Random-Singlet phase in two-dimensional disordered SU(2) spin system

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outline

- Introduction

  Deconfine quantum criticality and the JQ model

  Infinite-randomness fixed point and the random-singlet state in 1D

- 2D disordered *J*-*Q* model

  Methods

  Valence-bond glass state

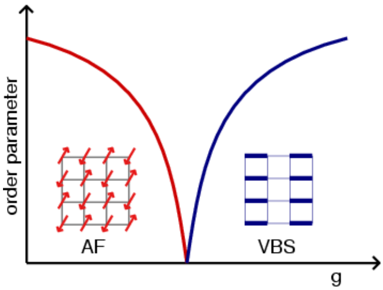
  Transition from Néel to VBG

  Dynamic exponents of the VBG

- Conclusions

Deconfined Quantum Criticality

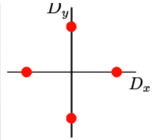
describes the direct continuous transition from Néel to VBS in 2D



- Neither 3D O(3) universality class (Néel-param)
- Nor 3D O(2) universality class (away from VBS).

  (Z_4 anisotropy is dangerously irrelevant

  Okubo et al, PRB 2015; Léonard and Delamotte, PRL 2015)



Néel order parameter

$\mathbf{m}_s = \frac{1}{N}\sum_i (-1)^{x_i+y_i}\mathbf{S}_i$

$\langle \mathbf{S}_i \cdot \mathbf{S}_j \rangle$ breaks lattice symm

VBS order parameter (D_x, D_y)

New physics: Deconfined quantum criticality

Senthil, Vishwanath, Balents, Sachdev, Fisher; Science (2004)

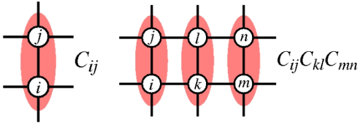
- Order parameters of the Néel state and the VBS state are NOT the fundamental objects, they are composites of fractional quasiparticles carrying $S = 1/2$

Deconfined quantum criticality

2D J-Q_3 model is a Heisenberg model with additional multispin interactions

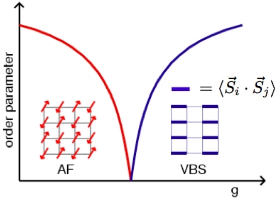
$$H = -J \sum_{\langle ij \rangle} C_{ij} - Q \sum_{\langle ijklmn \rangle} C_{ij} C_{kl} C_{mn},$$

$$C_{ij} = (\frac{1}{4} - \mathbf{S}_i \cdot \mathbf{S}_j)$$



Sandvik, PRL 98, 227202(2007)

- large Q, columnar VBS
- small Q, Néel
- No sign problem
- Ideal for QMC study of the DQC physics
- Scaling violation was resolved recently



Shao, Guo and Sandvik, Science, 352,213(2016)

What about disorder effect to this model?

Introduce randomness to the 2D J-Q_3 model

$$H = -\sum_{\langle i,j \rangle} J_{ij} \boldsymbol{C}_{ij} - \sum_{\langle ijklmn \rangle} Q_{ijklmn} \boldsymbol{C}_{ij} \boldsymbol{C}_{kl} \boldsymbol{C}_{mn}$$

in three ways

- dilute sites with probability P
- Q_{ijklmn} is randomly set to 0 and $2Q$, $J_{ij} = J$ constant $\rightarrow$, random Q model
- $J_{ij} \in [1-\Delta, 1+\Delta], Q_{ijklmn} = Q$ constant $\rightarrow$ random J model

randomness can be relevant

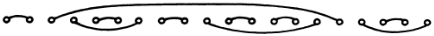
- When randomness is a relevant perturbation under RG, fixed points of GS phases and critical points appear beyond those realised in pure systems

- The randomness can increase without bounds in the RG flow: infinite-randomness fixed point (**IRFP**)
    - dynamic exponent z infinite.
    - Mean and typical correlations are different

An example: 1D Random-singlet phase

Occurs in spin-1/2 Heisenberg chains with random exchange couplings

- Each spin is paired with one other spin that maybe very far away

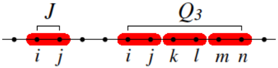


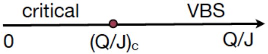
- using SDRG, properties of the Infinite Randomness Fixed Point are found
  - $\xi_t \sim \exp(\xi^{\psi})$; meaning $z \to \infty$
  - mean spin correlation $C(r) \propto 1/r^2$, dominated by rare long VBs
  - But the typical pair $C^{typ}(r) \propto \exp(-cr^{1/2})$

Dasgupta and Ma, PRB 22, 1305(1980); D. Fisher, PRB 50, 3799 (1994)

Random-singlet phase: obtained from random Q chain

Clean J-Q_3 chain dimerizes spontaneously from critical AF at $(Q/J)_c \approx 0.16$





- With randomness, the phase transition destroyed
- Disorder Q ($J = 0$): random singlets form between spinons localized at domain walls, called amorphous VBS, **asymptotically a RS state**

- With random J or random Q, despite the very different local properties, both exhibit RS properties in long-distance correlations

Y.-R. Shu, et al Phys. Rev. B 94, 174442(2016)

What about two-dimensional systems?

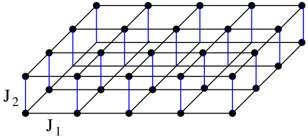
- IRFP was identified in 2D transverse-field Ising models
- However, no Random-singlet state found in 2D

People has been searching RS in various disordered systems

For example, diordered Heisenberg bilayer model

Clean model

$$H = J_1 \sum_{\langle i,j \rangle} (\mathbf{S}_{1i} \cdot \mathbf{S}_{1j} + \mathbf{S}_{2i} \cdot \mathbf{S}_{2j}) + J_2 \sum_i \mathbf{S}_{1i} \cdot \mathbf{S}_{2i}$$



- Néel to singlet product state at $J_2/J_1 \approx 2.522$
- 3D O(3) universality class;
  Landau-Ginzburg-Wilson framework

Disorder models

- Site-diluted randomness: effective interactions form an unfrustrated network which induces AF order in the dimerized phase
  Roscilde and S. Haas, PRL 207206 (2005); Rosclide, PRB 74, 144418(2006)
- Bond-diluted randomness: Mott glass is found, a Griffiths phase
  Ma, Sandvik and Yao, arxiv: 1511.07895
- No infinite-disorder fixed point is observed
  Y.-C. Lin et al PRB 74, 024427(2006)

- To find RS state, one expects frustrated systems
- In this work, we will show an RS state (Quantum spin liquid state) with finite dynamic exponent z found in disordered J-Q model

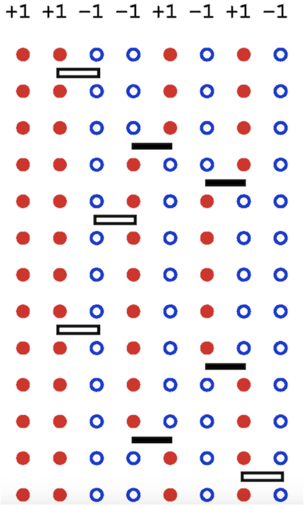
Methods

- SSE Quantum Monte Carlo simulation
  Sandvik, PRB, 59, R14157(1999)
- Projector Quantum Monte Carlo simulation with VB basis
  Sandvik, PRL 2005; Sandvik and Evertz, PRB 2010

SSE Quantum Monte Carlo method: finite temperature

$$\langle A \rangle = \frac{\mathrm{Tr}\{Ae^{-\beta H}\}}{\mathrm{Tr}\, e^{-\beta H}} \to \frac{\sum_c A_c W_c}{\sum_c W_c}$$

- An SSE configuration



- S_z basis
- diagonal and loop updates
- observables and estimators
  - energy estimator : number of operators, $H_c = -n/\beta$
  - spin stiffness estimator : winding number fluctuations

$$\rho_s = \frac{\langle W_\alpha^2 \rangle}{L^{d-2}\beta}$$

  - staggered magnetization $m_{sz} = \sum_i (-1)^{i_x+i_y} s_{iz}/N$
  - uniform suceptibility χ_u and local suseptibility χ_l

Projector Quantum Monte Carlo method: ground state $S = 0$

- Obtain ground state: apply the imaginary time evolution operator to an initial state

$$U(\tau \to \infty)|\Psi_0\rangle \to |0\rangle$$

where $U(\tau) = (-H)^\tau$ or $U(\tau) = \exp(-H\tau)$

- translate average in ground state to classical partition:

$$\boxed{\langle A \rangle = \frac{\langle \Psi_0 | U(\tau) A U(\tau) | \Psi_0 \rangle}{\langle \Psi_0 | U(\tau) U(\tau) | \Psi_0 \rangle} \to \frac{\sum_c A_c W_c}{\sum_c W_c}}$$

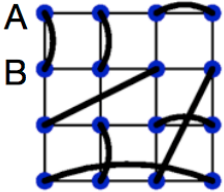
A_c is the estimator of A

Projector Quantum Monte Carlo method: ground state $S = 0$

This is done by

- using VB basis (in the singlet sector $S = 0$)
  - Valence-bonds between sublattice A, B sites $(i, j) = (|\uparrow_i \downarrow_j\rangle - |\downarrow_i \uparrow_j\rangle)/\sqrt{2}$
  - Basis states are products of Valence-bonds

$$|V\rangle = \prod_{b=1}^{N/2} (i_b, j_b) = |(a_1, b_1) \cdots (a_{N/2}, b_{N/2})\rangle$$



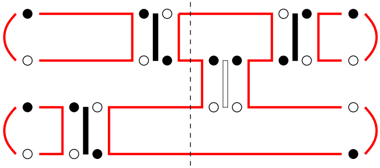
there are $N/2!$ basis states

  - expansion of arbitrary singlet state

$$|\Psi\rangle = \sum_r w_r |V_r\rangle,$$

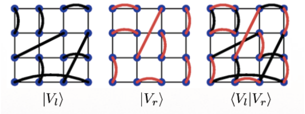
Projector Quantum Monte Carlo method: ground state $S=0$

- take $U(\tau) = \exp(-\tau H)$, in SSE representation; or simply take $U(\tau) = (-H)^{\tau}$, we translate the quantum groundstate expectation to a classical partition $Z = \sum_c W_c$



- loop update algorithm are used

- expectation values: transition graphs



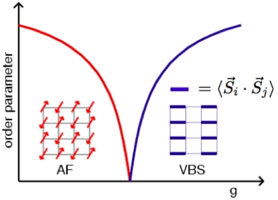
  - Spin correlations from loop structure

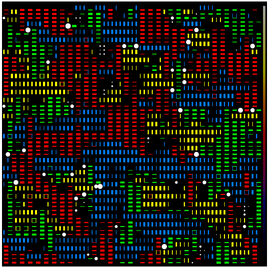
$$\langle \mathbf{S}_i \cdot \mathbf{S}_j \rangle = \begin{cases} 0, & (i)_L (j)_L \\ \frac{3}{4}\phi_{ij}, & (i,j)_L, \end{cases}$$

$\phi_{ij} = \pm 1$, i,j on the same/different sublattice

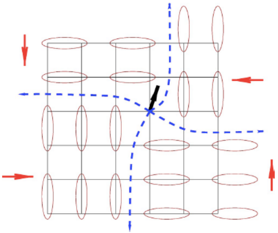
  - dimer correlatoin, Binder cumulant are also related to the loop structure Beach and Sandvik, Nucl. Phys. B 750, 142(2006)

The disordered *J*-*Q* model



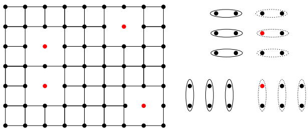


- Imry-Ma argument: VBS can not exist $\rightarrow$ Valence-bond glass (VBG)

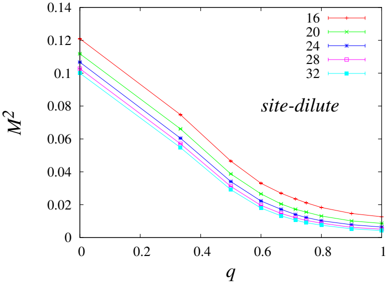


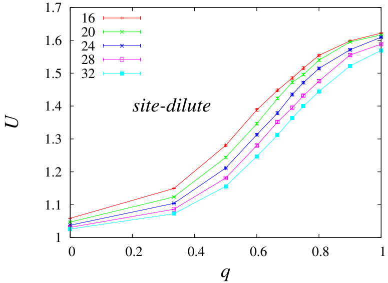
- ▸ different columnar patterns being energetically favored in different parts of the lattice
- ▸ spinon appears at the site domain walls meet
- ▸ But it's not clear if the VBG is paramagnetic!

Site-diluted J-Q_3 model



vacancies: not involved in J and Q interactions



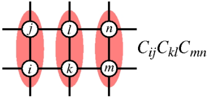


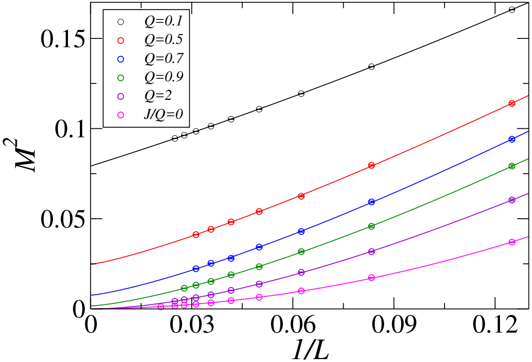
For a given dilution $P = 0.1$ $\quad (q = Q/(J+Q))$

- Néel order persists for whole range of Q/J, DQC destroyed
- Similar to site-diluted Heisenberg bilayer:
  effective couplings between spinons lead to Néel order
- No Random-Singlet state

Random Q model

Q_{ijklmn} **is randomly set to 0 and** $2Q$, $J_{ij} = J$ **constant**



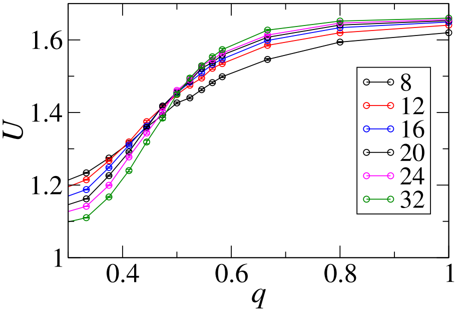


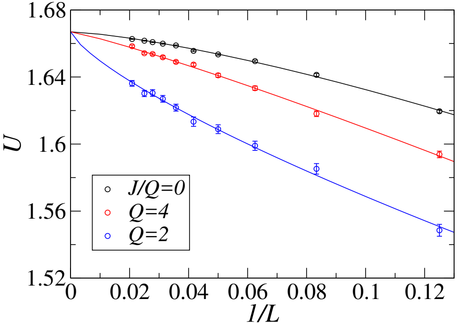
- A phase transition is clear seen when Q is adapted
  $Q \geq 2, M^2(L = \infty) \to 0$

Locate the transition point

Binder ratio

$$U = \frac{[\langle M^4 \rangle]}{[\langle M^2 \rangle]^2}$$

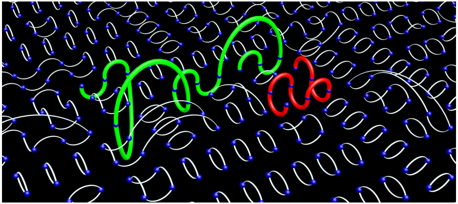




- The transition away from a long-range ordered phase
- Crossing points of the Binder ratio converge to the transition point, but drift a lot.

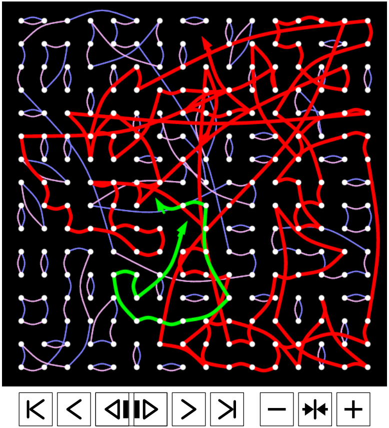
Locate the transition point: spinons

- Spinons play crucial role in DQC
- PQMC by extending valence-bond basis to $S = 1$: put in 2 unpaired ‘up’ spins
  - two spinons are two strings in a background of valence bond loops in the valence-bond transition-graphs

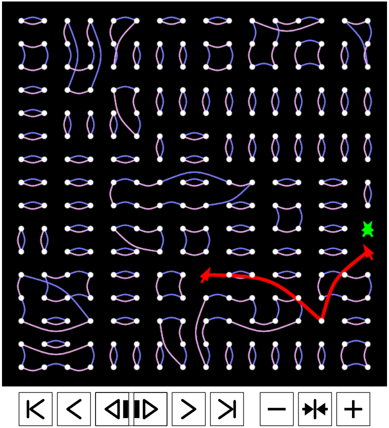


- Spinon size λ: the average number of sites visited by the string

Two movies show λ depending on q



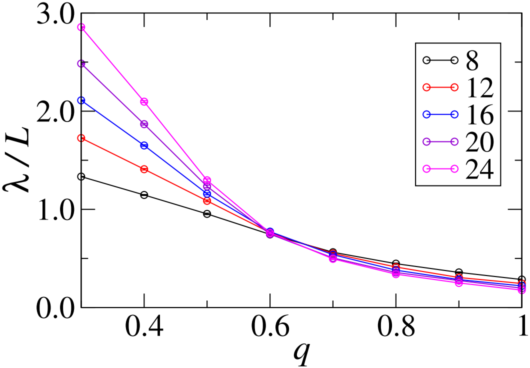
Néel state



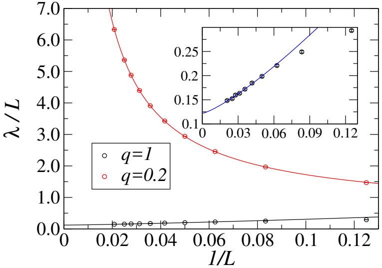
Random singlet state

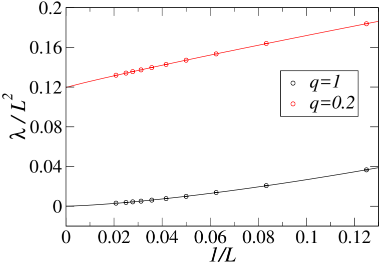
Locate the transition point: spinons

- λ/L cross at transition point

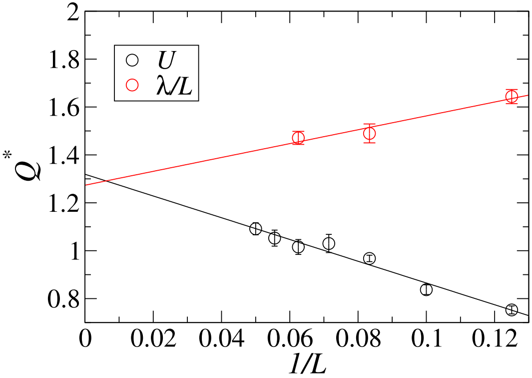


- RS state $\lambda \propto L$; in Néel state $\lambda \propto L^2$





Crossing-points of size pairs$(L, 2L)$ for both U and λ



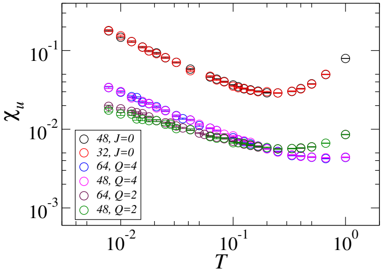
- The behaviors of both quantities appear to be roughly in $1/L$.
- $(Q/J)_c \approx 1.3$

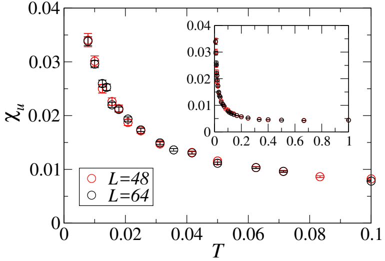
Dynamic exponent z

To investigate the dynamic exponent z of the proposed RS state, we calculate the uniform susceptibility χ_u of the system

- Due to rare long Valenc-bonds, finite-temperature behavior
  - $\chi_u \propto T^{-\alpha}$ with $\alpha = 1 - d/z$
  - χ_u diverges if $z > d$
- We found $z > d$ in the RS phase
  - α changes with Q, with z finite, in agreement with proposed VGB on the kagome lattice

    Singh, PRL 104, 177203(2010)
  - For $Q = 4$, we found $\alpha = 0.62(4)$, which means $z \approx 5.3$



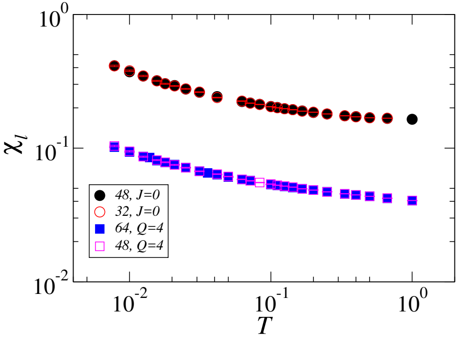


Local susceptibility

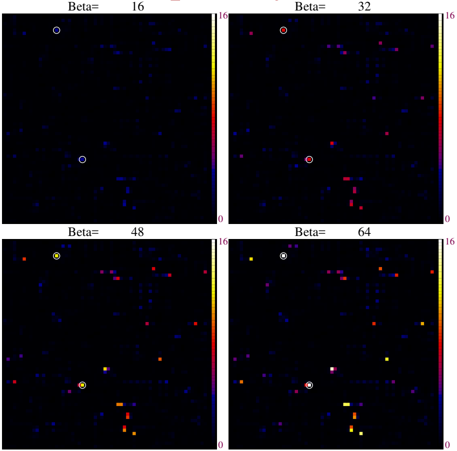
- Such VBG behavior can also be shown with the local susceptibility χ_l

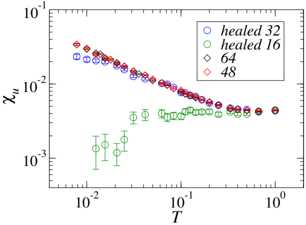
$$\chi_l \propto T^{-\alpha}$$

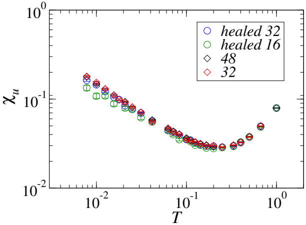
- Similar behavior to χ_u but lower temperature is needed for showing up



local susceptibility distribution of a typical realization



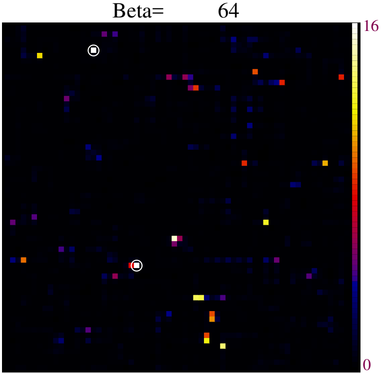


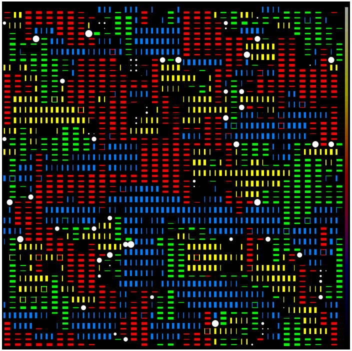


$J/Q = 0$, four temperatures

- There are spins not involved in any Q bond contribute strongly to χ_l
- but Not the main reason of the divergence of χ_u: put randomly a Q bond on the spin to 'heal' it

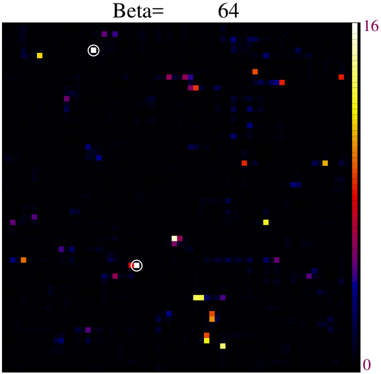
local susceptibility distribution of a typical realization

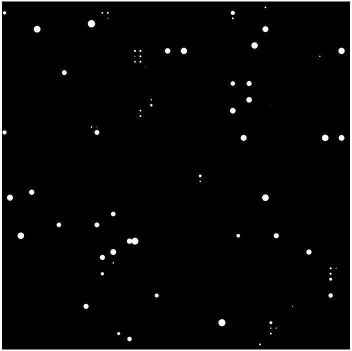




- spinons lead to large χ_l
- bright dots in the rightside are spins with small $\langle S_i \cdot S_j \rangle$ to all 4 neighbours

local susceptibility distribution of a typical realization





- spinons lead to large χ_l
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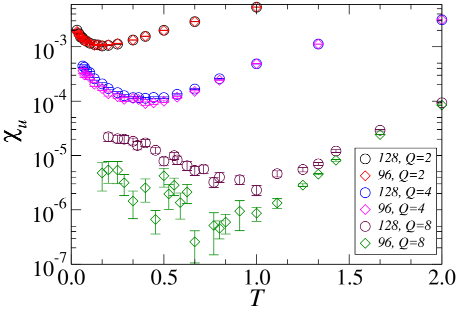
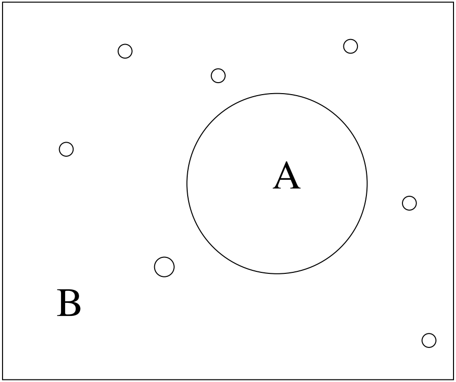
Random J model

$$H = -\sum_{\langle i,j \rangle} J_{ij} \boldsymbol{C}_{ij} - \sum_{\langle ijklmn \rangle} Q_{ijklmn} \boldsymbol{C}_{ij} \boldsymbol{C}_{kl} \boldsymbol{C}_{mn}$$

- $J_{ij} \in [1-\Delta, 1+\Delta] \rightarrow$ random J model
- $Q_{ijklmn} = Q$ constant
- similar, but weaker divergence behaviors



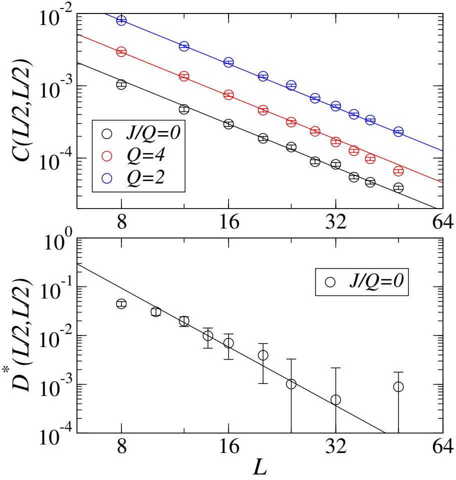
Is the VBG state a Griffiths phase?



- a phase A within a limited part of a system that is overall in a phase B
- suceptibility also diverges as $T^{-\alpha}$ due to rare events of large size A

Is the VBG state a Griffiths phase?

The long distance spin correlation C and staggered dimer correlation D^*



The dimer correlation

$$D_x(\mathbf{r}_{ij}) = \langle B_x(\mathbf{r}_i) B_x(\mathbf{r}_j) \rangle,$$

$$B_x(\mathbf{r}_i) = \mathbf{S}(\mathbf{r}_i) \cdot \mathbf{S}(\mathbf{r}_i + \mathbf{x})$$

The staggered dimer correlation D_x^*

$$D_x^*(\mathbf{r}) = D_x(\mathbf{r}) - \frac{1}{2}[D_x(\mathbf{r} - \mathbf{x}) + D_x(\mathbf{r} + \mathbf{x})]$$

- For large Q/J: $C(L/2, L/2) \propto 1/L^2$
- $D(L/2, L/2) \propto 1/L^4$

- Correlations in Griffiths phases decay exponentially with distance, which is a fundamental consequence of the rare-event mechanism
- Griffiths phase is ruled out

conclusions

- By introducing disorder in the 2D J-Q model we report and characterize a 2D RS state with finite dynamic exponent.
- This is a spin liquid state without frustration
- The JQ model mimicks frustrated quantum spin models, such as the J_1-J_2 Heisenberg model, the RS state found here may correspond to the same fixed point as that investigated in the $S = 1/2$ Heisenberg model on frustrated 2D lattices

Thank you !