi \to i\sigma_2\chi$$

$$\text{Operator} : \ \Psi \quad \leftrightarrow \quad \mathcal{M}_a^2$$

Application: Half-filled Landau level



- Finite magnetic field: half-filled Landau level
- Same as 2DEG at $\nu = 1/2$
- Particle-hole symmetry unbroken

$$\mathcal{CT} : \Psi \to i\sigma_2 \Psi^\dagger$$

- Traditional HLR approach does not keep particle-hole **manifestly**

Dual picture: $i\bar{\chi}\not{D}_a\chi - \frac{1}{4\pi}adA$

Finite field = finite vortex density

$$n_\chi = \frac{B}{4\pi}$$

Simplest solution: a Fermi surface of dual Dirac fermions!

Particle-hole acts like time-reversal

$$\mathcal{T}:\chi \to i\sigma_2\chi,\ \mathcal{T}^2 = (-1)^{N_\chi}$$

Dirac Composite Fermi Liquid

- Composite fermions look like TI surface!
- Compared with HLR:
  no Chern-Simons term, but a Berry phase of π
- Numerical evidence:
  suppression of certain $2k_f$ singularity
  (Geraedts, et. al, 2015)

Open Issues

- Dirac vs. HLR theory: really different or not?

- Could HLR be secretly PH symmetric?
  - Maybe: many observables “unreasonably” PH symmetric ($\boldsymbol{\sigma}_{xy}$, commensurability oscillations) (**CW**, Cooper, Halperin, Stern; Cheung, Raghu, Mulligan)
  - $m_e \to 0$ limit not needed: relevant for experiments! e.g. Weiss oscillation (Kamburov, et. al, PRL 2014)
  - But some other quantities not automatically PH symmetric — still not fully understood (Levin, Son)

Part III: Deconfined criticality and QED_3

- **CW**, Nahum, Metlitski, Xu, Senthil, 1703.02426

A two-component duality

$$|D_b z_1|^2 + |D_b z_2|^2 - |z_1|^4 - |z_2|^4 \quad \Longleftrightarrow \quad \bar{\psi}_1 i \not{D}_a \psi_1 + \bar{\psi}_2 i \not{D}_a \psi_2$$

Karch, Tong, 2016; CW, Nahum, Metlitski, Xu, Senthil, 2017

- Easy-plane CP^1 = QED_3 with two Dirac fermions

- IR fate controversial for both:
  continuity vs. chiral symmetry breaking

- Assume they flow to nontrivial fixed point — promising evidence from recent numerics (Karthik, Narayanan, 2016; Qin, et. al; Zhang, et, al, 2017)

Why care about these theories?

- QED_3 with N_f=2: SPT (boson IQHE) to trivial insulator transition (Grover, Vishwanath; Lu, Lee; 2013)

$$\bar{\psi}_1 i \not{D}_a \psi_1 + \bar{\psi}_2 i \not{D}_a \psi_2 + m(\bar{\psi}_1\psi_1 + \bar{\psi}_2\psi_2) + \frac{1}{2\pi} adA + \frac{1}{4\pi} AdA$$

Integrate out ψ → $\frac{\mathrm{sgn}(m)}{4\pi} ada + \frac{1}{2\pi} adA + \frac{1}{4\pi} AdA$

Integrate out a → $\frac{1 - \mathrm{sgn}(m)}{4\pi} AdA$

Why care about these theories?

- CP_1: deconfined criticality — Neel to valence-bond-solid (VBS) transition (Senthil, Vishwanath, Balents, Sachdev, Fisher 2004)

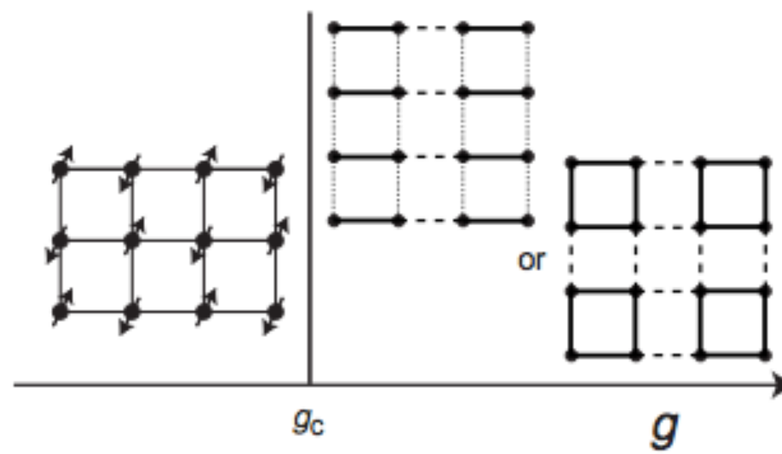
$$|D_b z_1|^2 + |D_b z_2|^2 - m(|z_1|^2 + |z_2|^2) - |z_1|^4 - |z_2|^4$$

m>0: free photon = "superfluid"

m<0: photon gapped (Higgs), but

$$\langle z_1^\dagger z_2 \rangle \neq 0$$

Two nearby phases break completely different symmetries



- Duality → the two transitions are the same!

- Many testable predictions

- For example: $3 - \dfrac{1}{\nu_{jq}} = \dfrac{1 + \eta_{\bar{\psi}\sigma^z\psi}}{2}$

- Broadly consistent with recent numerics from several lattice models:

$\eta \approx 1.0$ (Karthik, Narayanan, 2016; Qin, et. al. 1705.10670)

$\nu \approx 0.5$ (Qin, et. al. 1705.10670; Zhang, et. al. 1706.05414)

A mini-web of dualities

$$|D_b z_1|^2 + |D_b z_2|^2 - |z_1|^4 - |z_2|^4 \qquad \bar{\psi}_1 i \not{D}_a \psi_1 + \bar{\psi}_2 i \not{D}_a \psi_2$$

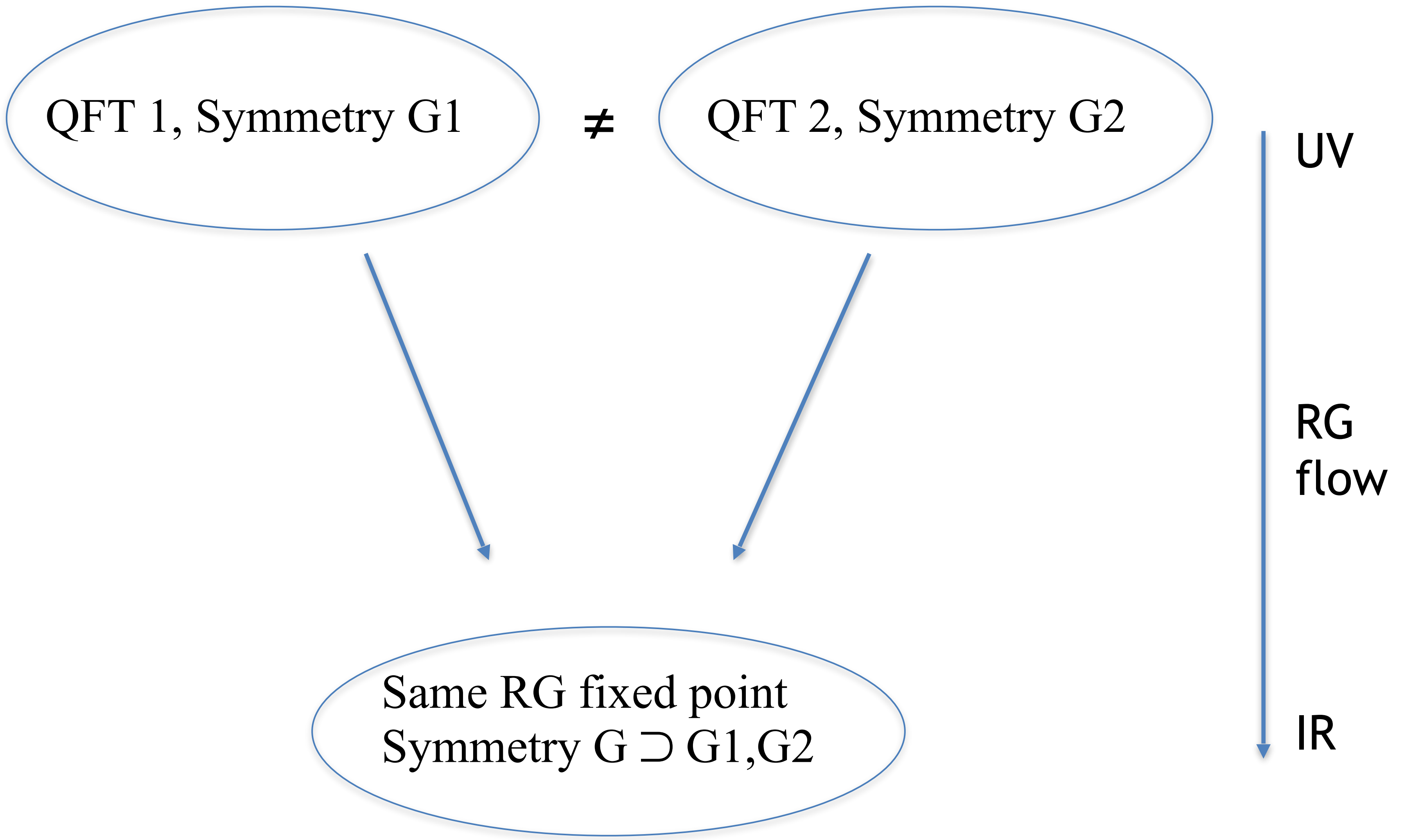
$$|D_{\tilde{b}} w_1|^2 + |D_{\tilde{b}} w_2|^2 - |w_1|^4 - |w_2|^4 \qquad \bar{\chi}_1 i \not{D}_{\tilde{a}} \chi_1 + \bar{\chi}_2 i \not{D}_{\tilde{a}} \chi_2$$

- Self-dualities for both theories
  (Motrunich, Vishwanath, 2003; Xu, You, 2015)

- Same Lagrangian, different symmetry actions
  e.g. a conserved U(1) current:

$$j^\mu_{z_1} - j^\mu_{z_2} \sim \frac{1}{2\pi} \epsilon^{\mu\nu\lambda} \partial_\nu \tilde{b}_\lambda$$

Emergent Symmetry



Emergent O(4) symmetry

$$|D_b z_1|^2 + |D_b z_2|^2 - |z_1|^4 - |z_2|^4$$

$$|D_{\tilde{b}} w_1|^2 + |D_{\tilde{b}} w_2|^2 - |w_1|^4 - |w_2|^4$$

$$\bar{\psi}_1 i \not{D}_a \psi_1 + \bar{\psi}_2 i \not{D}_a \psi_2$$

$$\bar{\chi}_1 i \not{D}_{\tilde{a}} \chi_1 + \bar{\chi}_2 i \not{D}_{\tilde{a}} \chi_2$$

- Fixed point must have $SU(2)_\psi$, $SU(2)_\chi$, Z_2: $z_1 \leftrightarrow z_2$

- Total symmetry: O(4)

- Only a subgroup manifest in any single theory — need the duality web to reveal the full structure!

Another (conjectured) duality

$$\sum_{\alpha=1,2} |D_b z_\alpha|^2 - \left(|z_1|^2 + |z_2|^2\right)^2$$

$$\Longleftrightarrow \quad \sum_{j=1,2} \bar{\psi}_j i \not{D}_a \psi_j + \phi \sum_{j=1,2} \bar{\psi}_j \psi_j + V(\phi)$$

- CP^1 with full SO(3) spin rotation symmetry
  ↔ QED_3-Gross-Neveu
- Both theories have their own self-dual: same Lagrangian, different symmetry action
- Emergent symmetry from the duality web: SO(5)! Numerically observed (Nahum, et, al)

Summary

- A web of field theory dualities in (2+1)d
- A particle-hole symmetric theory of half-filled Landau level: Dirac composite fermions
- Dualities and symmetries in deconfined quantum criticality and QED_3

Thank you!