Topological Phases in Driven Quantum Systems

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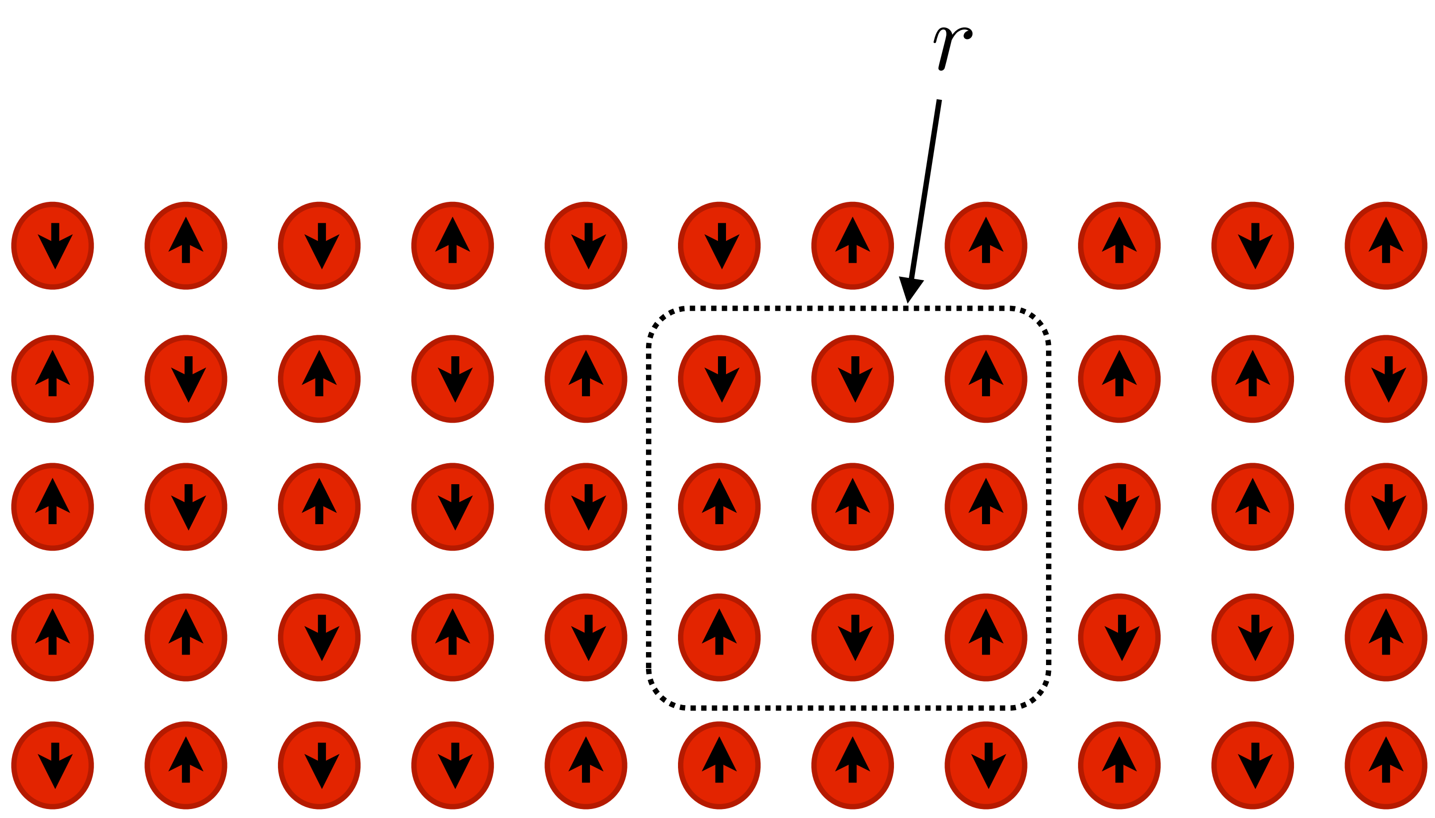
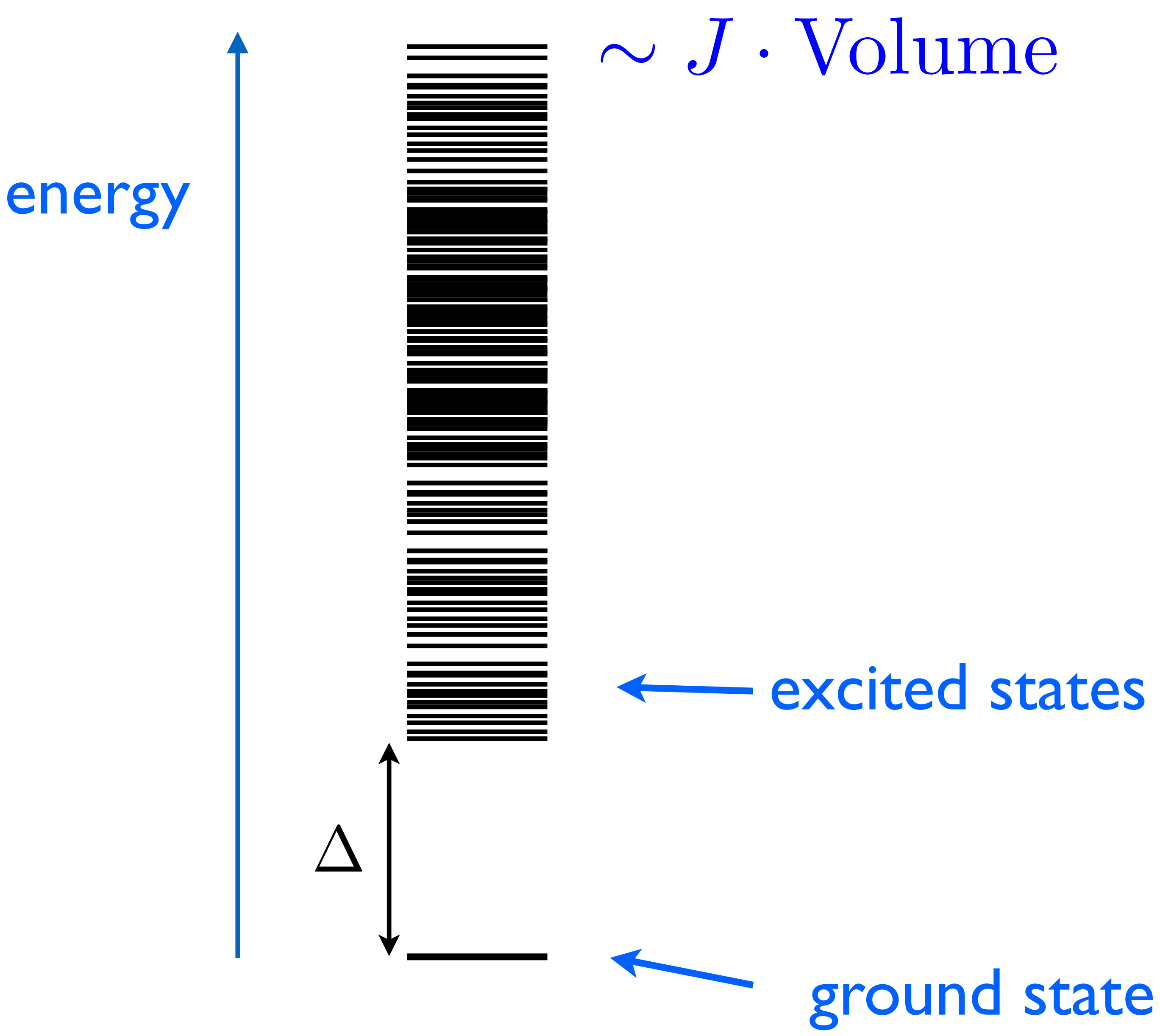
Quantum many body systems:

Hilbert space: $\mathcal{H} = \bigotimes_{\mathrm{sites\ i}} \mathcal{H}_i$

Hamiltonian: $$H = \sum_r H_r \qquad J = \mathrm{max_r |H_r|}$$



Spectrum of gapped Hamiltonian:

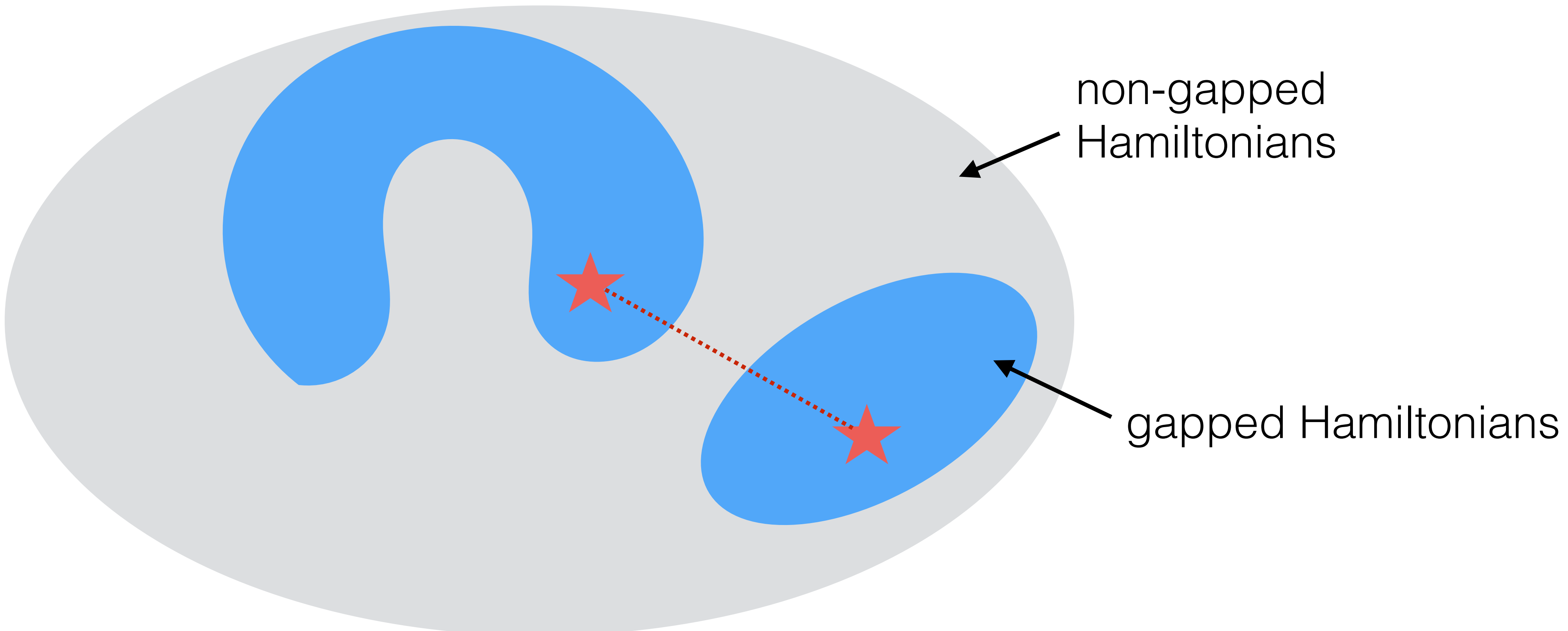


Gapped:

- exponentially decaying correlations in ground state

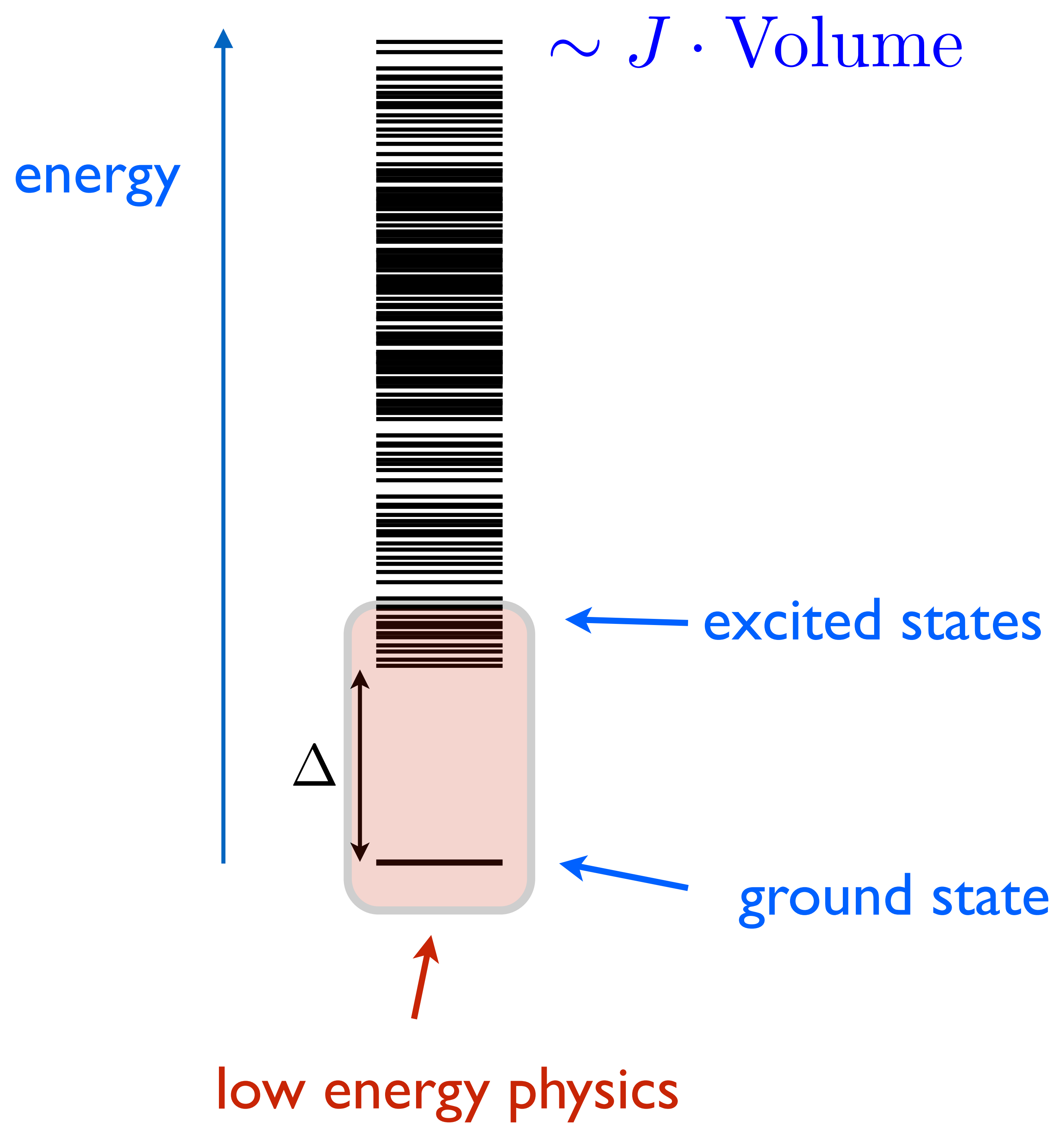
- area law entanglement entropy in ground state

Topological phases



- different phases distinguished by many-body invariants: e.g. quantized Hall conductivity, quasiparticle statistics, etc.

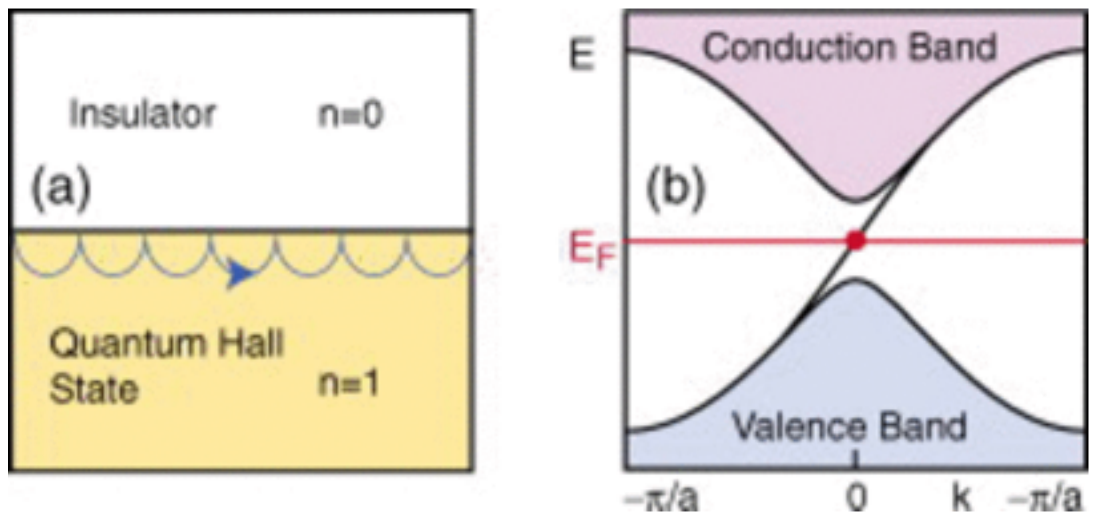
Spectrum of gapped Hamiltonian:



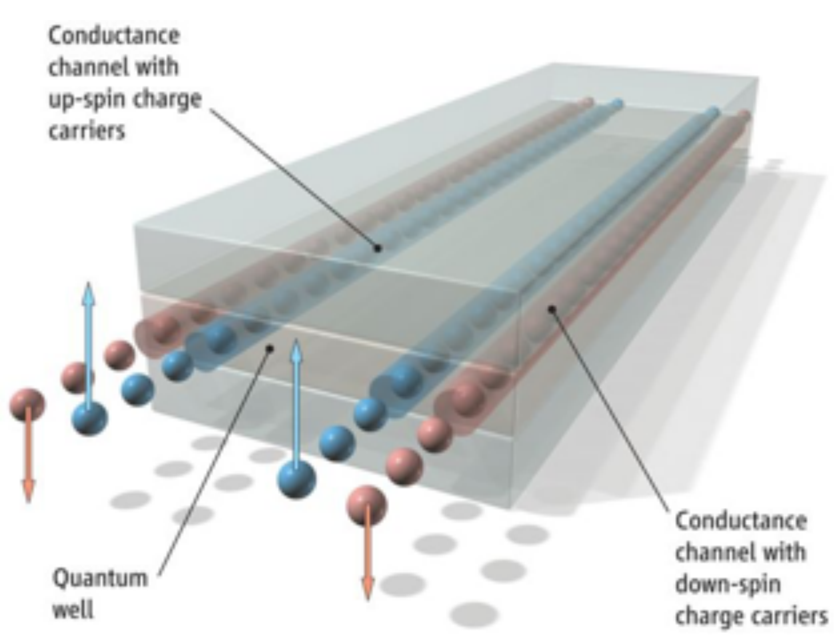
Topological phases

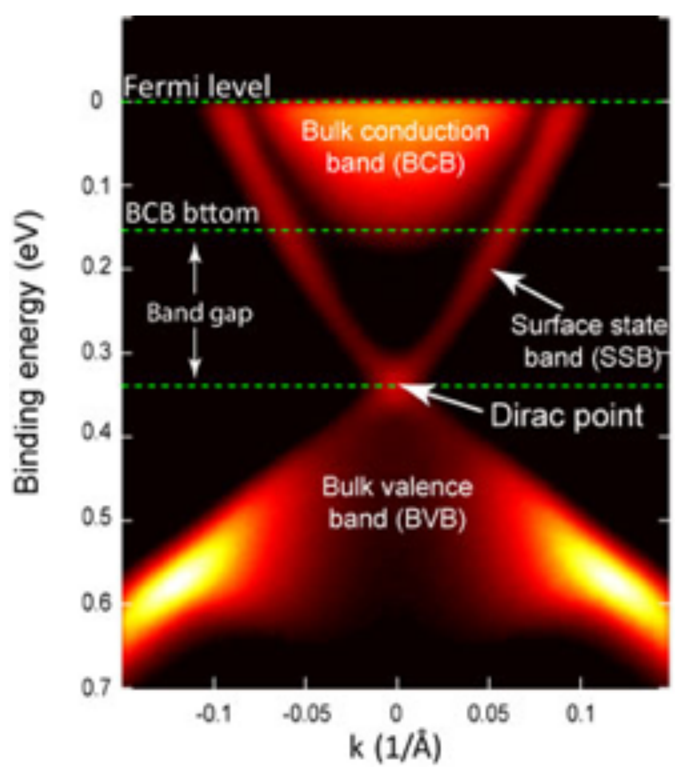
T=0 quantum phases: physics of the ground state

- quantum Hall effect (1980s):



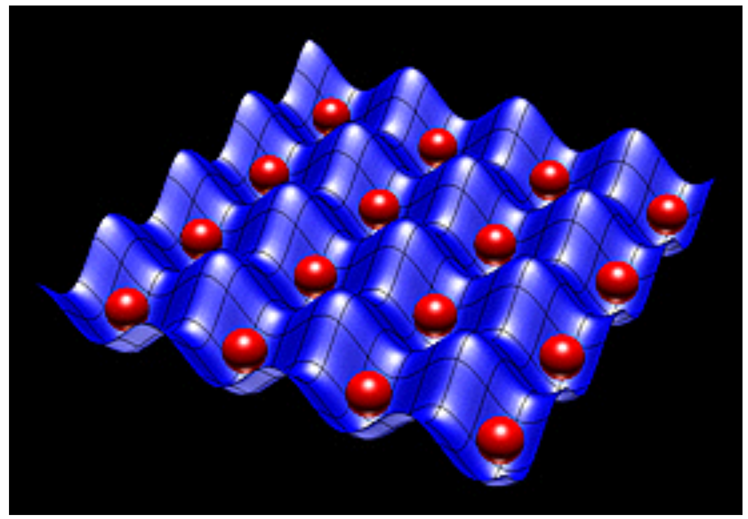
- quantum spin Hall effect, 3d topological insulator (2006-):





New designer quantum systems

Alkalai atoms in optical lattice:

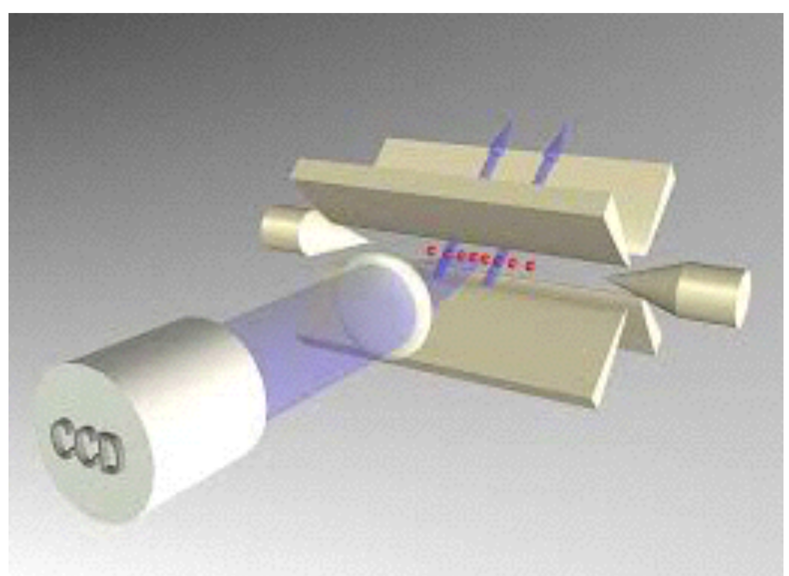


nist.gov

→ Mott insulator - superfluid transition

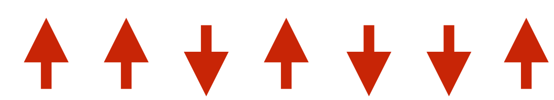
Quantum anti-ferromagnetism (Greiner 2017)

Trapped ions:

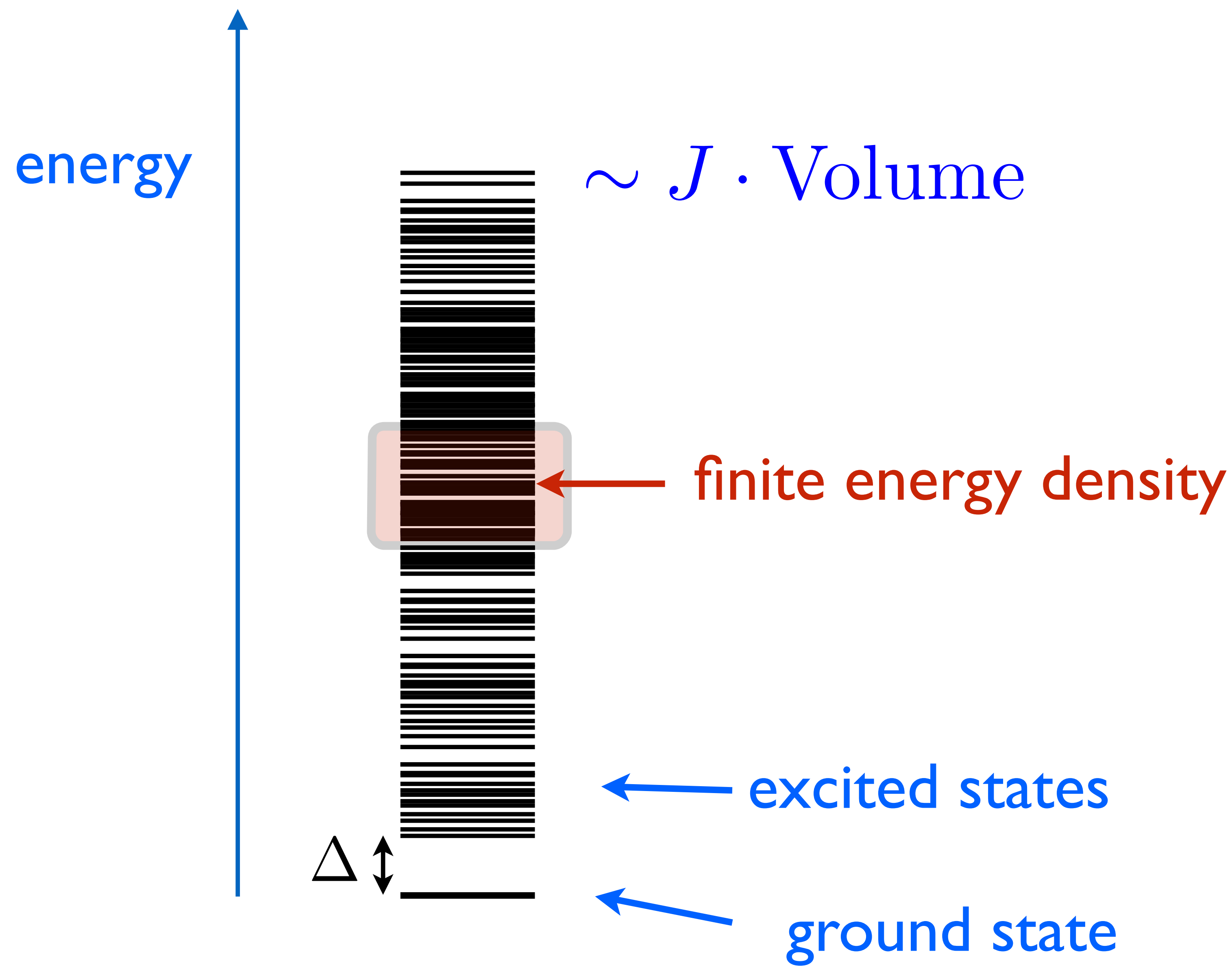


quantumoptics.at

→ effective Ising spin chain:



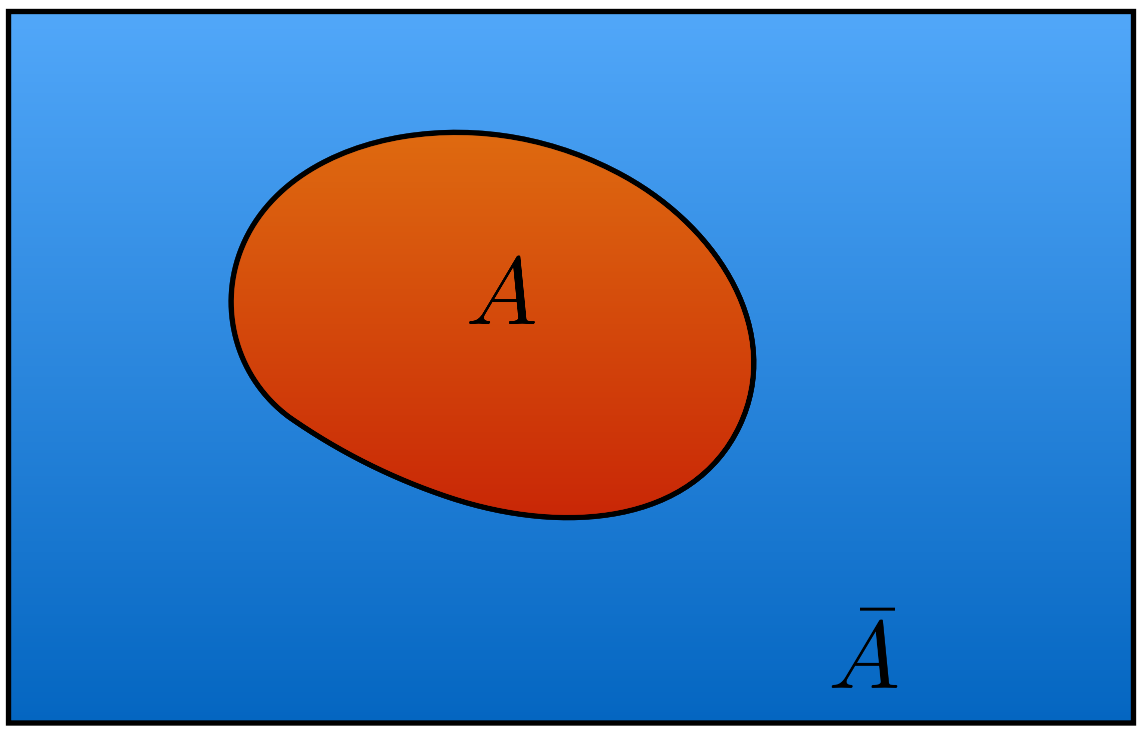
Can we observe topological properties at finite energy density?



Generic expectation: subsystems are thermalized

$$\langle\Psi(0)|H|\Psi(0)\rangle = E \qquad E > E_{g.s.}$$

$$=> \langle\Psi(t)|H|\Psi(t)\rangle = E$$



$$\rho_A(t) = \mathrm{Tr}_{\bar{\mathrm{A}}}\, |\Psi(\mathrm{t})\rangle\langle\Psi(\mathrm{t})|$$

- becomes thermal:

$$\rho_A(t) \to \frac{1}{Z}\,\mathrm{Tr}_{\bar{\mathrm{A}}}\, \mathrm{e}^{-\mathrm{H}/(\mathrm{k_B T})}$$

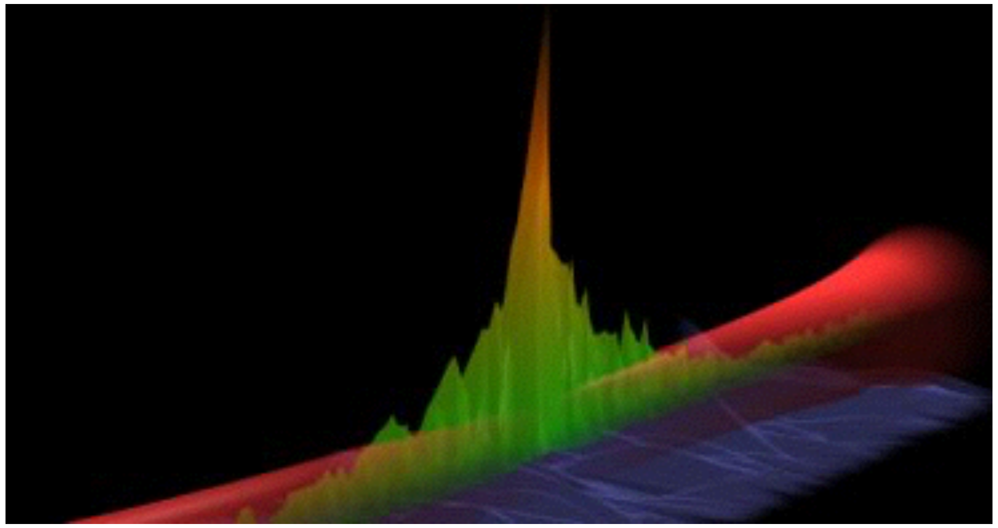
- finite energy density eigenstates have volume law entanglement entropy (Eigenstate Thermalization Hypothesis)

Avoiding thermalization: Many-body localization (MBL)

- with strong disorder, can fail to achieve thermal equilibrium, even with interactions

Basko, Aleiner, Altschuler; Pal, Huse, Gopalakrishnan, Nandkishore, Oganesyan, …
experiments by Bloch group (cold atoms) and Monroe group (trapped ions)

- recall non-interacting Anderson insulator: all single-particle eigenstates localized



physicsworld.com

$$n_j = a_j^\dagger a_j$$

complete set of quasi-local conserved quantities

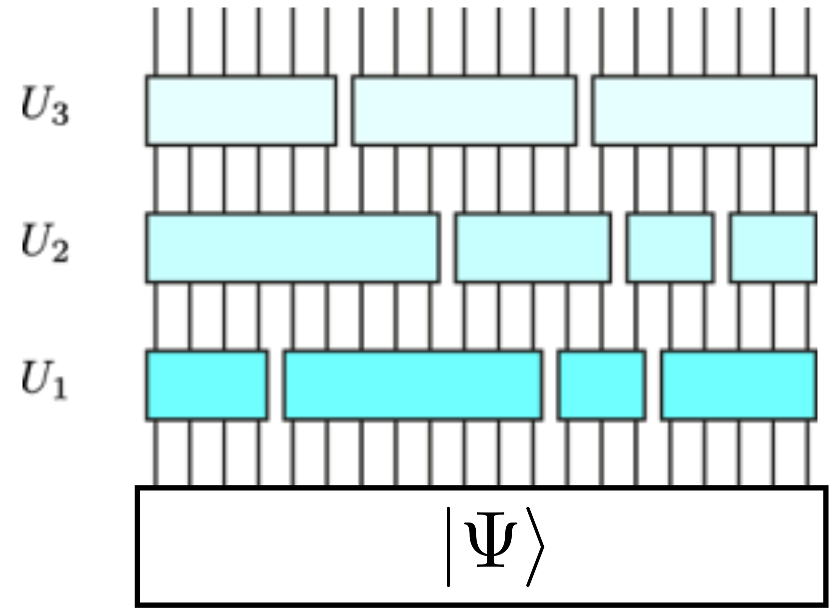
- existence of complete set of quasi-local conserved quantities remains remains true with weak interactions

J. Imbrie

Finite depth quantum circuit of local unitaries:

- unitary operator on a many body Hilbert space built out of local gates:

U =



- preserves notion of locality: if X is a local operator, then $U^\dagger X U$ is a 'nearby' or 'dressed' quasi-local operator.

- Floquet unitaries are approximate quantum circuits disordered couplings

$$H_0 = \sum_i h_i \sigma_i^z + \sum_{i,j} J_{i,j} \sigma_i^z \sigma_j^z + \dots$$

perturb with $H_1 = \sum_i c_i \sigma_i^x$: $\qquad H = H_0 + H_1$

then there exists a finite depth circuit of unitaries U that approximately diagonalizes H in the z basis:

(Imbrie)

$$U^\dagger H U = \sum_i h_i' \sigma_i^z + \sum_{i,j} J_{i,j}' \sigma_i^z \sigma_j^z + \dots$$

$\{U \sigma_i^z U^\dagger\}$ then forms a complete set of quasi-local commuting conserved quantities

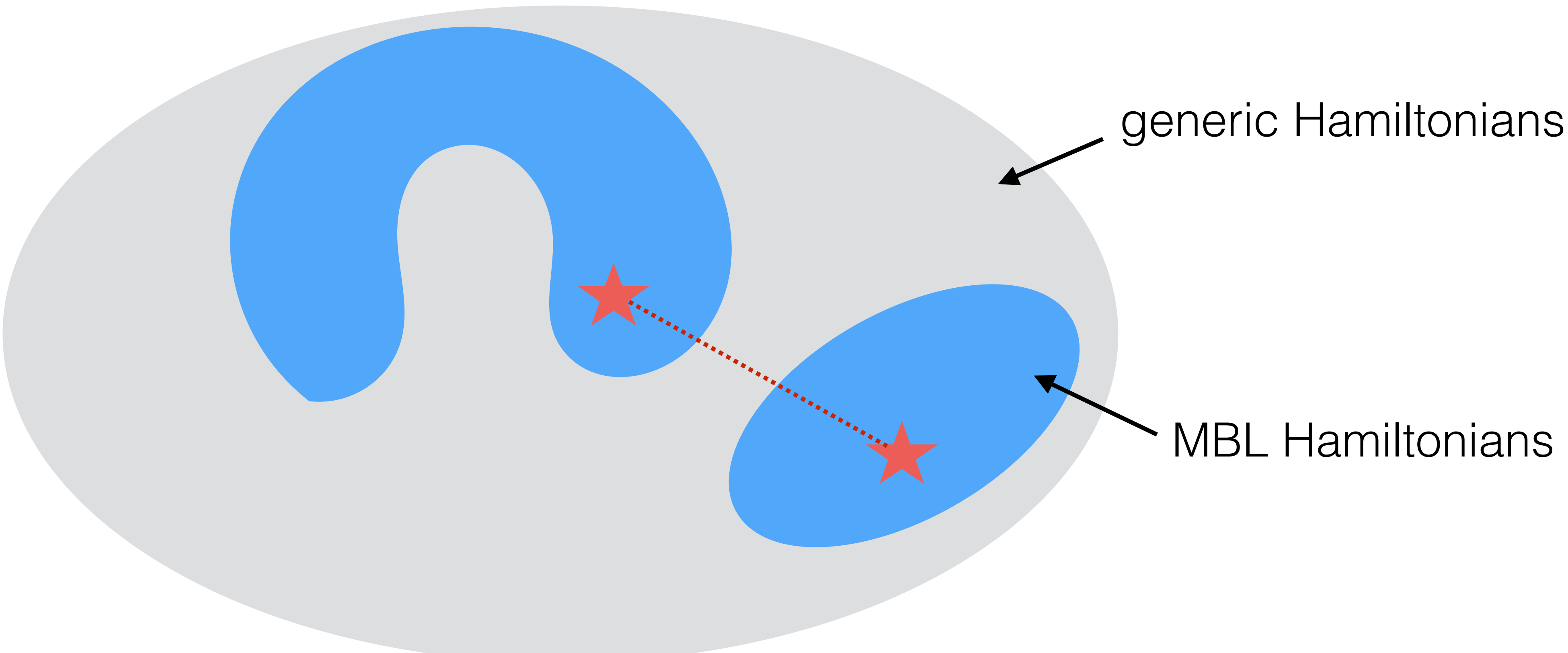
`l-bits’

(Huse, Nandkishore, Oganesyan)

=> area law entanglement entropy in all eigenstates

Many-body localization and topological order

- replace 'gapped' with 'many-body localized' (MBL)



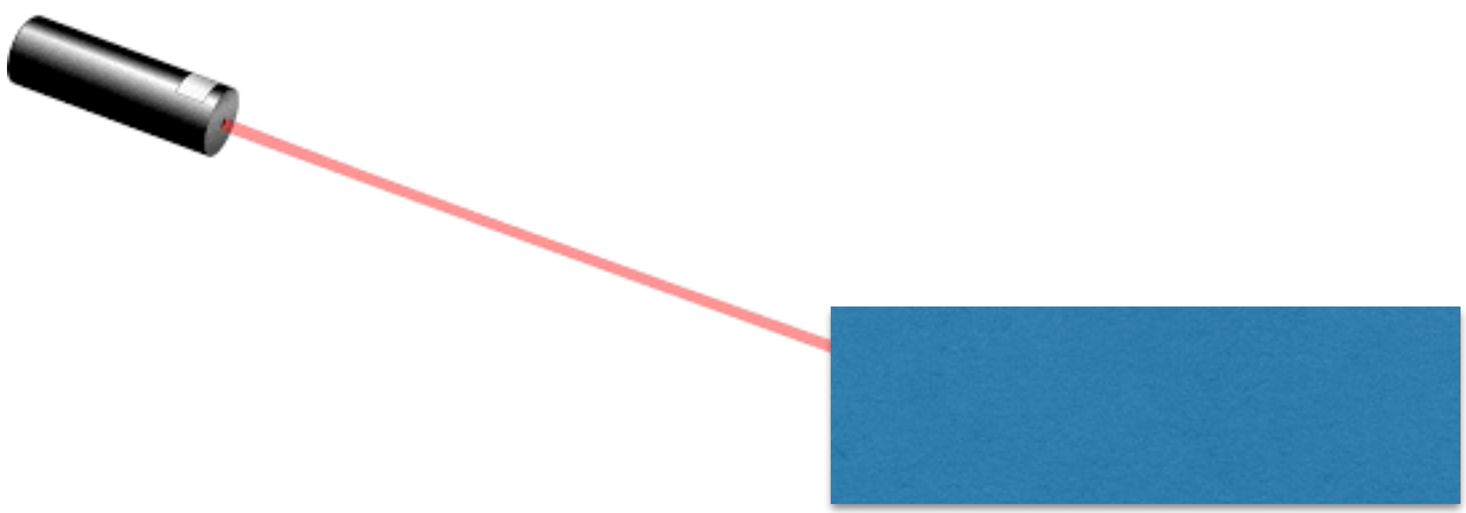
- topological order at finite energy density

(Bahri, Vosk, Altman, Vishwanath; Nandishore, Huse)

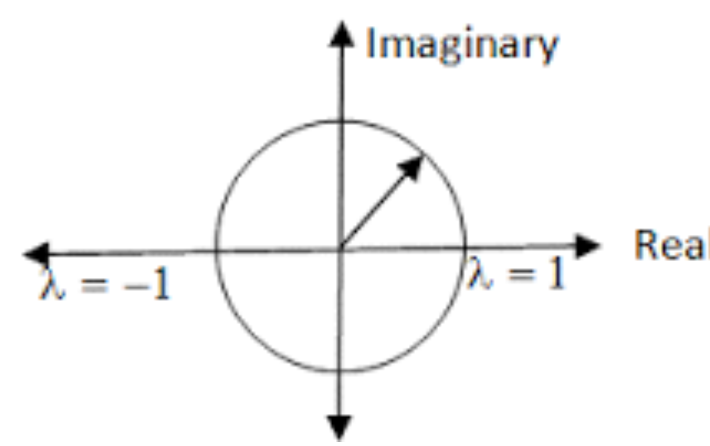
Floquet driving

- periodic time dependent Hamiltonian:

$$H(t+T) = H(t) \qquad U_F = T\exp\left(i\int_0^T H(t)dt\right)$$

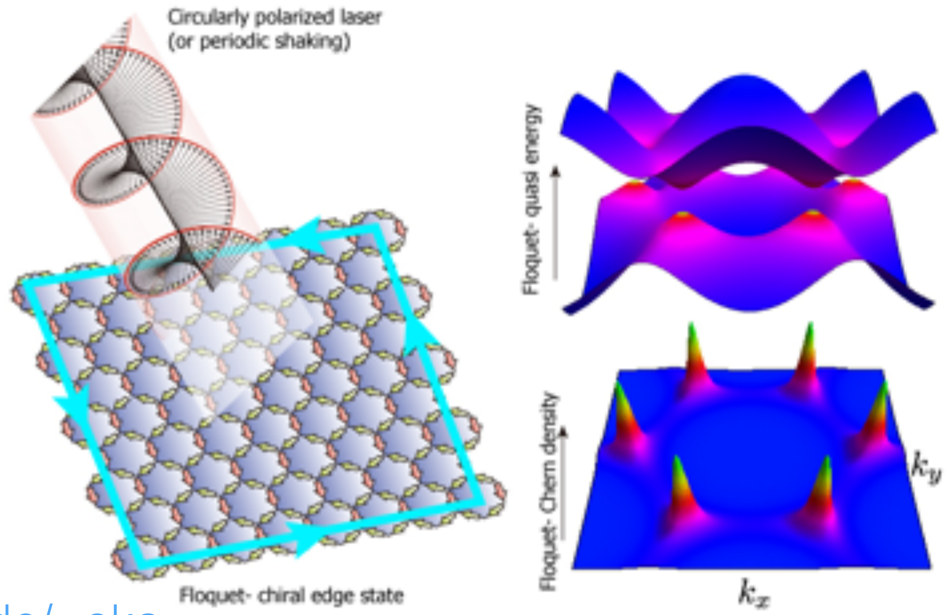


- diagonalize the 'Floquet unitary' U_F



Floquet band structure engineering

- free fermions: trivial -> topological:



pks.mpg.de/~oka

- proposal for Floquet Engineering 2d TI from trivial band structure (Lindner, Refael, Galitski, Nat. Phys. '11)

- photo-induced quantum Hall state on TI surface (Gedik et al, Science 2013)

Floquet band structure engineering

- *Intrinsically* Floquet band structures

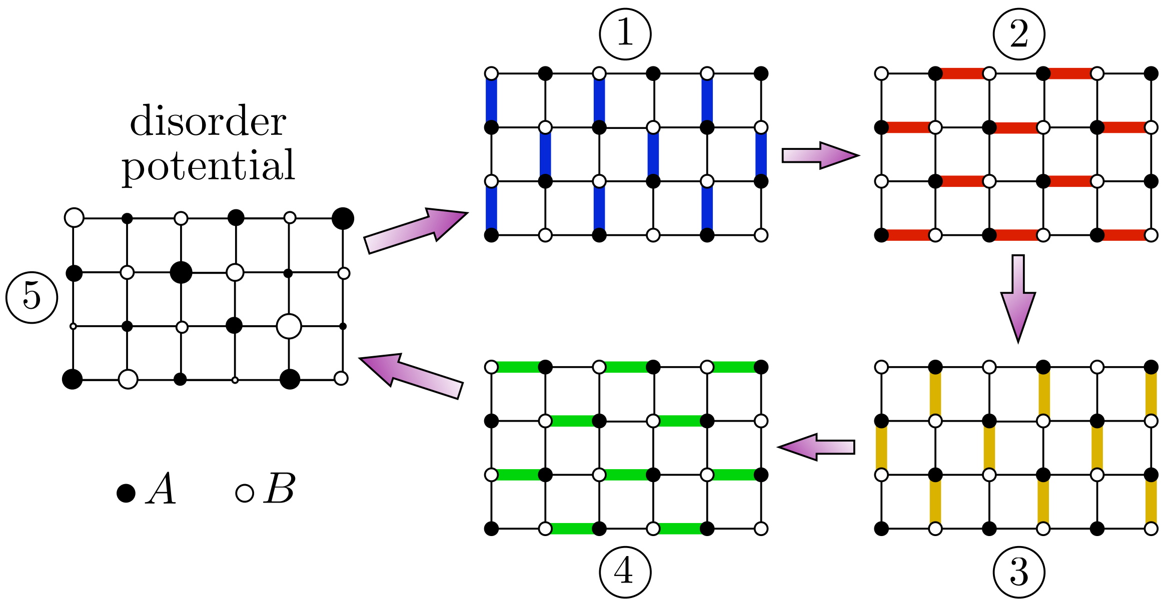


figure from Titum, Berg, Rudner, Refael, Lindner, Phys. Rev. X 6, 021013 (2016)

Rudner, Lindner, Berg, Levin ’12; Titum, Berg, Rudner, Refael, Lindner ’16; …

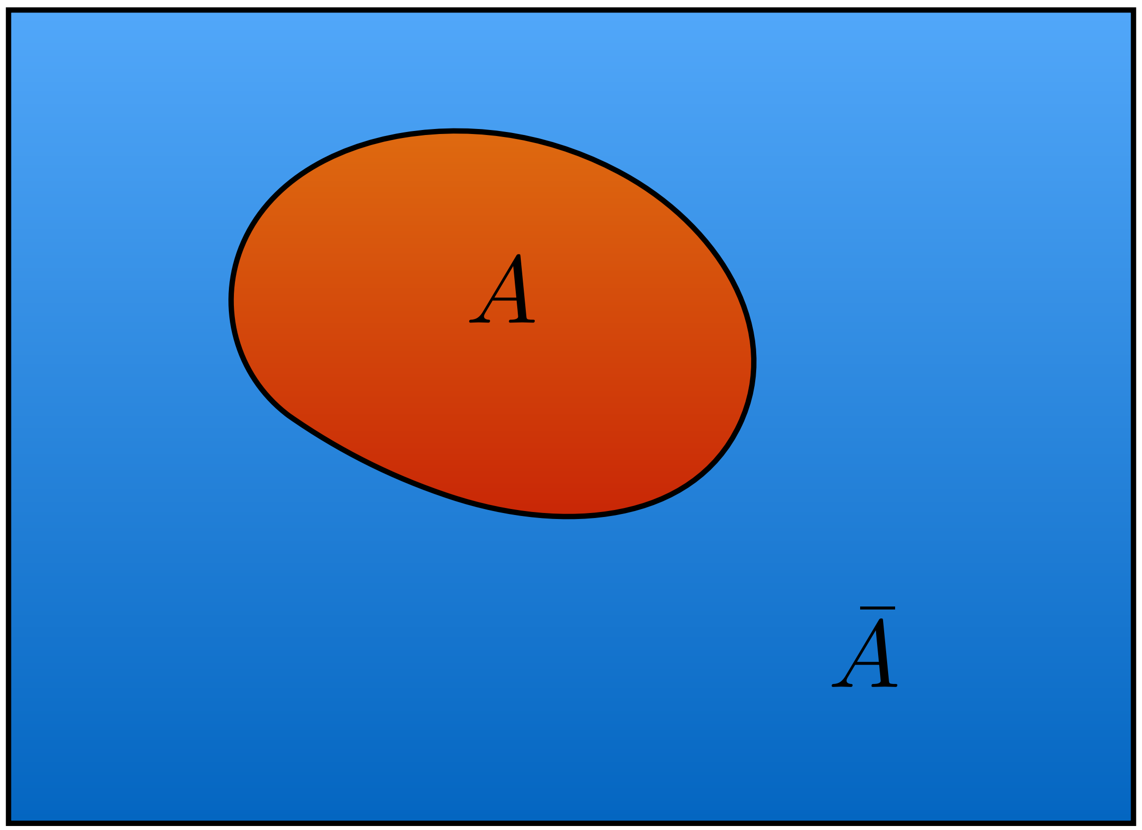
- interactions?

Dehgani, Oka, Mitra PRB ’14, PRB ‘15

Torres, Perez-Piskunow, Balseiro, Usaj, PRL ‘14

Floquet systems: heating problem without MBL

- generically, system will absorb energy until it is at infinite temperature:



$$\rho_A(t) = \mathrm{Tr}_{\bar{\mathrm{A}}} \, |\Psi(\mathrm{t})\rangle\langle\Psi(\mathrm{t})|$$

$$\rho_A(t) \to \mathbf{1} \quad \left(= \frac{1}{Z} e^{-\beta H}\right)$$

$$\text{as } \beta \to 0$$

- entropy has been maximized, system 'blows up'

- MBL in Floquet systems:

- MBL can be stable upon turning on a time dependent periodic perturbation:

Ponte, Papic, Huveneers, Abanin; Lazarides, Das, Moessner

$$H(t) = H_0 + H_1(t)$$

$$(H_1(t+T) = H_1(t))$$

$$U_F = T \exp\left(i \int_0^T H(t) dt \right)$$

$$\Downarrow$$

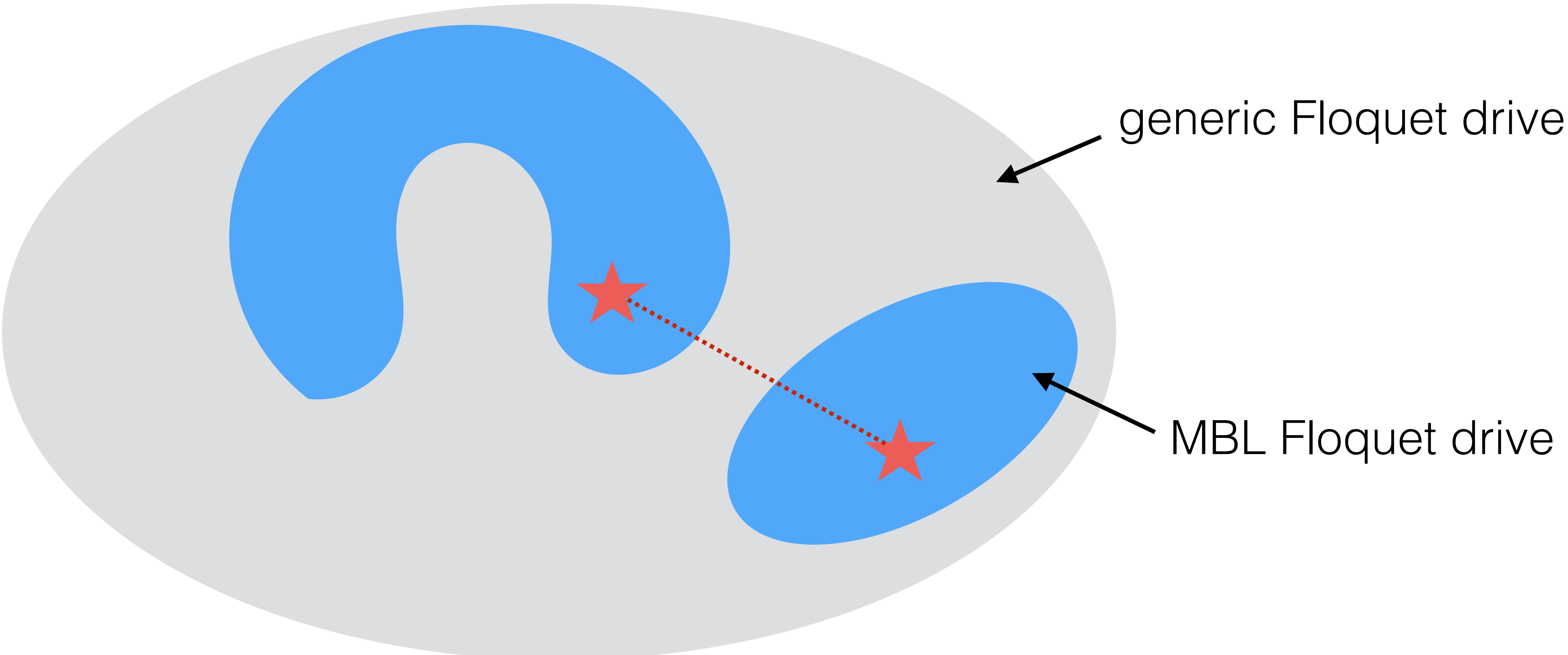
$U_F = e^{-iTH_{\text{eff}}}$, with $U^\dagger H_{\text{eff}} U = \sum_i h_i' \sigma_i^z + \sum_{i,j} J_{i,j}' \sigma_i^z \sigma_j^z$

- schematically,

$$U_F = \prod_\alpha U_\alpha$$

$\leftarrow$ quasi-local commuting unitaries

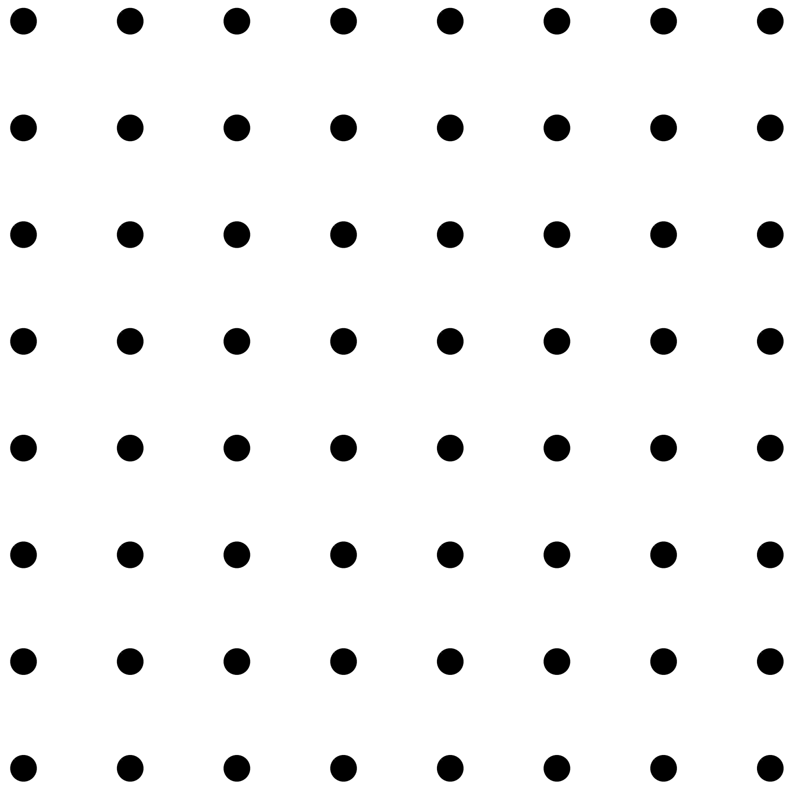
Floquet MBL phases



- many body interacting invariants for Floquet MBL phases?

Chiral Floquet MBL phases

Po, LF, Potter, Vishwanath, '16



$$H(t) = \sum_j H_j(t)$$ ← local terms

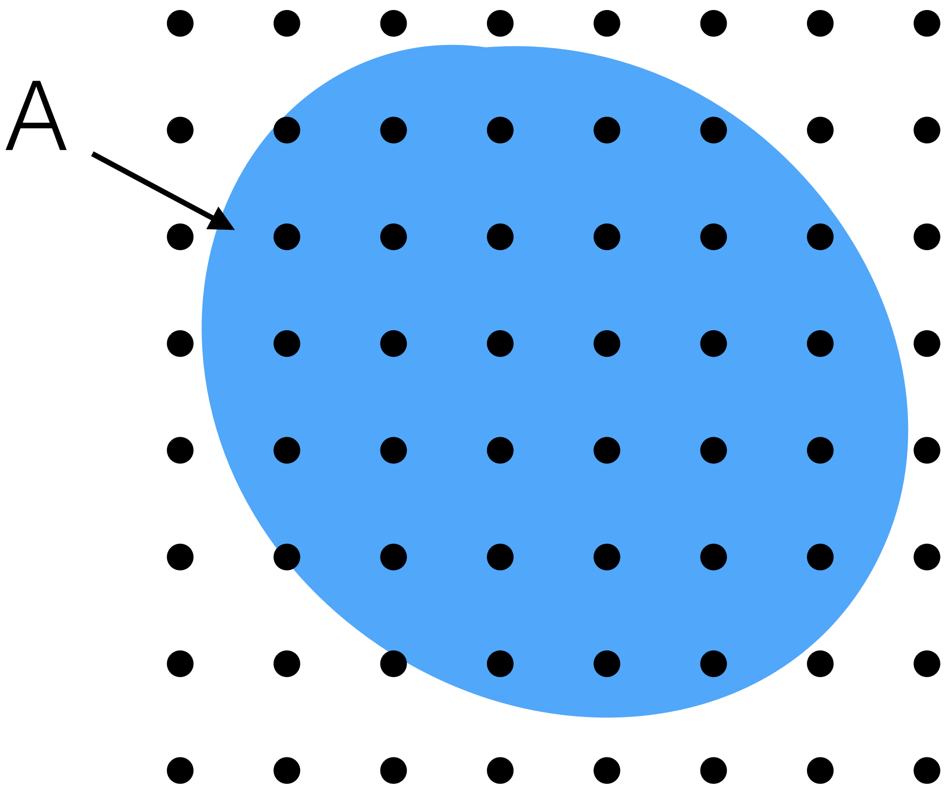
$$U_F = T \exp(i \int_0^T H(t)\, dt)$$

$$V^\dagger U_F V = e^{-iTH_{MBL}}$$

true in bulk of system

$$H_{MBL} = \sum_i h_i \sigma_i^z + \sum_{i,j} J_{i,j} \sigma_i^z \sigma_j^z + \ldots$$

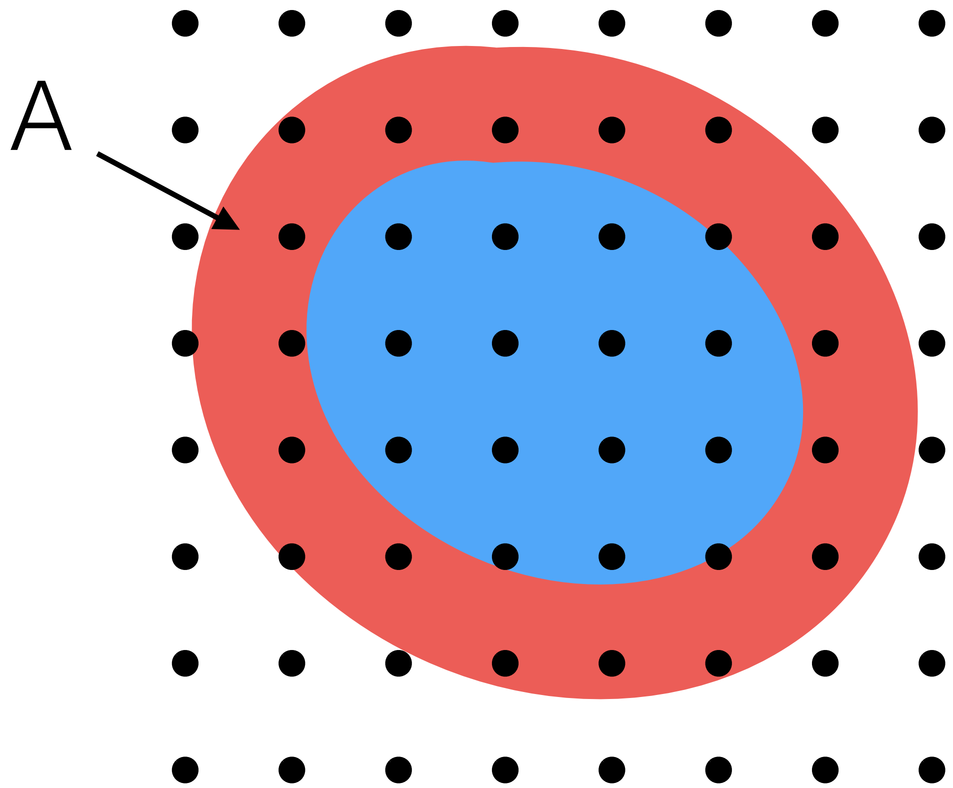
- Expose an edge:



$$H_A = \sum_{j \in A} H_j(t)$$

$$U_F^A = T \exp(i \int_0^T H_A(t)\, dt)$$

- Expose an edge:



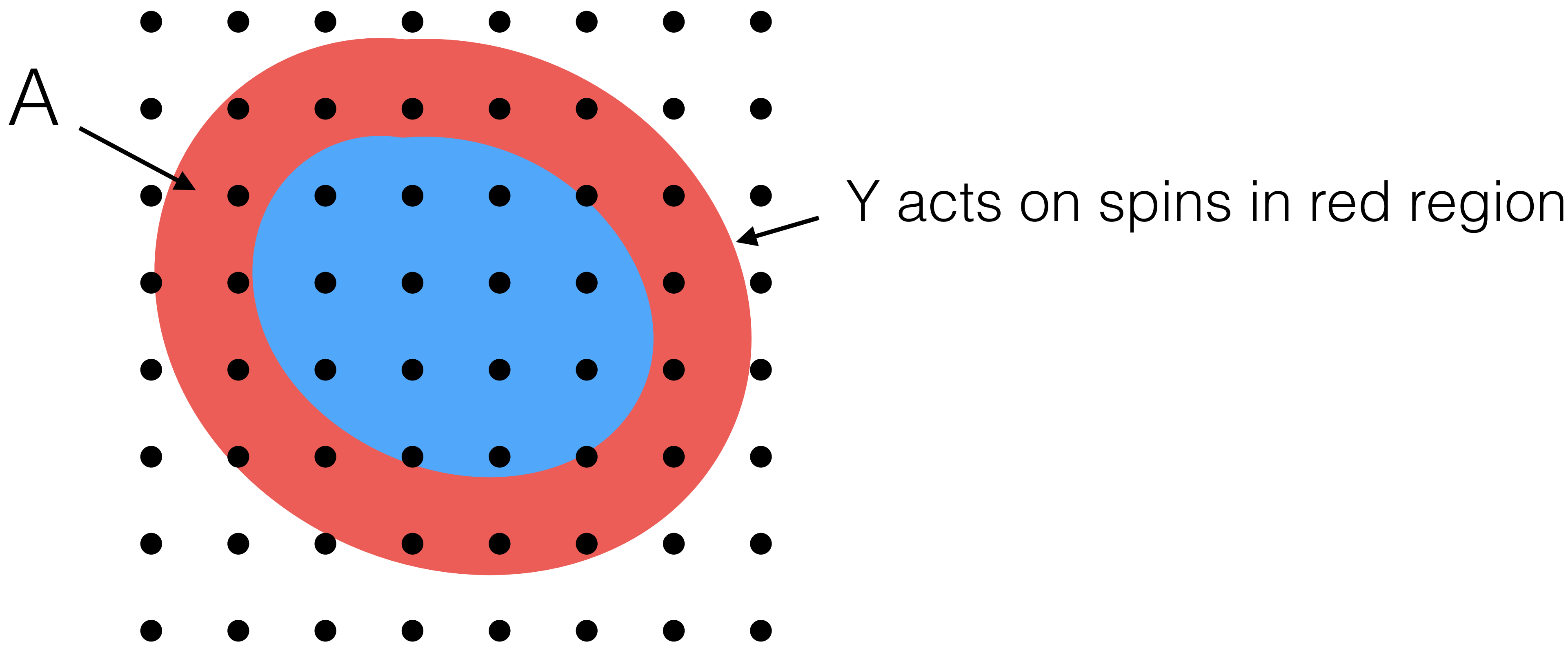
$$H_A = \sum_{j \in A} H_j(t)$$

$$U_F^A = T \exp(i \int_0^T H_A(t)\, dt)$$

$$V_A^\dagger U_F^A V_A = e^{-iTH'_{MBL}}$$

MBL in bulk of system, but not at edge.

- freeze bulk conserved quantities, get effective edge evolution Y acting on red spins: quasi 1d system.



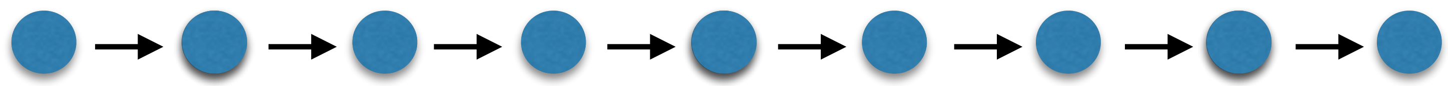
- furthermore, Y is locality preserving: for any local operator $\mathcal{O}$, $Y^\dagger \mathcal{O} Y$ is a (quasi)-local operator supported nearby.

- is Y the Floquet operator of some (quasi)-local 1d Hamiltonian?

Analogy

zero temperature 2d topological phase		MBL Floquet system
Bulk gap	⟷	Bulk MBL
Low energy field theory for the 1d edge	⟷	Locality preserving unitary Y on the 1d edge
lack of 1d UV completion for low energy edge theory (e.g. chiral anomaly)	⟷	Impossibility of writing Y as the Floquet evolution of a 1d driving Hamiltonian

E.g. uniform translation:



-there exists a rational quantized `GNVW' index associated to Y

$$\mathrm{ind}(Y) = \frac{p}{q}$$

Gross, Nesme, Vogts, Werner '09

- Y is the Floquet operator of a 1d system if and only if ind(Y)=1.

- $\nu(Y) = \log(\mathrm{ind}(\mathrm{Y}))$ characterizes the chiral flow of quantum information along the edge, and is a quantized invariant distinguishing different Floquet-MBL phases

Example of Chiral Floquet model:

- free fermion `anomalous Floquet-Anderson insulator'

Rudner, Lindner, Berg, Levin '12;
Titum, Berg, Rudner, Refael, Lindner '16

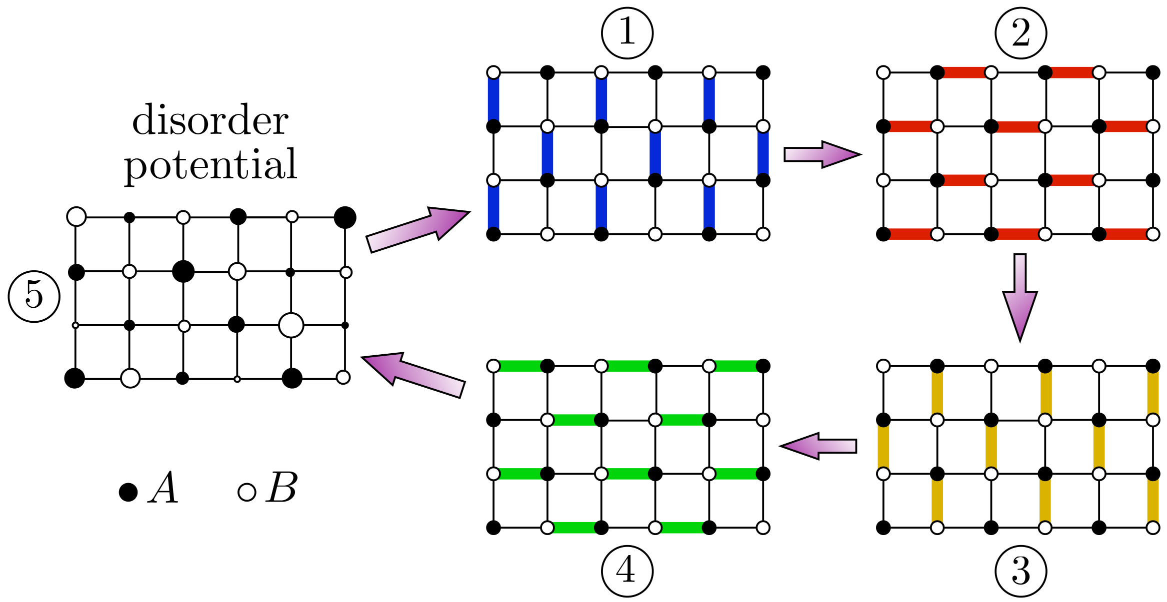


figure from Titum, Berg, Rudner, Refael, Lindner, Phys. Rev. X 6, 021013 (2016)

- after one time step nothing happens in the bulk, but a translation occurs on the edge

- replace fermion sites by bosonic spins (of arbitrary Hilbert space dimension p) and hopping by swap gates => get ind(Y)=p

- stable to interactions and all symmetry breaking in Floquet-MBL setting

Derivation of index

- coarse graining allows one to assume that Y is one-site local

$$Y^\dagger(\mathcal{O}_x \otimes \mathcal{O}_{x+1})Y \subset (\mathcal{O}_{x-1} \otimes \mathcal{O}_x) \otimes (\mathcal{O}_{x+1} \otimes \mathcal{O}_{x+2})$$

- in fact, the algebra $Y^\dagger(\mathcal{O}_x \otimes \mathcal{O}_{x+1})Y$ must be of the form $\mathcal{R}_L \otimes \mathcal{R}_R$

$$\mathcal{R}_L \subset \mathcal{O}_{x-1} \otimes \mathcal{O}_x$$
$$\mathcal{R}_R \subset \mathcal{O}_{x+1} \otimes \mathcal{O}_{x+2}$$

with simple bosonic 'support algebras' $\mathcal{R}_L, \mathcal{R}_R$

- $\mathcal{O}_x = \mathcal{M}_p(\mathbb{C})$, $\mathcal{R}_L = \mathcal{M}_q(\mathbb{C})$, index = p/q

- **fermionic generalization**: Z2 graded simple algebras => sqrt(2) index, corresponding to Majorana translation

Future directions

- fractional models

Po, LF, Vishwanath, Potter, arXiv 1701.01440

- incorporate fermions and symmetries (e.g. e <-> m symmetry in toric code)

LF, Po, Potter, Vishwanath, arXiv 1703.07360

- higher dimensions? connections to quantum cellular automata

- experimental realizations? c.f. discrete Floquet time crystals

von Keyserlingk, Khemani, Sondhi; Else, Nayak; Yao, Potter, Potirniche, Vishwanath

experiment: Monroe group (trapped ions), Lukin group (NV centers)